DESIGN EXAMPLES
Version 13.0

AMERICAN INSTITUTE
OF
STEEL CONSTRUCTION
INC.
PREFACE

The AISC Design Examples CD provides examples on the application of the 2005 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-05) and the AISC Steel Construction Manual, 13th Edition. The examples found herein illustrate how the Specification and Manual can be used to determine solutions to common engineering problems efficiently, and outline the background to many of the tabulated values found in the Manual.

The design examples on this CD do not represent a stand-alone document. They are intended to be used in conjunction with the Specification, its Commentary, and the Manual.

Part I of these examples is organized to correspond with the organization of the Specification and the Chapters are referred to by their corresponding letter reference from the Specification.

Part II is devoted primarily to connection examples that draw on the tables from the Manual, recommended design procedures, and the breadth of the Specification. The chapters of Part II are labeled II-A, II-B, II-C, etc.

Part III addresses aspects of design that are linked to the performance of a building as a whole. This includes coverage of lateral stability and second order analysis, illustrated through a four-story braced-frame and moment-frame building.

The Design Examples are arranged with LRFD and ASD designs presented side by side, for consistency with the Manual. Design with ASD and LRFD are based on the same nominal strength for each element so that the only difference between the approaches is which set of load combinations from ASCE 7 are used for design and whether the resistance factor for LRFD or the safety factor for ASD should be used.

CONVENTIONS

The following conventions are used throughout these examples:


2. The source of equations or tabulated values taken from the Specification or Manual is noted along the right-hand edge of the page.

3. When the design process differs between LRFD and ASD, the designs equations are presented side-by-side. This rarely occurs, except when the resistance factor, \( \phi \), and the safety factor, \( \Omega \), are applied.

4. The results of design equations are presented to 3 significant figures throughout these calculations.

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APPENDIX A. CROSS-REFERENCE LIST FOR THE 2005 AISC SPECIFICATION
CHAPTER A
GENERAL PROVISIONS

A1. SCOPE

All of the examples on this CD are intended to illustrate the application of the 2005 AISC Specification for Structural Steel Buildings (ANSI/AISC 360-05) and the AISC Steel Construction Manual, 13th Edition in low-and moderate-seismic applications, (i.e. with $R$ equal to or less than 3). For information on design applications involving $R$ greater than 3, the AISC Seismic Design Manual, available at www.aisc.org, should be consulted.

A2. REFERENCED SPECIFICATIONS, CODES AND STANDARDS

Section A2 includes a detailed list of the specifications, codes and standards referenced throughout the Specification.

A3. MATERIAL

Section A3 includes a list of the steel materials that are acceptable for use in accordance with the Specification. The complete ASTM standards for the most commonly used steel materials can be found in Selected ASTM Standards for Structural Steel Fabrication, available at www.aisc.org.
CHAPTER B
DESIGN REQUIREMENTS

B1. GENERAL PROVISIONS

B2. LOADS AND LOAD COMBINATIONS

In the absence of a building code to provide otherwise, the default load combinations to be used with this Specification are taken from ASCE7-02.

B3. DESIGN BASIS

Chapter B of the Specification and Part 2 of the Manual, describe the basis of design, for both LRFD and ASD.

This Section describes three basic types of connections: Simple Connections, Fully Restrained (FR) Moment Connections, and Partially Restrained (PR) Moment Connections. Several examples of the design of each of these types of connection are given in Part II of these design examples.

Information on the application of serviceability and ponding criteria may be found in Specification Chapter L, and its associated commentary. Design examples and other useful information on this topic are given in AISC Design Guide 3, Serviceability Design Consideration for Steel Buildings, Second Edition.

Information on the application of fire design criteria may be found in Specification Appendix 4, and its associated commentary. Design examples and other useful information on this topic are presented AISC Design Guide19, Fire Resistance of Structural Steel Framing.

Corrosion protection and fastener compatibility are discussed in Chapter 2 of the Manual.

B4. CLASSIFICATION OF SECTIONS FOR LOCAL BUCKLING

Specification Table B4.1 gives the complete list of limiting width-thickness ratios for all compression and flexural members defined by the Specification.

Except for one section, the W-shapes presented in the compression member selection tables as column sections meet the criteria as non-slender element sections. The W-shapes presented in the flexural member selection tables as beam sections meet the criteria for compact sections, except for 10 specific shapes. When non-compact or slender element members are tabulated in the design aids, local buckling criteria are accounted for in the tabulated design values.

The shapes listing and other member-design tables in the Manual also include footnoting to highlight sections that exceed local buckling limits in their most commonly available material grades. These footnotes include the following notations:

- Shape is slender in compression
- Shape exceeds compact limit for flexure
- The actual size, combination, and orientation of fastener components should be compared with the geometry of the cross-section to ensure compatibility
- Flange thickness greater than 2 in. Special requirements may apply per AISC Specification Section A3.1c.
- Shape does not meet the $h/t_w$ limit for shear in Specification Section G2.1
CHAPTER C
STABILITY ANALYSIS AND DESIGN

C1. STABILITY DESIGN REQUIREMENTS

The Specification has, for some time, required that design account for both the stability of the structural system as a whole, and the stability of individual elements. Thus, the lateral analysis used to assess stability should include consideration of the combined effect of gravity and lateral loads, including the resulting second-order effects, $P-\Delta$ and $P-\delta$. The effects of “leaning columns” should also be considered, as illustrated in the four-story building design example in Part III of AISC Design Examples.

$P-\Delta$ and $P-\delta$ effects are illustrated in Commentary Figure C-C1.1. Several methods for addressing stability, including $P-\Delta$ and $P-\delta$ effects are discussed in Section C2.

C2. CALCULATION OF REQUIRED STRENGTHS

The calculation of required strengths is illustrated in the four-story building design example in Part III of AISC Design Examples.
CHAPTER D
DESIGN OF MEMBERS FOR TENSION

INTRODUCTION

D1. SLENDERNESS LIMITATIONS

Section D1 does not establish a slenderness limit for tension members, but recommends limiting \( L/r \) to a maximum of 300. This is not an absolute requirement, and rods and hangers are specifically excluded from this recommendation.

D2. TENSILE STRENGTH

Both tensile yield strength and tensile rupture strengths must be considered for the design of tension members. It is not unusual for tensile rupture strength to govern the design of a tension member, particularly for small members with holes or heavier sections with multiple rows of holes.

For preliminary design, tables are provided in Part 5 of the Manual for W-shapes, L-shapes, WT shapes, Rectangular HSS, Square HSS, Round HSS, Pipe and 2L-shapes. The calculations in these tables for available tensile rupture strength assume an effective area, \( A_e \), of 0.75\( A_g \). If the actual effective area is greater than 0.75\( A_g \), the tabulated values will be conservative and manual calculations can be performed to obtain higher available strengths. If the actual effective area is less than 0.75\( A_g \), the tabulated values will be unconservative and manual calculations are necessary to determine the available strength.

D3. AREA DETERMINATION

The gross area, \( A_g \), is the total cross-sectional area of the member.

In computing net area, \( A_n \), an extra \( \frac{1}{16} \) in. is added to the bolt hole diameter and an allowance of \( \frac{1}{16} \) in. is added to the width of slots in HSS gusset connections.

A computation of the effective area for a chain of holes is presented in Example D.9.

Unless all elements of the cross-section are connected, \( A_e = A_n U \), where \( U \) is a reduction factor to account for shear lag. The appropriate values of \( U \) can be obtained from Table D3.1 of the Specification.

D4. BUILT-UP MEMBERS

The limitations for connections of built-up members are discussed in Section D4 of the Specification.

D5. PIN-CONNECTION MEMBERS

An example of a pin-connected member is given in Example D.7.

D6. EYEBARS

An example of an eyebar connection is given in Example D.8.
**Example D.1  W-Shape Tension Member**

**Given:**

Select an 8 in. W-shape, ASTM A992, to carry a dead load of 30 kips and a live load of 90 kips in tension. The member is 25 ft long. Verify the member strength by both LRFD and ASD with the bolted end connection shown. Verify that the member satisfies the recommended slenderness limit.

**Solution:**

Calculate the required tensile strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1.2(30 \text{ kips}) + 1.6(90 \text{ kips})$</td>
<td>$P_a = 30 \text{ kips} + 90 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>180 kips</td>
<td>120 kips</td>
</tr>
</tbody>
</table>

Try a W8×21

**Material Properties:**

<table>
<thead>
<tr>
<th>W8×21</th>
<th>ASTM A992</th>
<th>$F_y = 50 \text{ ksi}$</th>
<th>$F_u = 65 \text{ ksi}$</th>
</tr>
</thead>
</table>

**Geometric Properties:**

<table>
<thead>
<tr>
<th>W8×21</th>
<th>$A_g = 6.16 \text{ in.}^2$</th>
<th>$b_f = 5.27 \text{ in.}$</th>
<th>$t_f = 0.400 \text{ in.}$</th>
<th>$d = 8.28 \text{ in.}$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$r_f = 1.26 \text{ in.}$</td>
<td>$\bar{y} = 0.831 \text{ in.}$ (for WT4×10.5)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Check tensile yield limit state using tabulated values**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>277 kips</td>
<td>&gt; 180 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Check the available tensile rupture strength at the end connection**

*Verify the table assumption that $A_y / A_g \geq 0.75$ for this connection*
Calculate $U$ as the larger of the values from Table D3.1 case 2 or case 7

**Case 2 – Check as 2 WT-shapes**

$$U = 1 - \frac{\frac{x}{l}}{T} = 1 - \frac{0.831 \text{ in.}}{9.00 \text{ in.}} = 0.908$$

**Case 7**

$b_f = 5.27 \text{ in.} \quad d = 8.28 \text{ in.} \quad b_f < 2/3d$

$$U = 0.85$$

*Use $U = 0.908$*

**Calculate $A_n$**

$$A_n = A_g - 4(d_h + \frac{\sqrt{3}}{2} \text{ in.})t_f$$

$$= 6.16 \text{ in.}^2 - 4(\frac{\sqrt{3}}{2} \text{ in.} + \frac{\sqrt{3}}{2} \text{ in.})(0.400 \text{ in.}) = 4.76 \text{ in.}^2$$

**Calculate $A_e$**

$$A_e = A_n U = 4.76 \text{ in.}^2(0.908) = 4.32 \text{ in.}^2$$

$$A_e / A_g = 4.32 \text{ in.}^2 / 6.16 \text{ in.}^2 = 0.701 < 0.75 \text{ tabulated values for rupture n.a.}$$

$$P_n = F_u A_e = (65 \text{ ksi})(4.32 \text{ in.}^2) = 281 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$\phi_t P_n = 0.75(281 \text{ kips}) = 211 \text{ kips}$</td>
<td>$P_u / \Omega_t = (281 \text{ kips})/2.00 = 141 \text{ kips}$</td>
</tr>
<tr>
<td>211 kips &gt; 180 kips <strong>o.k.</strong></td>
<td>141 kips &gt; 120 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Check the non-mandatory slenderness limit**

$$L / r = \left(\frac{25.0 \text{ ft}}{12.0 \text{ in.}}\right) \left(\frac{12.0 \text{ in.}}{1.26 \text{ in.}}\right) = 238 < 300 \text{ o.k.}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>225 kips &gt; 180 kips <strong>o.k.</strong></td>
<td>150 kips &gt; 120 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

The W8×21 available tensile strength is governed by the tensile rupture limit state at the end connection.

**Commentary**

Fig. C-D3.1

Table D3.1

Section D3.2

Eqn. D3-1

Section D3.3

Eqn. D2-2

Section D2

Table D1

Manual

Table 5-1
Example D.2  Single-Angle Tension Member

Given:

Verify, by both ASD and LRFD, the strength of an L4×4×½, ASTM A36, with one line of (4) ¾ in. diameter bolts in standard holes. The member carries a dead load of 20 kips and a live load of 60 kips in tension. Calculate at what length this tension member would cease to satisfy the recommended slenderness limit.

Solution:

Material Properties:
L4×4×½  ASTM A36  \( F_y = 36 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  

Geometric Properties:
L4×4×½  \( A_g = 3.75 \text{ in.}^2 \)  \( r_z = 0.776 \text{ in.} \)  \( \gamma = 1.18 \text{ in.} = \overline{\gamma} \)  

Calculate the required tensile strength

\[
P_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) = 120 \text{ kips} \]

\[
P_a = 20 \text{ kips} + 60 \text{ kips} = 80.0 \text{ kips} \]

Calculate the available tensile yield strength

\[
P_n = F_y A_g = (36 \text{ksi})(3.75\text{in.}^2) = 135 \text{kips} \]

\[
\phi_t P_n = 0.90(135 \text{kips}) = 122 \text{kips} \]

\[
\Omega_t = 1.67 \frac{P_n}{\Omega_t} = 135 \text{kips}/1.67 = 80.8 \text{kips} \]

Calculate the available tensile rupture strength

Calculate \( U \) as the larger of the values from Table D3.1 case 2 or case 8

Case 2

\[
U = 1 - \frac{\overline{\gamma}}{l} = 1 - \frac{1.18 \text{ in.}}{9.00 \text{ in.}} = 0.869 \]

Case 8 with 4 or more fasteners per line in the direction of loading

\[
U = 0.80 \]

Use \( U = 0.869 \)
Calculate $A_n$

$$A_n = A_g - (d_h' + \frac{z}{16})t$$

$$= 3.75 \text{ in.}^2 - (1\frac{1}{16}\text{ in.} + \frac{z}{16}\text{ in.})(\frac{1}{2}\text{ in.}) = 3.31 \text{ in.}^2$$

Calculate $A_e$

$$A_e = A_n U = 3.31 \text{ in.}^2 (0.869) = 2.88 \text{ in.}^2$$

$$P_n = F_u A_e = (58 \text{ ksi})(2.88 \text{ in.}^2) = 167 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td>$\Omega_t = 2.00$</td>
</tr>
<tr>
<td>$\phi_t P_n = 0.75(167 \text{ kips}) = 125 \text{ kips}$</td>
<td>$P_n/\Omega_t = (167 \text{ kips})/2.00 = 83.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

The L4×4×1/2 available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t P_n = 125 \text{ kips}$</td>
<td>$P_n/\Omega_t = 83.5 \text{ kips}$</td>
</tr>
<tr>
<td>125 kips &gt; 120 kips</td>
<td>83.5 kips &gt; 80.0 kips</td>
</tr>
</tbody>
</table>

Calculate recommended $L_{max}$

$$L_{max} = 300r_z = (300)(0.776 \text{ in.}) \left(\frac{\text{ft}}{12.0 \text{ in.}}\right) = 19.4 \text{ ft}$$

Note: The $L/r$ limit is a recommendation, not a requirement.
Example D.3 WT-Shape Tension Member

Given:

A WT6×20, ASTM A992, member has a length of 30 ft and carries a dead load of 40 kips and a live load of 120 kips in tension. Assume the end connection is fillet welded and has a length of 16 in. Verify the member strength by both LRFD and ASD. Assume that the gusset plate and the weld have been checked and are satisfactory.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>WT6×20</th>
<th>ASTM A992</th>
<th>$F_y = 50$ ksi</th>
<th>$F_u = 65$ ksi</th>
</tr>
</thead>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>WT6×20</th>
<th>$A_g = 5.84$ in.$^2$</th>
<th>$r_x = 1.57$ in.</th>
<th>$\bar{y} = 1.09$ in. = $\bar{x}$ (in equation for $U$)</th>
</tr>
</thead>
</table>

Manual Table 2-3

Manual Table 1-8

Calculate the required tensile strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a = 1.2(40.0 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips}$</td>
<td>$P_a = 40.0 \text{ kips} + 120 \text{ kips} = 160 \text{ kips}$</td>
</tr>
</tbody>
</table>

Check tensile yielding limit state using tabulated values

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_a = 263 \text{ kips} &gt; 240 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$P_a/\Omega_t = 175 \text{ kips} &gt; 160 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Check tensile rupture limit state using tabulated values

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>214 kips &lt; 240 kips</td>
<td>n.g.</td>
</tr>
<tr>
<td>142 kips &lt; 160 kips</td>
<td>n.g.</td>
</tr>
</tbody>
</table>

The tabulated available rupture strengths may be conservative for this case, therefore calculate the exact solution.

Calculate $U$

\[
U = 1 - \frac{\bar{y}}{l} = 1 - \frac{1.09 \text{ in.}}{16.0 \text{ in.}} = 0.932
\]

Table D3.1 Case 2
Calculate $A_n$

$A_n = A_g = 5.84 \text{ in}^2$ (because there are no holes)

Calculate $A_e$

$A_e = A_n U$
$= 5.84 \text{ in}^2 (0.932) = 5.44 \text{ in}^2$

Calculate $P_n$

$P_n = F_u A_e$
$= 65 \text{ ksi} (5.44 \text{ in}^2) = 354 \text{ kips}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi t = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi t P_n = 0.75 (354 \text{ kips}) = 266 \text{ kips}$</td>
<td>$P_n/\Omega = (354 \text{ kips})/2.00 = 177 \text{ kips}$</td>
</tr>
</tbody>
</table>

266 kips $>$ 240 kips \text{ o.k.}  
177 kips $>$ 160 kips \text{ o.k.}

Alternately, the available tensile rupture strengths can be determined by modifying the tabulated values. The available tensile rupture strengths published in the tension member selection tables are based on the assumption that $A_e = 0.75A_g$. The actual available strengths can be determined by adjusting the table values as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi t P_n = 214 \text{ kips}$</td>
<td>$P_n/\Omega = 142 \text{ kips}$</td>
</tr>
<tr>
<td>$= 214 \text{ kips} \left( \frac{5.44 \text{ in}^2}{0.75 (5.84 \text{ in}^2)} \right) = 266 \text{ kips}$</td>
<td>$= 142 \text{ kips} \left( \frac{5.44 \text{ in}^2}{0.75 (5.84 \text{ in}^2)} \right) = 176 \text{ kips}$</td>
</tr>
</tbody>
</table>

The WT6×20 available tensile strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi t P_n = 263 \text{ kips}$</td>
<td>$P_n/\Omega = 175 \text{ kips}$</td>
</tr>
<tr>
<td>263 kips $&gt;$ 240 kips \text{ o.k.}</td>
<td>175 kips $&gt;$ 160 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>

**Check the non-mandatory slenderness limit**

$$L/\rho = \left( \frac{30.0 \text{ ft}}{1.57 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right) = 229 < 300 \text{ o.k.}$$
Example D.4  Rectangular HSS Tension Member

Given:
Verify, by LRFD and ASD, the strength of an HSS 6×4×3/8, ASTM A500 grade B, with a length of 30 ft. The member is carrying a dead load of 35 kips and a live load of 105 kips in tension. Assume the end connection is fillet welded to a 1/2 in. thick single concentric gusset plate and has a length of 16 in.

Solution:

Material Properties:
HSS 6×4×3/8  ASTM A500 grade B  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  \( \) Manual Table 2-3

Member Geometric Properties:
HSS 6×4×3/8  \( A_g = 6.18 \text{ in.}^2 \)  \( r_y = 1.55 \text{ in.} \)  \( t = 0.349 \text{ in.} \)  \( \) Manual Table 1-11

Calculate the required tensile strength
\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
P_a = 1.2(35.0 \text{ kips}) + 1.6(105 \text{ kips}) & P_a = 35.0 \text{ kips} + 105 \text{ kips} \\
= 210 \text{ kips} & = 140 \text{ kips} \\
\end{array}
\]

Check available tensile yield strength
\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi P_a = 256 \text{ kips} > 210 \text{ kips} & P_a/\Omega = 170 \text{ kips} > 140 \text{ kips} \\
\text{o.k.} & \text{o.k.} \\
\end{array}
\]

Check available tensile rupture strength
\[
\begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
201 \text{ kips} < 210 \text{ kips} & 134 \text{ kips} < 140 \text{ kips} \\
\text{n.g.} & \text{n.g.} \\
\end{array}
\]

The tabulated available rupture strengths may be conservative in this case, therefore calculate the exact solution.

Calculate \( U \)
\[
\bar{x} = \frac{B^2 + 2BH}{4(B + H)} = \frac{(4.00 \text{ in.})^2 + 2(4.00 \text{ in.})(6.00 \text{ in.})}{4(4.00 \text{ in.} + 6.00 \text{ in.})} = 1.60 \text{ in.} \\
U = 1 - \frac{\bar{x}}{l} = 1 - \frac{1.60 \text{ in.}}{16.0 \text{ in.}} = 0.900
\]

Manual Table D3.1 Case 6
Allowing for a \( \frac{1}{16} \) in. gap in fit-up between the HSS and the gusset plate, 

\[
A_e = A_n - 2(t_p + \frac{1}{16} \text{ in.})t \\
= 6.18 \text{ in.}^2 - 2(\frac{1}{2} \text{ in.} + \frac{1}{16} \text{ in.})(0.349 \text{ in.}) = 5.79 \text{ in.}^2
\]

**Calculate** \( A_e \)  

\[
A_e = A_n U \\
= 5.79 \text{ in.}^2(0.900) = 5.21 \text{ in.}^2
\]

**Calculate** \( P_n \)  

\[
P_n = F_u A_e \\
= 58 \text{ ksi}(5.21 \text{ in.}^2) = 302 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_f = 0.75 )</td>
<td>( \Omega_f = 2.00 )</td>
<td>D2</td>
</tr>
<tr>
<td>( \phi_P = 0.75(302 \text{ kips}) = 227 \text{ kips} )</td>
<td>( P_e/\Omega_f = 302 \text{ kips}/2.00 = 151 \text{ kips} )</td>
<td>D1</td>
</tr>
</tbody>
</table>

227 kips > 210 kips \quad o.k. \quad 151 kips > 140 kips \quad o.k.

The HSS available tensile strength is governed by the tensile rupture limit state.

**Check the non-mandatory slenderness limit**  

\[
L/r = \left( \frac{30.0 \text{ ft}}{1.55 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right) = 232 < 300 \quad o.k.
\]
Example D.5  Round HSS Tension Member

Given:

See Figure D-5 below. An HSS6.000×0.500, ASTM A500 grade B, has a length of 30 ft. The member carries a dead load of 40 kips and a live load of 120 kips in tension. Assume the end connection is a fillet welded ½ in. thick single concentric gusset plate that has a length of 16 in. Verify the strength by both LRFD and ASD.

Solution:

Material Properties:

HSS6.000×0.500  ASTM A500 grade B  \( F_y = 42 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  Manual

Table 2-3

Member Geometric Properties:

HSS6.000×0.500  \( A_g = 8.09 \text{ in.}^2 \)  \( r = 1.96 \text{ in.} \)  \( t = 0.465 \text{ in.} \)  Manual

Table 1-13

Calculate the required tensile strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(40.0 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips} )</td>
<td>( P_u = 40.0 \text{ kips} + 120 \text{ kips} = 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check available tensile yield strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_u = 306 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>( P_u/\Omega_t = 203 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check available tensile rupture strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_u = 264 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>( P_u/\Omega_t = 176 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check that \( A_g \geq 0.75A_e \) as assumed in table

\( L = 16.0 \text{ in.} \)  \( D = 6.00 \text{ in.} \)  \( L/D = 16.0 \text{ in.}/(6.00 \text{ in.}) = 2.67 > 1.3 \)  Manual

Table D3.1  Case 5

Allowing for a \( \frac{1}{16} \text{ in.} \) gap in fit-up between the HSS and the gusset plate,

\[
A_n = A_g - 2(t_p + \frac{1}{16} \text{ in.})t
\]
\[
= 8.09 \text{ in.}^2 - 2(0.500 \text{ in.} + \frac{1}{16} \text{ in.})(0.465 \text{ in.}) = 7.57 \text{ in.}^2
\]
Calculate $A_e$

\[ A_e = A_u U \]
\[ = 7.57 \text{ in.}^2 \times 1.0 = 7.57 \text{ in.}^2 \]

\[ A_e / A_g = 7.57 \text{ in.}^2 / (8.09 \text{ in.}^2) = 0.936 \text{ in.}^2 > 0.75 \text{ A}_g \quad \text{o.k.} \]

Check the non-mandatory slenderness limit

\[ L / r = \left( \frac{30.0 \text{ ft}}{1.96 \text{ in.}} \right) \left( \frac{12.0 \text{ in.}}{\text{ft}} \right) = 184 < 300 \quad \text{o.k.} \]
Example D.6  Double-Angle Tension Member

Given:

A 2L4×4×1/2 (¾-in. separation), ASTM A36, has one line of (8) ¾-in. diameter bolts in standard holes and is 25 ft in length. The double angle is carrying a dead load of 40 kips and a live load of 120 kips in tension. Verify the strength by both LRFD and ASD.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>2L4×4×1/2</th>
<th>ASTM A36</th>
<th>$F_y = 36$ ksi</th>
<th>$F_u = 58$ ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Manual Table 2-3</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

For a single L4×4×1/2

- $A_g = 3.75$ in.$^2$
- $r_y = 1.83$ in.
- $x = 1.18$ in.

Manual Table 1-7

Calculate the required tensile strength

LRFD ASD

<table>
<thead>
<tr>
<th>$P_n = 1.2(40.0 \text{ kips}) + 1.6(120 \text{ kips})$</th>
<th>$P_n = 40.0 \text{ kips} + 120 \text{ kips}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>= 240 kips</td>
<td>= 160 kips</td>
</tr>
</tbody>
</table>

Calculate the available tensile yield strength

$$P_n = F_y A_g = (36 \text{ksi})(2)(3.75\text{in.}^2) = 270 \text{ kips}$$

Eqn. D2-1

LRFD ASD

<table>
<thead>
<tr>
<th>$\phi t = 0.90$</th>
<th>$\omega_t = 1.67$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_n = 0.90(270 \text{ kips}) = 243 \text{ kips}$</td>
<td>$P_n/\omega_t = (270 \text{ kips})/1.67 = 162 \text{ kips}$</td>
</tr>
</tbody>
</table>

Section D2

Calculate the available tensile rupture strength

Calculate $U$

$$U = 1 - \frac{x}{l} = 1 - \frac{1.18 \text{ in.}}{21.0 \text{ in.}} = 0.944$$

Table D3-1 Case 2
Calculate $A_n$

$$A_n = A_g - 2(d_h + \frac{1}{16} \text{ in.})t$$

$$= 2(3.75 \text{ in.}^2) - 2(\frac{1}{16} \text{ in.} + \frac{1}{16} \text{ in.})(\frac{1}{2} \text{ in.}) = 6.63 \text{ in.}^2$$

Calculate $A_e$

$$A_e = A_n U = 6.63 \text{in.}^2 (0.944) = 6.26 \text{ in.}^2$$

$$P_n = F_u A_e = (58 \text{ksi})(6.26 \text{ in.}^2) = 363 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi_t P_n = 0.75(363 \text{ kips}) = 272 \text{ kips}$</td>
<td>$P_n/\Omega = (363 \text{ kips})/2.00 = 182 \text{ kips}$</td>
</tr>
</tbody>
</table>

The available strength is governed by the tensile yielding limit state.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>243 kips &gt; 240 kips</td>
<td>162 kips &gt; 160 kips</td>
</tr>
</tbody>
</table>

Check the non-mandatory slenderness limit

$$L/r = \left(\frac{25.0 \text{ ft}}{1.21 \text{ in./ft}}\right) = 248 < 300 \quad \text{ o.k.}$$
Example D.7  Pin-Connected Tension Member

Given:

An ASTM A36 pin connected tension member with the dimensions shown below carries a dead load of 12 kips and a live load of 4 kips in tension. The diameter of the pin is 1 inch, in a \( \frac{3}{8} \)-in. oversized hole. Assume that the pin itself is adequate. Verify the strength by both LRFD and ASD.

![Image of a pin-connected tension member]

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Plate</th>
<th>ASTM A36</th>
<th>( F_y = 36 \text{ ksi} )</th>
<th>( F_u = 58 \text{ ksi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Manual</td>
<td>Manual</td>
<td>Table 2-4</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

\( w = 4.25 \text{ in.} \)  \( t = 0.500 \text{ in.} \)  \( d = 1.00 \text{ in.} \)  \( a = 2.25 \text{ in.} \)  \( c = 2.50 \text{ in.} \)

Check dimensional requirements:

1) \( b_{eff} = 2t + 0.63 \text{ in.} = 2(0.500 \text{ in.}) + 0.63 \text{ in.} = 1.63 \text{ in.} \)

2) \( a \geq 1.33b_{eff} = 2.25 \text{ in.} \geq (1.33)(1.63 \text{ in.}) = 2.17 \text{ in.} \)  \text{o.k.}

3) \( w \geq 2b_{eff} + d = 4.25 \text{ in.} \geq 2(1.63 \text{ in.}) + 1.00 \text{ in.} = 4.26 \text{ in.} \geq 4.25 \text{ in.} \)  \text{o.k.}

4) \( c \geq a = 2.50 \text{ in.} \geq 2.25 \text{ in.} \)  \text{o.k.}

Calculate the required tensile strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(12.0 \text{ kips}) + 1.6(4.00 \text{ kips}) = 20.8 \text{ kips} )</td>
<td>( P_u = 12.0 \text{ kips} + 4.00 \text{ kips} = 16.0 \text{ kips} )</td>
<td></td>
</tr>
</tbody>
</table>
Calculate the available tensile rupture strength on the net effective area

\[ P_n = 2t_{beff}F_u = (2)(0.500 \text{ in.})(1.63 \text{ in.})(58 \text{ ksi}) = 94.5 \text{ kips} \]  \hspace{1cm} \text{Eqn. D5-1}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.75 )</td>
<td>( \Omega_t = 2.00 )</td>
</tr>
<tr>
<td>( \phi_t P_n = 0.75(94.5 \text{ kips}) = 70.9 \text{ kips} )</td>
<td>( P_n/\Omega_t = (94.5 \text{ kips}) / 2.00 = 47.3 \text{ kips} )</td>
</tr>
</tbody>
</table>

Calculate the available shear rupture strength

\[ A_y = 2(a + d/2) = 2(0.500 \text{ in.})(2.25 \text{ in.} + (1.00 \text{ in.} / 2)) = 2.75 \text{ in.}^2 \]  \hspace{1cm} \text{Section D5.1}

\[ P_n = 0.6F_uA_y = (0.6)(58 \text{ ksi})(2.75 \text{ in.}^2) = 95.7 \text{ kips} \]  \hspace{1cm} \text{Eqn. D5-2}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_y = 0.75 )</td>
<td>( \Omega_y = 2.00 )</td>
</tr>
<tr>
<td>( \phi_y P_n = 0.75(95.7 \text{ kips}) = 71.8 \text{ kips} )</td>
<td>( P_n/\Omega_y = (95.7 \text{ kips}) / 2.00 = 47.9 \text{ kips} )</td>
</tr>
</tbody>
</table>

Calculate the available bearing strength

\[ A_{pb} = 0.500 \text{ in.}(1.00 \text{ in.}) = 0.500 \text{ in.}^2 \]

\[ R_n = 1.8F_yA_{pb} = 1.8(36 \text{ ksi})(0.500 \text{ in.}^2) = 32.4 \text{ kips} \]  \hspace{1cm} \text{Eqn. J7.1}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.75 )</td>
<td>( \Omega_t = 2.00 )</td>
</tr>
<tr>
<td>( \phi_t P_n = 0.75(32.4 \text{ kips}) = 24.3 \text{ kips} )</td>
<td>( P_n/\Omega_t = (32.4 \text{ kips}) / 2.00 = 16.2 \text{ kips} )</td>
</tr>
</tbody>
</table>

Calculate the available tensile yielding strength

\[ A_g = 4.25 \text{ in.} \times (0.500 \text{ in.}) = 2.13 \text{ in.}^2 \]  \hspace{1cm} \text{Section D2}

\[ P_n = F_yA_g = 36 \text{ ksi}(2.13 \text{ in.}^2) = 76.5 \text{ kips} \]  \hspace{1cm} \text{Eqn. D2.1}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_t = 0.90 )</td>
<td>( \Omega_t = 1.67 )</td>
</tr>
<tr>
<td>( \phi_t P_n = 0.90(76.5 \text{ kips}) = 68.9 \text{ kips} )</td>
<td>( P_n/\Omega_t = (76.5 \text{ kips}) / 1.67 = 45.8 \text{ kips} )</td>
</tr>
</tbody>
</table>

The available tensile strength is governed by the bearing strength limit state

\[ P_n/\Omega_t = 16.2 \text{ kips} > 16.0 \text{ kips} \]  \hspace{1cm} \text{O.K.}

\[ P_n/\Omega_t = 16.2 \text{ kips} > 16.0 \text{ kips} \]  \hspace{1cm} \text{O.K.}
Example D.8 Eyebar Tension Member

Given:

See Figure D-8 below. A ¾ in. thick eyebar member, ASTM A36, carries a dead load of 25 kips and a live load of 15 kips in tension. The pin diameter \(d\) is 3 in. Verify the strength by both LRFD and ASD.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Material</th>
<th>(F_y)</th>
<th>(F_u)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

\[
\begin{align*}
 w &= 3 \text{ in.} & b &= 2.23 \text{ in.} & t &= 0.625 \text{ in.} \\
 d_b &= 3 \text{ in.} & d_h &= 3.03 \text{ in.} & R &= 8.00 \text{ in.}
\end{align*}
\]

Check dimensional requirements

\[
\begin{align*}
1) & \quad t \geq \frac{1}{2} \text{ in.} \quad \text{0.625 in.} \geq 0.500 \text{ in.} \quad \text{o.k.} \\
2) & \quad w \leq 8t \quad \text{3.00 in.} \leq 8(0.625 \text{ in.}) = 5 \text{ in.} \quad \text{o.k.} \\
3) & \quad d \geq \frac{7}{8}w \quad \text{3.00 in.} \geq \frac{7}{8}(3.00 \text{ in.}) = 2.63 \text{ in.} \quad \text{o.k.} \\
4) & \quad d_h \leq d + \frac{1}{32} \text{ in.} \quad \text{3.03 in.} \leq 3.00 \text{ in.} + (\frac{1}{32} \text{ in.}) = 3.03 \text{ in.} \quad \text{o.k.} \\
5) & \quad R \geq d_h + 2b \quad \text{8.00 in.} \geq 3.03 \text{ in.} + 2(2.23 \text{ in.}) = 7.50 \text{ in.} \quad \text{o.k.} \\
6) & \quad \frac{2}{3}w \leq b \leq \frac{3}{4}w \quad \frac{2}{3}(3.00 \text{ in.}) \leq 2.23 \text{ in.} \leq \frac{3}{4}(3.00 \text{ in.}) \quad \text{2.00 in.} \leq 2.23 \text{ in.} \leq 2.25 \text{ in.} \quad \text{o.k.}
\end{align*}
\]
Calculate the required tensile strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(25.0 \text{ kips}) + 1.6(15.0 \text{ kips})$</td>
<td>$P_a = 25.0 \text{ kips} + 15.0 \text{ kips}$</td>
</tr>
<tr>
<td>$= 54.0 \text{ kips}$</td>
<td>$= 40.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

Calculate the available tensile yield strength at the eyebar body (at w)

$A_g = 3.00 \text{ in.} \times (0.625 \text{ in.}) = 1.88 \text{ in.}^2$

$P_a = F_y A_g = (36 \text{ ksi})(1.88 \text{ in.}^2) = 67.7 \text{ kips}$

Eqn. D2-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_t = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi_t P_a = 0.90(67.7 \text{ kips}) = 60.9 \text{ kips}$</td>
<td>$P_a/\Omega = (67.7 \text{ kips})/1.67 = 40.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

60.9 kips > 54.0 kips \textbf{o.k.} \quad 40.3 kips > 40.0 kips \textbf{o.k.}

Section D2

The eyebar tension member available strength is governed by the tension yield limit state.

Note: The eyebar detailing limitations ensure that the tensile yielding limit state at the eyebar body will control the strength of the eyebar itself. The pin should also be checked for shear yielding, and if the material strength is lower than that of the eyebar, bearing.
Example D.9  Find \( A_e \) of a Plate with Staggered Bolts

**Given:**

See Fig. D-9 below. A 14 in. wide and \( \frac{1}{2} \) in. thick plate subject to tensile loading has staggered holes as shown. Compute \( A_n \) and \( A_e \)

**Solution:**

*Calculate net hole diameter*

\[
d_{net} = d_h + \frac{1}{16} \text{ in.} = 0.875 \text{ in.}
\]

*Compute the net width for all possible paths across the plate*

Because of symmetry, many of the net widths are identical and need not be calculated

\[
w = 14.0 \cdot \sum d_{net} + \sum \frac{s^2}{4g}
\]

Line A-B-E-F: 
\[
w = 14.0 \text{ in.} \cdot 2(0.875 \text{ in.}) = 12.3 \text{ in.}
\]

Line A-B-C-D-E-F: 
\[
w = 14.0 \text{ in.} \cdot 4(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} = 11.5 \text{ in.} \quad \text{(controls)}
\]

Line A-B-C-D-G: 
\[
w = 14.0 \text{ in.} \cdot 3(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} = 11.9 \text{ in.}
\]

Line A-B-D-E-F: 
\[
w = 14.0 \text{ in.} \cdot 3(0.875 \text{ in.}) + \frac{(2.50 \text{ in.})^2}{4(7.00 \text{ in.})} + \frac{(2.50 \text{ in.})^2}{4(3.00 \text{ in.})} = 12.1 \text{ in.}
\]

Therefore, \( A_n = (11.5 \text{ in.})(0.500 \text{ in.}) = 5.75 \text{ in.}^2 \)

*Calculate \( U \)*

Since tension load is transmitted to all elements by the fasteners

\[U = 1.0\]

\[A_e = A_n(1.0) = 5.75 \text{ in.}^2\]
CHAPTER E
DESIGN OF MEMBERS FOR COMPRESSION

This chapter covers the design of compression members, the most common of which are columns. The Manual includes design tables for the following compression member types in their most commonly available grades:

- wide-flange column shapes
- HSS
- double angles
- single angles

LRFD and ASD information is presented side by side for quick selection, design or verification. All of the tables account for the reduced strength of sections with slender elements.

The design and selection method for both LRFD and ASD designs is similar to that of previous specifications, and will provide similar designs. In the new Specification, ASD and LRFD will provide identical designs when the live load is approximately three times the dead load.

The design of built-up shapes with slender elements can be tedious and time consuming, and it is recommended that standard rolled shapes be used, when possible.

E1. GENERAL PROVISIONS

The design compressive strength, \( \phi_cP_n \), and the allowable compressive strength, \( P_n/\Omega_c \), are determined as follows:

\[
P_n = \text{nominal compressive strength based on the controlling buckling mode}
\]

\[
\phi_c = 0.90 \quad (\text{LRFD})
\]

\[
\Omega_c = 1.67 \quad (\text{ASD}).
\]

Because \( F_{cr} \) is used extensively in calculations for compression members, it has been tabulated in Table 4-22 for all of the common steel yield strengths.

E2. SLENDERNESS LIMITATIONS AND EFFECTIVE LENGTH

In this edition of the Specification, there is no limit on slenderness, \( KL/r \). Per the Commentary, it is recommended that \( KL/r \) not exceed 200, as a practical limit based on professional judgment and construction economics.

Although there is no restriction on the unbraced length of columns, the tables of the Manual are stopped at common or practical lengths for ordinary usage. For example, a double 3\( \times \)3\( \times \)\( \frac{3}{8} \) angle, with a \( \frac{3}{8} \)-in. separation has an \( r_c \) of 1.25 in. At a \( KL/r \) of 200, this strut would be 20'-10" long. This is thought to be a reasonable limit based on fabrication and handling requirements.

Throughout the Manual, shapes that contain slender elements when supplied in their most common material grade are footnoted with the letter “c”. For example, see a W14\( \times \)22c.

E3. COMPRESSIVE STRENGTH FOR FLEXURAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

Non-slender (compact and non-compact) sections, including non-slender built-up I-shaped columns and non-slender HSS columns, are governed by these provisions. The general design curve for critical stress versus \( KL/r \) is shown in Figure E-1.
E4. COMPRESSIVE STRENGTH FOR TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING OF MEMBERS WITHOUT SLENDER ELEMENTS

This section is most commonly applicable to double angles and Tee sections, which are singly symmetric shapes subject to torsional and flexural-torsional buckling. The available strength in axial compression of these shapes is tabulated in Part 4 of the Manual and examples on the use of these tables have been included in this chapter, for the shapes with $KL_z = KL_y$.

E5. SINGLE ANGLE COMPRESSION MEMBERS

The available strength of single-angle compression members is tabulated in Part 4 of the Manual.

E6. BUILT-UP MEMBERS

There are no tables for built-up shapes in the Manual, due to the number of possible geometries. This section makes suggestions as to how select built-up members to avoid slender elements, thereby making the analysis relatively straightforward.

### Table: Transition Point Limiting Values of $KL/r$

<table>
<thead>
<tr>
<th>$F_y$ ksi (MPa)</th>
<th>Limiting $KL/r$</th>
<th>$0.44F_y$ ksi (MPa)</th>
</tr>
</thead>
<tbody>
<tr>
<td>36 (248)</td>
<td>134</td>
<td>15.8 (109)</td>
</tr>
<tr>
<td>50 (345)</td>
<td>113</td>
<td>22.0 (152)</td>
</tr>
<tr>
<td>60 (414)</td>
<td>104</td>
<td>264 (182)</td>
</tr>
<tr>
<td>70 (483)</td>
<td>96</td>
<td>30.8 (212)</td>
</tr>
</tbody>
</table>

**Figure E-1 Standard Column Curve**

BRACING AND BRACE POINTS

The term $L$ is used throughout this chapter to describe the length between points that are braced against lateral and/or rotational displacement.
E7. MEMBERS WITH SLENDER ELEMENTS

The design of these members is similar to members without slender elements except that the formulas are modified by a reduction factor for slender elements, $Q$. Note the similarity of Equation E7-2 with Equation E3-2, and the similarity of Equation E7-3 with Equation E3-3.

The Tables of Part 4, incorporate the appropriate reductions in available strength to account for slender elements.

Design examples have been included in this Chapter for built-up I-shaped members with slender webs and slender flanges. Examples have also been included for a double angle, $WT$, and an HSS shape with slender elements.
Example E.1a  W-Shape Column Design with Pinned Ends

Given:
Select an ASTM A992 ($F_y = 50$ ksi) W-shape column to carry an axial dead load of 140 kips and live load of 420 kips. The column is 30 feet long, and is pinned top and bottom in both axes. Limit the column size to a nominal 14 in. shape.

Solution:

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips}) = 840 \text{ kips}$</td>
<td>$P_u = 140 \text{ kips} + 420 \text{ kips} = 560 \text{ kips}$</td>
</tr>
</tbody>
</table>

Select a column using Manual Table 4-1

For a pinned-pinned condition, $K = 1.0$

Because the unbraced length is the same in both the x-x and y-y directions and $r_x$ exceeds $r_y$ for all W-shapes, y-y axis bucking will govern.

Enter the table with an effective length, $KL_y$, of 30 ft, and proceed across the table until reaching the least weight shape with an available strength that equals or exceeds the required strength. Select a W14×132.
The available strengths in axial compression for a y-y axis effective length of 30 ft are:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_Pw = 892 \text{ kips} &gt; 840 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$P_n/\Omega_c = 594 \text{ kips} &gt; 560 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Manual Table 4-1
Example E.1b  **W-Shape Column Design with Intermediate Bracing**

**Given:**
Redesign the column from Example E.1a assuming the column is laterally braced about the y-y axis and torsionally braced at the midpoint.

**Solution:**

*Calculate the required strength*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_a = 1.2(140 \text{ kips}) + 1.6(420 \text{ kips}) = 840 \text{ kips}$</td>
<td>$P_a = 140 \text{ kips} + 420 \text{ kips} = 560 \text{ kips}$</td>
</tr>
</tbody>
</table>

*Select a column using Manual Table 4-1.*

For a pinned-pinned condition, $K = 1.0$

Since the unbraced lengths differ in the two axes, select the member using the y-y axis then verify the strength in the x-x axis.

Enter Table 4-1 with a y-y axis effective length, $K_L$, of 15 ft and proceed across the table until reaching a shape with an available strength that equals or exceeds the required strength. Try a $W14 \times 90$. A 15 ft long $W14 \times 90$ provides an available strength in the y-y direction of

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_n = 1000 \text{ kips}$</td>
<td>$P_n/\Omega = 667 \text{ kips}$</td>
</tr>
</tbody>
</table>

The $r_x/r_y$ ratio for this column, shown at the bottom of Manual Table 4-1, is 1.66. The equivalent y-y axis effective length for strong axis buckling is computed as

$$KL = \frac{30.0 \text{ ft}}{1.66} = 18 \text{ ft}$$
From the table, the available strength of a W14×90 with an effective length of 18 ft is

<table>
<thead>
<tr>
<th>Design</th>
<th>( P_n/O_c ) <em>LRFD</em></th>
<th>( P_n/O_c ) <em>ASD</em></th>
<th>( \phi_P P_n ) <em>LRFD</em></th>
<th>( \phi_P P_n ) <em>ASD</em></th>
<th>( P_n/O_c ) <em>LRFD</em></th>
<th>( P_n/O_c ) <em>ASD</em></th>
<th>( \phi_P P_n ) <em>LRFD</em></th>
<th>( \phi_P P_n ) <em>ASD</em></th>
<th>( P_n/O_c ) <em>LRFD</em></th>
<th>( P_n/O_c ) <em>ASD</em></th>
<th>( \phi_P P_n ) <em>LRFD</em></th>
<th>( \phi_P P_n ) <em>ASD</em></th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>1280</td>
<td>1920</td>
<td>1160</td>
<td>1740</td>
<td>1060</td>
<td>1590</td>
<td>959</td>
<td>1440</td>
<td>872</td>
<td>1310</td>
<td>792</td>
<td>1190</td>
</tr>
<tr>
<td>6</td>
<td>1250</td>
<td>1870</td>
<td>1130</td>
<td>1700</td>
<td>1030</td>
<td>1550</td>
<td>934</td>
<td>1400</td>
<td>849</td>
<td>1280</td>
<td>771</td>
<td>1160</td>
</tr>
<tr>
<td>7</td>
<td>1240</td>
<td>1860</td>
<td>1120</td>
<td>1680</td>
<td>1020</td>
<td>1530</td>
<td>924</td>
<td>1390</td>
<td>840</td>
<td>1260</td>
<td>763</td>
<td>1150</td>
</tr>
<tr>
<td>8</td>
<td>1220</td>
<td>1840</td>
<td>1110</td>
<td>1660</td>
<td>1010</td>
<td>1510</td>
<td>914</td>
<td>1370</td>
<td>831</td>
<td>1250</td>
<td>754</td>
<td>1140</td>
</tr>
<tr>
<td>9</td>
<td>1210</td>
<td>1820</td>
<td>1090</td>
<td>1640</td>
<td>995</td>
<td>1500</td>
<td>902</td>
<td>1360</td>
<td>820</td>
<td>1230</td>
<td>745</td>
<td>1120</td>
</tr>
<tr>
<td>10</td>
<td>1200</td>
<td>1800</td>
<td>1080</td>
<td>1620</td>
<td>981</td>
<td>1470</td>
<td>889</td>
<td>1340</td>
<td>808</td>
<td>1210</td>
<td>734</td>
<td>1100</td>
</tr>
<tr>
<td>11</td>
<td>1180</td>
<td>1770</td>
<td>1060</td>
<td>1590</td>
<td>965</td>
<td>1450</td>
<td>875</td>
<td>1320</td>
<td>795</td>
<td>1200</td>
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</tr>
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<td>709</td>
<td>1070</td>
</tr>
<tr>
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<td>1020</td>
<td>1540</td>
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<td>1150</td>
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</tr>
<tr>
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<td>982</td>
<td>1480</td>
<td>893</td>
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<td>809</td>
<td>1220</td>
<td>734</td>
<td>1100</td>
<td>667</td>
<td>1000</td>
</tr>
<tr>
<td>16</td>
<td>1080</td>
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<td>1440</td>
<td>872</td>
<td>1310</td>
<td>790</td>
<td>1190</td>
<td>717</td>
<td>1080</td>
<td>651</td>
<td>978</td>
</tr>
<tr>
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<td>1060</td>
<td>1580</td>
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<td>1410</td>
<td>851</td>
<td>1280</td>
<td>771</td>
<td>1160</td>
<td>699</td>
<td>1050</td>
<td>635</td>
<td>954</td>
</tr>
<tr>
<td>18</td>
<td>1030</td>
<td>1550</td>
<td>912</td>
<td>1370</td>
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<td>751</td>
<td>1130</td>
<td>681</td>
<td>1020</td>
<td>618</td>
<td>928</td>
</tr>
<tr>
<td>19</td>
<td>1000</td>
<td>1510</td>
<td>887</td>
<td>1330</td>
<td>806</td>
<td>1210</td>
<td>730</td>
<td>1100</td>
<td>662</td>
<td>995</td>
<td>600</td>
<td>902</td>
</tr>
<tr>
<td>20</td>
<td>970</td>
<td>1470</td>
<td>862</td>
<td>1290</td>
<td>783</td>
<td>1180</td>
<td>700</td>
<td>1070</td>
<td>639</td>
<td>968</td>
<td>583</td>
<td>873</td>
</tr>
</tbody>
</table>

\[ \phi_P P_n = 928 \text{ kips} > 840 \text{ kips} \quad \text{o.k.} \quad \frac{P_n}{O_c} = 618 \text{ kips} > 560 \text{ kips} \quad \text{o.k.} \]

The available compression strength is governed by the \( x-x \) axis flexural buckling limit state.
The available strengths of the columns described in Examples E.1a and E.1b are easily selected directly from the Manual Tables. The available strengths can also be verified by hand calculations, as shown in the following Examples E.1c and E.1d.

**Example E.1c  W-Shape Available Strength Calculation**

Calculate the available strength of a W14×132 column with unbraced lengths of 30 ft in both axes. The material properties and loads are as given in Example E.1a.

**Material properties:**
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)  

**Geometric Properties:**
W14×132  \( A_g = 38.8 \text{ in.}^2 \)  \( r_x = 6.28 \text{ in.} \)  \( r_y = 3.76 \text{ in.} \)  

**Calculate the available strength**

For a pinned-pinned condition, \( K = 1.0 \)

Since the unbraced length is the same for both axes, the \( y-y \) axis will govern.

\[
\frac{K L_e}{r_y} = \frac{1.0 (30.0 \text{ ft}) 12.0 \text{ in.}}{3.76 \text{ in.} \text{ ft}} = 95.7
\]

For \( F_y = 50 \text{ ksi} \), the available critical stresses, \( \phi_c F_{cr} \) and \( F_{cr}/\Omega_c \) for \( KL/r = 95.7 \) are interpolated from Manual Table 4-22 as

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c F_{cr} = 23.0 \text{ ksi} )</td>
<td>( F_{cr}/\Omega_c = 15.3 \text{ ksi} )</td>
</tr>
<tr>
<td>( \phi_c P_e = 38.8 \text{ in.}^2 \times (23.0 \text{ ksi}) )</td>
<td>( P_e/\Omega_e = 38.8 \text{ in.}^2 \times (15.3 \text{ ksi}) )</td>
</tr>
<tr>
<td>= 892 kips &gt; 840 kips  o.k.</td>
<td>= 594 kips &gt; 560 kips  o.k.</td>
</tr>
</tbody>
</table>

Note that the calculated values match the tabulated values.
Example E.1d  W-Shape Available Strength Calculation

Calculate the available strength of a W14×90 with a strong axis unbraced length of 30 ft and weak axis and torsional unbraced lengths of 15 ft. The material properties and loads are as given in Example E.1b.

Geometric Properties:
W14×90  \( A_g = 26.5 \text{ in.}^2 \)  \( r_x = 6.14 \text{ in.} \)  \( r_y = 3.70 \text{ in.} \)

Check both slenderness ratios

\[ K = 1.0 \]

\[ \frac{KL}{r_x} = 1.0 \left( \frac{30.0 \text{ ft}}{12 \text{ in.}} \right) = 58.6 \left( \frac{6.14 \text{ in}}{\text{ft}} \right) \text{ governs} \]

\[ \frac{KL}{r_y} = 1.0 \left( \frac{15.0 \text{ ft}}{12 \text{ in.}} \right) = 48.6 \left( \frac{3.70 \text{ in.}}{\text{ft}} \right) \]

The available critical stresses may be interpolated from Manual Table 4-22 or calculated directly as follows.

Calculate the elastic critical buckling stress, \( F_e \)

\[ F_e = \frac{\pi^2 E}{(KL/r^2)} = \frac{\pi^2 29,000 \text{ ksi}}{(58.6)^2} = 83.3 \text{ ksi} \]

Calculate flexural buckling stress, \( F_{cr} \)

Check limit

\[ 4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 113 > 58.6 \]

Because \( \frac{KL}{r} < 4.71 \sqrt{\frac{E}{F_y}} \),

\[ F_{cr} = 0.658 \frac{F_y}{r_y} = \left[ 0.658 \frac{38.9 \text{ ksi}}{\text{ksi}} \right] \frac{50.0 \text{ ksi} = 38.9 \text{ ksi}}{\text{ksi}} \]

\[ P_n = F_{cr} A_g = 38.9 \text{ ksi} \left( 26.5 \text{ in.}^2 \right) = 1030 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c F_n = 0.90(1030 \text{ kips}) = 928 \text{ kips} &gt; 840 \text{ kips} ) o.k.</td>
<td>( P_n/\Omega_c = (1030 \text{ kips}) / 1.67 = 618 \text{ kips} &gt; 560 \text{ kips} ) o.k.</td>
</tr>
</tbody>
</table>
Example E.2  Built-up Column with a Slender Web

Given:
Verify that a built-up, ASTM A572 grade 50, column with PL\(1\text{in.} \times 8\text{in.}\) flanges and a PL\(4\text{in.} \times 15\text{in.}\) web is sufficient to carry a dead load of 70 kips and live load of 210 kips in axial compression. The column length is 15 ft and the ends are pinned in both axes.

Solution:

Material Properties:
ASTM A572 Grade 50 \(F_y = 50 \text{ ksi}\) \(F_u = 65 \text{ ksi}\) Manual Table 2-3

Geometric Properties:
Built-up Column \(d = 17.0\) in. \(b_f = 8.00\) in. \(t_f = 1.00\) in. \(h = 15.0\) in. \(t_w = 0.250\) in.

Calculate the required strength

\[
P_{u} = 1.2(70.0 \text{ kips}) + 1.6(210 \text{ kips}) = 420 \text{ kips}
\]

\[
P_{a} = 70.0 \text{ kips} + 210 \text{ kips} = 280 \text{ kips}
\]

Calculate built-up section properties (ignoring fillet welds)

\[A = 2(8.00 \text{ in.})(1.00 \text{ in.}) + (15.0 \text{ in.})(0.250 \text{ in.}) = 19.8 \text{ in.}^2\]

\[
I_y = \frac{2(1.00 \text{ in.})(8.00 \text{ in.})^3 + (15.0 \text{ in.})(0.250 \text{ in.})^3}{12} = 85.4 \text{ in.}^4
\]

\[
r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{85.4 \text{ in.}^4}{19.8 \text{ in.}^2}} = 2.08 \text{ in.}
\]

\[I_x = \sum Ad^2 + \sum I_x
\]

\[= 2(8.00 \text{ in.}^3)(8.00 \text{ in.})^2 + \frac{(0.250 \text{ in.})(15.00 \text{ in.})^3}{12} + \frac{2(8.0 \text{ in.})(1.0 \text{ in.})^3}{12} = 1100 \text{ in.}^4
\]
Calculate the elastic flexural buckling stress

For a pinned-pinned condition, $K = 1.0$

Since the unbraced length is the same for both axes, the $y$-$y$ axis will govern by inspection.

$$ \frac{KL_y}{r_y} = \frac{1.0(15.0 \text{ ft})}{2.08 \text{ in.}} = 86.6 $$

$$ F_e = \frac{\pi^2 E}{(KL_y/r_y)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(86.6)^2} = 38.2 \text{ ksi} $$

Calculate the elastic critical torsional buckling stress

Note: Torsional buckling will not govern if $KL_y > KL_z$, however, the check is included here to illustrate the calculation.

$$ C_w = \frac{I_y h^2}{4} = \frac{85.4 \text{ in.}^4(16.0 \text{ in.})^2}{4} = 5470 \text{ in.}^6 $$

$$ J = \sum \frac{bt^3}{3} = \frac{2(8.00 \text{ in.})(1.00 \text{ in.})^3 + (15.0 \text{ in.})(0.250 \text{ in.})^3}{3} = 5.41 \text{ in.}^4 $$

$$ F_e = \frac{\pi^2 E C_w}{(KL_y)^2} + GJ \left[ \frac{1}{I_z + I_y} \right] $$

$$ = \frac{\pi^2 (29,000 \text{ ksi})(5470 \text{ in.}^6)}{(180 \text{ in.}^2)^2} + (11,200 \text{ ksi})(5.41 \text{ in.}^4) \left[ \frac{1}{1100 \text{ in.}^4 + 85.4 \text{ in.}^4} \right] $$

$$ = 92.2 \text{ ksi} > 38.2 \text{ ksi} $$

Therefore, the flexural buckling limit state controls.

Use $F_e = 38.2 \text{ ksi}$

Check for slender elements using Specification Section E7

Determine $Q_s$, the unstiffened element (flange) reduction factor

Calculate $k_c$

$$ k_c = \frac{4}{\sqrt{h/i_w}} = \frac{4}{\sqrt{15.0/0.250}} = 0.516 \text{ which is between 0.35 and 0.76} $$

$$ b = \frac{4.00 \text{ in.}}{1.00 \text{ in.}} < 0.64 \sqrt{\frac{k_c E}{F_y}} = 0.64 \sqrt{\frac{0.516(29,000 \text{ ksi})}{50 \text{ ksi}}} = 11.1 $$
Therefore, the flange is not slender.

\[ Q_s = 1.0 \]

**Determine \( Q_a \), the stiffened element (web) reduction factor**

\[ \frac{h}{t} = \frac{15.0 \text{ in.}}{0.250 \text{ in.}} = 60.0 > 1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 35.9 \], therefore the web is slender.

\[ Q_a = \frac{A_{\text{eff}}}{A} \] where \( A_{\text{eff}} \) is effective area based on the reduced effective width of the web, \( b_e \).  

Section E7.2

For equation E7-17, take \( f \) as \( F_{cr} \) with \( Q = 1.0 \)

\[ KL/r = 86.6 \text{ from above} \]

\[ 4.71 \sqrt{\frac{E}{Q F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{1.0(50 \text{ ksi})}} = 113 > 86.6 \]

When \( \frac{KL}{r} \leq 4.71 \sqrt{\frac{E}{Q F_y}} \)

\[ F_{cr} = Q \left[ 0.658 \frac{Q F_y}{E} \right] F_y = 1.0 \left[ 0.658 \frac{1.0(50 \text{ ksi})}{29,000 \text{ ksi}} \right] (50 \text{ ksi}) = 28.9 \text{ ksi} \]

Eqn. E7-2

\[ b_e = 1.92 t \left[ \frac{E}{f} \left( \frac{1}{(b/t)/\sqrt{F}} \right) \right] \leq b, \text{ where } b = h \]

Eqn. E7-17

\[ = 1.92(0.250 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{28.9 \text{ ksi}}} \left[ 1 - \frac{0.34}{(15.0 \text{ in.}/0.250 \text{ in.})} \right] \leq 15.0 \text{ in.} \]

\[ = 12.5 \text{ in.} < 15.0 \text{ in.}, \text{ therefore compute } A_{\text{eff}} \text{ with reduced effective web width.} \]

\[ A_{\text{eff}} = b_{se} + 2b_f \theta_f = (12.5 \text{ in.})(0.250 \text{ in.}) + 2(8.00 \text{ in.})(1.00 \text{ in.}) = 19.1 \text{ in.}^2 \]

\[ Q_a = \frac{A_{\text{eff}}}{A} = \frac{19.1 \text{ in.}^2}{19.8 \text{ in.}^2} = 0.966 \]

Eqn. E7-16

\[ Q = Q_s Q_a = (1.00)(0.966) = 0.966 \]

**Determine whether Specification Equation E7-2 or E7-3 applies**

Section E7

\[ KL/r = 86.6 \text{ from above} \]

\[ 4.71 \sqrt{\frac{E}{Q F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{0.966(50 \text{ ksi})}} = 115.4 > 86.6 \]

Therefore, Specification Equation E7-2 applies.
Calculate the nominal compressive strength

\[ P_n = F_{cr} A_y = 28.5 \text{ ksi} \left(19.8 \text{ in}^2\right) = 562 \text{ kips} \]  

Eqn. E7-1

\[ F_{cr} = Q \left[ 0.658 \frac{QF}{F_y} \right] = 0.966 \left[ 0.658 \frac{0.966(50 \text{ ksi})}{38.2 \text{ ksi}} \right] (50 \text{ ksi}) = 28.5 \text{ ksi} \]  

Eqn. E7-2

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_n ) = 0.90 (562 kips)</td>
<td>( P_n / \Omega_c = 562 \text{ kips} / 1.67 )</td>
</tr>
<tr>
<td>= 505 kips &gt; 420 kips, o.k.</td>
<td>= 336 kips &gt; 280 kips, o.k.</td>
</tr>
</tbody>
</table>

Section E1
Example E.3  Built-up Column with Slender Flanges

Given:

Determine if a built-up, ASTM A572 grade 50 column with PL\(\frac{3}{8}\) in.\(\times\)10\(\frac{1}{2}\) in. flanges and a PL\(\frac{1}{4}\) in.\(\times\)7\(\frac{3}{4}\) in. web has sufficient available strength to carry a dead load of 40 kips and a live load of 120 kips in axial compression. The column unbraced length is 15 ft in both axes and the ends are pinned.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A572 Gr. 50</td>
<td>(F_y)</td>
<td>50 ksi</td>
</tr>
<tr>
<td>ASTM A572 Gr. 50</td>
<td>(F_u)</td>
<td>65 ksi</td>
</tr>
</tbody>
</table>

Manual Table 2-3

Geometric Properties:

Built-up Column
- \(d\) = 8.00 in.
- \(b_f\) = 10.5 in.
- \(t_f\) = 0.375 in.
- \(h\) = 7.25 in.
- \(t_w\) = 0.250 in.

Calculate the required strength

\[
P'_{u} = 1.2(40.0 \text{ kips}) + 1.6(120 \text{ kips}) = 240 \text{ kips}
\]

\[
P'_{a} = 40.0 \text{ kips} + 120 \text{ kips} = 160 \text{ kips}
\]

Calculate built-up section properties (ignoring fillet welds)

\[
A = 2(10.5 \text{ in.})(0.375 \text{ in.}) + (7.25 \text{ in.})(0.250 \text{ in.}) = 9.69 \text{ in.}^2
\]

Since the unbraced length is the same for both axes, the weak axis will govern.

\[
I_y = 2\left[\frac{(0.375 \text{ in.})(10.5 \text{ in.})}{12}\right] + \frac{(7.25 \text{ in.})(0.250 \text{ in.})}{12} = 72.4 \text{ in.}^4
\]

\[
r_y = \frac{I_y}{A} = \frac{72.4 \text{ in.}^4}{9.69 \text{ in.}^2} = 2.37 \text{ in.}
\]

\[
I_s = 2(10.5 \text{ in.})(0.375 \text{ in.})(3.81 \text{ in.})^2 + \frac{(0.25 \text{ in.})(7.25 \text{ in.})}{12} + \frac{2(10.5 \text{ in.})(0.375 \text{ in.})}{12}
\]

\[
= 122 \text{ in.}^4
\]
Check web slenderness

For a stiffened element (web) in a doubly symmetric I-shaped section, under uniform compression,

\[
\lambda_w = 1.49 \frac{E}{F_y} = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 35.9
\]

Therefore, the web is not slender.

Note that the fillet welds are ignored in the calculation of \( h \) for built up sections.

Check flange slenderness

Calculate \( k_c \)

\[
k_c = 4 \frac{h}{t_w} = 4 \frac{7.25 \text{ in.}}{0.250 \text{ in.}} = 0.743 \quad \text{where} \quad 0.35 \leq k_c \leq 0.76 \quad \text{o.k.}
\]

Use \( k_c = 0.743 \)

For flanges of a built-up I-shaped section under uniform compression;

\[
\lambda_c = 0.64 \sqrt{\frac{k_c E}{F_y}} = 0.64 \sqrt{\frac{0.743(29,000 \text{ ksi})}{50 \text{ ksi}}} = 13.3
\]

Therefore, the flanges are slender.

For compression members with slender elements, Section E7 of the Specification applies. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional, and flexural-torsional buckling. Depending on the slenderness of the column, Specification Equation E7-2 or E7-3 applies. \( F_e \) is used in both equations and is calculated as the lesser of Specification Equations E3-4 and E4-4.

For a pinned-pinned condition, \( K = 1.0 \)

Since the unbraced length is the same for both axes, the weak axis will govern.

\[
K \frac{L_y}{r_y} = 1.0 \left( 15.0 \text{ ft} \right) \left( \frac{12 \text{ in.}}{2.73 \text{ in.}} \right) = 65.9
\]

Calculate the elastic critical stress, \( F_e \), for flexural buckling

\[
F_e = \frac{\pi^2 E}{KL^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(65.9)^2} = 65.9 \text{ ksi}
\]
Calculate the elastic critical stress, \( F_e \), for torsional buckling

Not likely to govern, but check for completeness

\[
C_w = \frac{l h^2}{4} = \frac{72.4 \text{ in.}^4(7.63 \text{ in.})^2}{4} = 1050 \text{ in.}^6
\]
\[
J = \sum \frac{b h^3}{3} = \frac{2(10.5 \text{ in.})(0.375 \text{ in.})^3 + 7.25 \text{ in.}(0.25 \text{ in.})^3}{3} = 0.407 \text{ in.}^4
\]
\[
F_e = \left[ \frac{\pi^2 E C_w}{(K, L)^2} + GJ \right] \frac{1}{I_x + I_y} = \left[ \frac{\pi^2 (29,000 \text{ ksi})(1050 \text{ in.}^6)}{(180 \text{ in.})^2} + (11,200 \text{ ksi})(0.407 \text{ in.}^4) \right] \frac{1}{122 \text{ in.}^4 + 72.4 \text{ in.}^4} = 71.2 \text{ ksi} > 65.9 \text{ ksi}
\]

Therefore, use \( F_e = 65.9 \text{ ksi} \)

Determine \( Q \), the slenderness reduction factor

\( Q = Q_a Q_s \), where \( Q_a = 1.0 \) because the web is not slender

Calculate \( Q_s \), the unstiffened element (flange) reduction factor

Determine the proper equation for \( Q_s \) by checking limits for Equations E7-7 to E7-9

\[
\frac{b}{t} = 14.0 \text{ from above}
\]
\[
0.64 \sqrt{\frac{E_k}{F_y}} = 0.64 \sqrt{\frac{29,000 \text{ ksi}(0.743)}{50 \text{ ksi}}} = 13.3
\]
\[
1.17 \sqrt{\frac{E_k}{F_y}} = 1.17 \sqrt{\frac{29,000 \text{ ksi}(0.743)}{50 \text{ ksi}}} = 24.3
\]
\[
0.64 \sqrt{\frac{E_k}{F_y}} < \frac{b}{t} \leq 1.17 \sqrt{\frac{E_k}{F_y}} \text{ therefore, Equation E7-8 applies.}
\]
\[
Q_s = 1.415 - 0.65 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E_k}}
\]
\[
= 1.415 - 0.65(14.0) \sqrt{\frac{50 \text{ ksi}}{(29,000 \text{ ksi})(0.743)}} = 0.977
\]
\[
Q = Q_s Q_a = (0.977)(1.0) = 0.977
\]
Calculate nominal compressive strength

$$4.71 \sqrt{\frac{E}{QF_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{0.977(50 \text{ ksi})}} = 115 > 65.9$$  therefore, Specification Eqn. E7-2 applies.

$$F_{cr} = Q \left[ 0.658 \frac{QF_y}{F_y} \right] F_y = 0.977 \left[ 0.658 \frac{0.977(50 \text{ ksi})}{65.9 \text{ ksi}} \right] (50 \text{ ksi}) = 35.8 \text{ ksi}$$  Eqn. E7-2

$$P_a = F_a A_y = (35.8 \text{ ksi})(9.69 \text{ in}^2) = 347 \text{ kips}$$  Eqn. E7-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega_c = 1.67$</td>
</tr>
<tr>
<td>$\phi P_a = 0.90(347 \text{ kips})$</td>
<td>$P_a/\Omega_c = (347 \text{ kips})/1.67$</td>
</tr>
<tr>
<td>$= 312 \text{ kips} &gt; 240 \text{ kips}$</td>
<td>$= 208 \text{ kips} &gt; 160 \text{ kips}$</td>
</tr>
</tbody>
</table>

Note: Built-up sections are generally more expensive than standard rolled shapes; therefore, a standard compact shape, such as a W8×35 might be a better choice even if the weight is somewhat higher. This selection could be taken directly from Manual Table 4-1.
Example E.4a  W-Shape Compression Member (Moment Frame)

This example is primarily intended to illustrate the use of the alignment chart for sidesway uninhibited columns.

**Given:**

The member sizes shown for the moment frame illustrated here (sidesway uninhibited in the plane of the frame) have been determined to be adequate for lateral loads. The material for both the column and the girders is ASTM A992 grade 50. The loads shown at each level are the accumulated dead loads and live loads at that story. The column is fixed at the base about the \( x \)-\( x \) axis of the column.

Determine if the column is adequate to support the gravity loads shown. Assume the column is continuously supported in the transverse direction (the \( y \)-\( y \) axis of the column).

**Material Properties:**

ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

**Geometric Properties:**

<table>
<thead>
<tr>
<th>Member Size</th>
<th>Moment of Inertia, ( I_x )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W18×50</td>
<td>800 in.(^4)</td>
</tr>
<tr>
<td>W24×55</td>
<td>1350 in.(^4)</td>
</tr>
<tr>
<td>W14×82</td>
<td>24.0 in.(^2)</td>
</tr>
</tbody>
</table>

Calculate the required strength for the column between the roof and floor

\[
P_a = 1.2(41.5 \text{ kips}) + 1.6(125 \text{ kips}) = 250 \text{ kips}
\]

Calculate the effective length factor, \( K \)

\[
\frac{P_a}{A_g} = \frac{250 \text{ kips}}{24.0 \text{ in.}^2} = 10.4 \text{ ksi} < 18 \text{ ksi}
\]

\( \tau = 1.00 \)

\[
\frac{P_a}{A_g} = \frac{167 \text{ kips}}{24.0 \text{ in.}^2} = 6.96 \text{ ksi} < 12 \text{ ksi}
\]

\( \tau = 1.00 \)

Therefore, no reduction in stiffness for inelastic buckling will be used.
Determine $G_{\text{top}}$ and $G_{\text{bottom}}$

\[
G_{\text{top}} = \tau \frac{\sum (I_c / L_c)}{\sum (I_g / L_g)} = (1.00) \frac{\left( \frac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2 \left( \frac{800 \text{ in.}^4}{35.0 \text{ ft}} \right)} = 1.38
\]

Commentary C2.2

\[
G_{\text{bottom}} = \tau \frac{\sum (I_c / L_c)}{\sum (I_g / L_g)} = (1.00) \frac{2 \left( \frac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2 \left( \frac{1350 \text{ in.}^4}{35.0 \text{ ft}} \right)} = 1.63
\]

Commentary C2.2 & Fig. C-C2.4

From the alignment chart, $K$ is slightly less than 1.5. Because the column available strength tables are based on the $KL$ about the y-y axis, the equivalent effective column length of the upper segment for use in the table is:

\[
KL = \frac{(KL)_e}{(r_e / r_i)} = \frac{1.5(14.0 \text{ ft})}{2.44} = 8.61 \text{ ft}
\]

Take the available strength of the W14x82 from Manual Table 4.1

At $KL = 9$ ft, the available strength in axial compression is:

\[
\phi \frac{P_e}{A_g} = 942 \text{ kips} > 250 \text{ kips} \quad \text{o.k.}
\]

Manual Table 4-1

Calculate the required strength for the column segment between the floor and the foundation

\[
P_a = 1.2(100 \text{ kips}) + 1.6 (300 \text{ kips}) = 600 \text{ kips}
\]

\[
P_a = 100 \text{ kips} + 300 \text{ kips} = 400 \text{ kips}
\]

Calculate the effective length factor, $K$

\[
\frac{P_a}{A_g} = \frac{600 \text{ kips}}{24.0 \text{ in.}^2} = 25.0 \text{ ksi}
\]

\[
tau = 0.890
\]

\[
G_{\text{top}} = \tau \frac{\sum (I_c / L_c)}{\sum (I_g / L_g)} = (0.890) \frac{2 \left( \frac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2 \left( \frac{1350 \text{ in.}^4}{35.0 \text{ ft}} \right)} = 1.45
\]

Manual Table 4-21

\[
G_{\text{bottom}} = \tau \frac{\sum (I_c / L_c)}{\sum (I_g / L_g)} = (0.890) \frac{2 \left( \frac{881 \text{ in.}^4}{14.0 \text{ ft}} \right)}{2 \left( \frac{1350 \text{ in.}^4}{35.0 \text{ ft}} \right)} = 1.45
\]

Commentary Section C2.2b
\[ G_{\text{bottom}} = 1 \text{ (fixed)} \]

From the alignment chart, \( K \) is approximately 1.42. Because the column available strengths are based on the \( KL \) about the \( y-y \) axis, the effective column length of the lower segment for use in the table is:

\[
KL = \frac{(KL)}{r_y} = \frac{1.42(14.0 \text{ ft})}{2.44} = 8.15 \text{ ft}
\]

*Take the available strength of the W14×82 from Manual Table 4-1*

at \( L = 9 \text{ ft} \) (conservative) the available strength in axial compression is:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_e = 942 \text{ kips} &gt; 600 \text{ kips} )</td>
<td>\textbf{o.k.}</td>
<td>( P_e / \Omega_e = 627 \text{ kips} &gt; 400 \text{ kips} )</td>
</tr>
</tbody>
</table>

A more accurate strength could be determined by interpolation from Manual Table 4-1
Example E.4b  W-Shape Compression Member (Moment Frame)

Determine the available strength of the column shown subject to the same gravity loads shown in Example E.4a with the column pinned at the base about the \(x\)-\(x\) axis. All other assumptions remain the same.

As determined in Example E.4a, for the column segment between the roof and the floor, the column strength is adequate.

As determined in Example E.4a, for the column segment between the floor and the foundation, \(G_{\text{top}} = 1.45\)

At the base, \(G_{\text{bot}} = 10\) (pinned)

Note: this is the only change in the analysis.

From the alignment chart, \(K\) is approximately equal to 2.0. Because the column available strength Tables are based on the effective length, \(KL\), about the \(y\)-\(y\) axis, the effective column length of the lower segment for use in the table is:

\[
KL = \left( \frac{KL}{r_y} \right)_{r_x} = \frac{2.0(14.0 \text{ ft})}{2.44} = 11.5 \text{ ft}
\]

Interpolate the available strength of the W14×82 from Manual Table 4-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_u = 863) kips &gt; 600 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>(P_u / \Omega = 574) kips &gt; 400 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
Example E.5    Double Angle Compression Member without Slender Elements

Given:
Verify the strength of a 2L4\times\frac{3}{2}\times\frac{3}{6} \text{ LLBB (\frac{3}{4}-\text{in. separation}) strut with a length of 8 ft and pinned ends carrying an axial dead load of 20 kips and live load of 60 kips. Also, calculate the required number of fully tightened or welded intermediate connectors required.}

Material Properties:
ASTM A36  \quad F_y = 36 \text{ ksi} \quad F_u = 58 \text{ ksi}

Geometric Properties:
2L4\times\frac{3}{2}\times\frac{3}{6} \text{ LLBB}
\quad r_x = 0.719 \text{ in. (single angle)}
\quad r_y = 1.25 \text{ in.}
\quad r_x = 1.55 \text{ in. for } \frac{3}{8} \text{ inch separation}
\quad r_y = 1.69 \text{ in. for } \frac{3}{4} \text{ inch separation}

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>P_u = 1.2(20 kips) + 1.6(60 kips) = 120 kips</td>
<td>P_a = 20 kips + 60 kips = 80.0 kips</td>
</tr>
</tbody>
</table>

Select a column using Manual Table 4-9

K = 1.0

For (KL)_x = 8 \text{ ft}, the available strength in axial compression is taken from the upper (x-x) portion of the table as

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>\phi_c P_a = 127 kips &gt; 120 kips</td>
<td>\phi_c P_a = 127 kips &gt; 120 kips</td>
</tr>
</tbody>
</table>

For buckling about the y-y axis, the values are tabulated for a separation of \frac{3}{8} \text{ in.}

To adjust to a spacing of \frac{3}{4} \text{ in.}, (KL)_y is multiplied by the ratio of the r_y for a \frac{3}{8}-\text{in. separation to the r_x for a \frac{3}{4}-\text{in. separation. Thus,}}

\[(KL)_y = 1.0(8.00 \text{ ft}) \left(\frac{1.55 \text{ in.}}{1.69 \text{ in.}}\right) = 7.34 \text{ ft}\]

The calculation of the equivalent (KL)_y above is a simplified approximation of Specification Section E6.1. To ensure a conservative adjustment for a \frac{3}{4} \text{ in. separation, take (KL)_y = 8 \text{ ft.}}
The available strength in axial compression is taken from the lower (y-y) portion of the table as

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi, P_x = 130$ kips &gt; 120 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>$P_y/\Omega_y = 86.5$ kips &gt; 80.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Therefore, $x-x$ axis flexural buckling governs.

**Determine the number of intermediate connectors required**

Per Table 4-12, at least two welded or pretensioned bolted intermediate connectors are required. This can be verified as follows:

$$a = \text{distance between connectors} = \frac{(8.00 \text{ ft})(12 \text{ in./ft})}{3 \text{ spaces}} = 32.0 \text{ in.}$$

Section E6.2

The effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, $a$, must not exceed three quarters of the controlling slenderness ratio of the overall built-up member.

Therefore:

$$\frac{Ka}{r} \leq \frac{3}{4} \left( \frac{KL}{r} \right)_{\text{max}}$$

Solving for $a$ gives,

$$a \leq \frac{3r}{4K} \left( \frac{KL}{r} \right)_{\text{max}}$$

Thus,

$$a \leq \frac{3r}{4K} \left( \frac{KL}{r} \right)_{\text{max}} = \frac{3(0.719 \text{ in.})(76.8)}{4(1.0)} = 41.4 \text{ in.} > 32.0 \text{ in.}$$

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below.

**Calculation Solution:**

**Geometric Properties:**

2L4×3\frac{1}{2}×\frac{3}{8} LLBB  
<table>
<thead>
<tr>
<th>$A_y$</th>
<th>5.34 in.$^2$</th>
<th>$r_y$</th>
<th>1.69 in.</th>
<th>$\bar{r}_o$</th>
<th>2.33 in.</th>
<th>$H = 0.813$</th>
<th>Manual Table 1-15</th>
</tr>
</thead>
<tbody>
<tr>
<td>$J$</td>
<td>0.132 in.$^4$ (single angle)</td>
<td>$r_y$</td>
<td>1.05 in. (single angle)</td>
<td>$\bar{x}$</td>
<td>0.947 in. (single angle)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\bar{r}$</td>
<td>0.375 in.</td>
<td>$E$</td>
<td>29,000 ksi</td>
<td>$F_y$</td>
<td>36 ksi</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Check for slender elements**

$$b = 4.0 \text{ in.}$$

$$\tau = 0.375 \text{ in.}$$

$$\lambda_n = 0.45 \sqrt{\frac{E}{F_y}} = 0.45 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 12.8 > 7.7$$

Therefore, there are no slender elements.
For compression members without slender elements, Specification Sections E3 and E4 apply. The nominal compressive strength, $P_n$, shall be determined based on the limit states of flexural, torsional, and flexural-torsional buckling.

**Check flexural buckling about the x-x axis**  
Section E3

$$\frac{KL}{r_x} = \frac{1.0(8.00 \text{ ft})(12 \text{ in./ft})}{1.25 \text{ in.}} = 76.8$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r_x}\right)^2} = \frac{\pi^2(29,000 \text{ ksi})}{(76.8)^2} = 48.5 \text{ ksi}$$

$$4.71 \sqrt{\frac{E}{F_e}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 134 > 76.8, \text{ therefore}$$

$$F_{cr} = \left[0.658 F_e\right] F_e = \left[0.658 \frac{36 \text{ ksi}}{29.000 \text{ ksi}}\right](36 \text{ ksi}) = 26.4 \text{ ksi} \text{ controls}$$

**Check torsional and flexural-torsional buckling**  
Section E4

For non-slender double angle compression members, Specification Equation E4-2 applies.

$F_{cry}$ is taken as $F_{cr}$, for flexural buckling about the y-y axis from Specification Equation E3-2 or E3-3 as applicable.

**Compute the modified $\frac{KL}{r_y}$ for built up members with fully tightened or welded connectors**  
Section E6

$a = 96.0 \text{ in.} / 3 = 32.0 \text{ in.}$

$r_{hb} = r_y (\text{single angle}) = 1.05 \text{ in.}$

$$\alpha = \frac{h}{2r_{hb}} = \frac{2x + 0.750 \text{ in.}}{2(0.947 \text{ in.} + 0.750 \text{ in.})} = \frac{2(1.05 \text{ in.})}{2(1.05 \text{ in.})} = 1.26$$

$$\left(\frac{KL}{r}\right)_m = \sqrt{\left(\frac{KL}{r}\right)_o^2 + 0.82 \left(\frac{\alpha^2}{(1 + \alpha)^2} \frac{a}{r_{hb}}\right)^2}$$

$$= \sqrt{(56.8)^2 + 0.82 \left(\frac{(1.26)^2}{1 + (1.26)^2} \frac{32.0 \text{ in.}}{1.05 \text{ in.}}\right)^2} = 60.7 < 134$$

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)_m} = \frac{\pi^2(29,000 \text{ ksi})}{(60.7)^2} = 77.7 \text{ ksi}$$

$$F_{cry} = \left[0.658 F_e\right] F_e = \left[0.658 \frac{36 \text{ ksi}}{77.7 \text{ ksi}}\right](36 \text{ ksi}) = 29.7 \text{ ksi}$$
\[
F_{ce} = \frac{GJ}{A_p r_s^2} = \frac{(11,200 \text{ ksi})(2 \text{ angles})(0.132 \text{ in.}^4)}{(5.34 \text{ in.})(2.33 \text{ in.})^3} = 102 \text{ ksi}
\]

Eqn. E4-3

\[
F_c = \left( \frac{F_{ce} + F_{ce}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{ce}F_{ce}H}{(F_{ce} + F_{ce})^2}} \right]
\]

Eqn. E4-2

\[
= \left( \frac{29.7 \text{ ksi} + 102 \text{ ksi}}{2(0.813)} \right) \left[ 1 - \sqrt{1 - \frac{4(29.7 \text{ ksi})(102 \text{ ksi})(0.813)}{(29.7 \text{ ksi} + 102 \text{ ksi})^2}} \right] = 27.8 \text{ ksi}
\]

does not control

\[
P_n = F_c A_x = (26.4 \text{ ksi})5.34 \text{ in}^2 = 141 \text{ kips}
\]

Eqn. E4-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_c = 0.90)</td>
<td>(\Omega_c = 1.67)</td>
</tr>
<tr>
<td>(\phi_c P_n = 0.90(141 \text{ kips}) = 127 \text{ kips})</td>
<td>(P_n / \Omega_c = \frac{141 \text{ kips}}{1.67} = 84.4 \text{ kips})</td>
</tr>
<tr>
<td>&gt; 120 kips o.k.</td>
<td>&gt; 80.0 kips o.k.</td>
</tr>
</tbody>
</table>

Section E1
Example E.6  Double Angle Compression Member with Slender Elements

Given:
Determine if a 2L5×3×¾ LLBB (¾-in. separation) strut with a length of 8 ft and pinned ends has sufficient available strength to support a dead load of 10 kips and live load of 30 kips in axial compression. Also, calculate the required number of fully tightened or welded intermediate connectors.

Material Properties:
ASTM A36  \( F_y = 36 \text{ ksi} \quad F_u = 58 \text{ ksi} \)  

Geometric Properties:
2L5×3×4 LLBB  
\( r_x = 0.652 \text{ in. (single angle)} \)  
\( r_x = 1.62 \text{ in.} \quad r_y = 1.19 \text{ in. for } \frac{1}{4} \text{ inch separation} \)  
\( r_y = 1.33 \text{ in. for } \frac{3}{4} \text{ inch separation} \)

Calculate the required strength

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
P_e = 1.2(10.0 \text{kips}) + 1.6(30.0 \text{kips}) = 60.0 \text{kips} & P_e = 10.0 \text{kips} + 30.0 \text{kips} = 40.0 \text{kips} \\
\hline
\end{array}
\]

Table Solution:

\( K = 1.0 \)

From the upper portion of Manual Table 4-9, the available strength for buckling about the x-x axis, with \((KL)_x = 8 \text{ ft}\) is

\[
\phi_xP_{ax} = 87.2 \text{kips} > 60.0 \text{kips} \quad \text{o.k.} \]

\[
P_{ax}/\Omega_x = 58.0 \text{kips} > 40.0 \text{kips} \quad \text{o.k.}
\]

For buckling about the y-y axis, the tabulated values are based on a separation of \(\frac{3}{4} \text{ in.}\). To adjust for a spacing of \(\frac{1}{4} \text{ in.}\), \((KL)_y\), is multiplied by the ratio of \(r_y\) for a \(\frac{3}{4}\)-in. separation to \(r_y\) for a \(\frac{1}{4}\)-in. separation.

\[
(KL)_y = 1.0(8.0 \text{ ft})\left(\frac{1.19 \text{ in.}}{1.33 \text{ in.}}\right) = 7.16 \text{ ft}
\]

This calculation of the equivalent \((KL)_y\) does not completely take into account the effect of section E6.1 and is slightly unconservative.

From the tabulated values in the lower portion of Manual Table 4-9, interpolate for a value at \((KL)_y = 7.16 \text{ ft},\)
The available strength in compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi P_{ny} = 65.1 \text{ kips} &gt; 60.0 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$P_{ny} / \Omega_c = 43.3 \text{ kips} &gt; 40.0 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Manual Table 4-9

These strengths are approximate due to the linear interpolation from the Table and the approximate value of the equivalent $(KL)_y$ noted above. These can be compared to the more accurate values calculated in detail below.

Determine the number of intermediate connectors required.

From the tabulated values, it is determined that at least two welded or pretensioned bolted intermediate connectors are required. This can be confirmed by calculation, as follows:

$$a = \text{distance between connectors} = \frac{(8.00 \text{ ft})(12 \text{ in.}/\text{ft})}{3 \text{ spaces}} = 32.0 \text{ in.}$$

The effective slenderness ratio of the individual components of the built-up member based upon the distance between intermediate connectors, $a$, must not exceed three quarters of the controlling slenderness ratio of the overall built-up member.

Therefore,

$$\frac{K a}{r_y} \leq \frac{3}{4} \left( \frac{K L}{r} \right)_{\text{max}}$$

Solving for $a$ gives,

$$a \leq \frac{3r_y \left( \frac{K L}{r} \right)_{\text{max}}}{4K}$$

$$r_y = r_x = 0.652 \text{ in.}$$

$$\frac{K L_x}{r_x} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in.}/\text{ft})}{1.62 \text{ in.}} = 59.3$$

$$\frac{K L_y}{r_y} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in.}/\text{ft})}{1.33 \text{ in.}} = 72.2 \quad \text{controls}$$

Thus,

$$a \leq \frac{3r_y \left( \frac{K L}{r} \right)_{\text{max}}}{4K} = \frac{3(0.652 \text{ in.})(72.2)}{4(1.0)} = 35.3 \text{ in.} > 32.0 \text{ in.} \quad \text{o.k.}$$

The governing slenderness ratio used in the calculations of the Manual Table include the effects of the provisions of Section E6.1 and is slightly higher as a result. See below for these calculations. As a result, the maximum connector spacing calculated here is slightly conservative.

Available strength values can be verified by hand calculations, as shown below.

**Calculation Solution:**

**Geometric Properties:**

$2\text{L5x3x3/4 LLBB} \quad A_g = 3.88 \text{ in.}^2 \quad r_x = 1.33 \text{ in.} \quad r_y = 2.59 \text{ in.} \quad H = 0.657$

$J = 0.0438 \text{ in.}^4 \quad (\text{single angle}) \quad r_y = 0.853 \text{ in.} \quad (\text{single angle})$

$\bar{x} = 0.648 \text{ in.} \quad (\text{single angle})$

Manual Tables 1-15, 1-7 and 4-9
Determine if the section is noncompact or slender

\[
\frac{b}{t} = \frac{5.00 \text{ in.}}{0.250 \text{ in.}} = 20.0
\]

Calculate the limiting width-thickness ratios

\[
\lambda_c = 0.45 \sqrt{\frac{E}{F_y}} = 0.45 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 12.8 < 20.0 \quad \text{Therefore, the angle has a slender element.}
\]

For a double angle compression member with slender elements, Specification Section E7 applies. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional, and flexural-torsional buckling. Depending on the elastic critical buckling stress, \( F_{cr} \) of the member, \( F_{cr} \) will be determined by Specification E7-2 or E7-3.

Determine \( Q \), the slender element reduction factor

\[
Q = Q_s (Q_a = 1.0) \quad \text{for members composed of unstiffened slender elements.}
\]

Calculate \( Q_s \) for the angles individually using Specification Section E7.1c

\[
0.45 \sqrt{\frac{E}{F_y}} = 0.45 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 12.8 < 20.0
\]

\[
0.91 \sqrt{\frac{E}{F_y}} = 0.91 \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 25.8 > 20.0
\]

Therefore, Specification Equation E7-11 applies.

\[
Q_s = 1.34 - 0.76 \left( \frac{b}{t} \right) \sqrt{\frac{F_y}{E}} = 1.34 - 0.76 \left( \frac{20.0}{36 \text{ ksi}} \sqrt{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} \right) = 0.804
\]

\( Q_s = 1.0 \) (no stiffened elements)

Therefore, \( Q_s Q_w = 0.804 \times (1.0) = 0.804 \)

Determine the applicable equation for the critical stress, \( F_{cr} \)

From above, \( K = 1.0 \)

Specification Equation E7-2 requires the computation of \( F_{cr} \). For singly symmetric members, Specification Equations E3-4 and E4-5 apply.
Check x-x axis flexural buckling

\[
\frac{K_x L}{r_x} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.62 \text{ in.}} = 59.3
\]

\[
F_x = \frac{\pi^2 E}{K_x L} \left(\frac{r_x}{K_x L}\right)^2 = \frac{\pi^2 (29,000 \text{ ksi})}{(59.3)^2} = 81.4 \text{ ksi}
\]

**does not govern**

Eqn. E3-4

Check torsional and flexural-torsional buckling

\[
\frac{K_y L}{r_y} = \frac{1.0(8.0 \text{ ft})(12.0 \text{ in./ft})}{1.33 \text{ in.}} = 72.2
\]

**Compute the modified** \( \frac{KL}{r_y} \) **for built up members with fully tightened or welded connectors**

Section E6

\[a = 96.0 \text{ in.} / 3 = 32.0 \text{ in.}\]

\[r_{ob} = r_y \text{ (single angle)} = 0.853 \text{ in.}\]

\[
\alpha = \frac{h}{2r_{ob}} = \frac{2\bar{x} + 0.750 \text{ in.}}{2r_y} = \frac{2(0.648 \text{ in.}) + 0.750 \text{ in.}}{2(0.853 \text{ in.})} = 1.20
\]

\[
\left(\frac{KL}{r_y}\right)_m = \sqrt{\left(\frac{KL}{r_y}\right)_a^2 + 0.82 \frac{\alpha^2}{(1 + \alpha)^2} \left(\frac{a}{r_{ob}}\right)^2}
\]

\[
= \sqrt{(72.2)^2 + 0.82 \frac{(1.20)^2}{1 + (1.20)^2} \left(\frac{32.0 \text{ in.}}{0.853 \text{ in.}}\right)^2} = 76.8
\]

\[
F_{ey} = \frac{\pi^2 E}{\left(\frac{KL}{r_y}\right)_m} = \frac{\pi^2 (29,000 \text{ ksi})}{(76.8)^2} = 48.6 \text{ ksi}
\]

Eqn. E4-10

\[
F_{ec} = \frac{\pi^2 E C_u + GJ}{K_y L} \left(\frac{1}{A_y F_{o}^2}\right)
\]

Eqn. E4-11

For double angles, omit term with \( C_u \) per User Note at end of Section E4.

\[
F_{ec} = \frac{GJ}{A_y F_{o}^2} = \frac{(11,200 \text{ ksi})(2 \text{ angles})(0.0438 \text{ in.}^4)}{3.88 \text{ in.}(2.59 \text{ in.})^2} = 37.7 \text{ ksi}
\]

\[
F_e = \left(\frac{F_{ey} + F_{ec}}{2H}\right) \left[1 - \frac{4F_{ey}F_{ec} H}{\left(F_{ey} + F_{ec}\right)^2}\right]
\]

Eqn. E4-5


\[
E = \left[ \frac{48.6 \text{ ksi} + 37.7 \text{ ksi}}{2(0.657)} \right] \left[ 1 - \sqrt{1 - \frac{4(48.6 \text{ ksi})(37.7 \text{ ksi})(0.657)}{(48.6 \text{ ksi} + 37.7 \text{ ksi})^2}} \right]
\]

= 26.6 ksi

\[
0.44QF_y = 0.44(0.804)(36 \text{ ksi}) = 12.7 \text{ ksi} < 26.6 \text{ ksi}, \text{ therefore Equation E7-2 applies.}
\]

\[
F_{cr} = Q \left[ 0.658 \frac{QF_y}{F_y} \right] = 0.804 \left[ 0.658 \frac{0.804 (36 \text{ ksi})}{26.6 \text{ ksi}} \right] = 18.4 \text{ ksi}
\]

\[
P_a = F_{cr}A_e = (18.4 \text{ ksi})3.88 \text{ in.}^2 = 71.3 \text{ kips}
\]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi_c = 0.90 & \Omega_c = 1.67 \\
\phi_cP_a = 0.90(71.3 \text{ kips}) & \frac{P_a}{\Omega_c} = \frac{71.3 \text{ kips}}{1.67} \\
= 64.1 \text{ kips} > 60.0 \text{ kips} & = 42.7 \text{ kips} > 40.0 \text{ kips} \\
\hline
\end{array}
\]

Section E1
Example E.7  Design of a WT Compression Member without Slender Elements

**Given:**
Select a WT-shape compression member with a length of 20 ft to support a dead load of 20 kips and live load of 60 kips in axial compression. The ends are pinned.

Because WT sections are cut from ASTM A992 W-shape beams, the material properties are:

**Material Properties:**
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

**Calculate the required strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips}) = 120 \text{ kips} )</td>
<td>( P_u = 20.0 \text{ kips} + 60.0 \text{ kips} = 80.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Table Solution:**

\( K = 1.0 \) Therefore \( (KL)_x = (KL)_y = 20.0 \text{ ft} \)

Select the lightest member from Table 4-7 with sufficient available strength about the both the \( x-x \) (upper portion of the table) and the \( y-y \) axis (lower portion of the table) to support the required strength.

Try a WT7×34

The available strength in compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_x P_{xx} = 128 \text{ kips} &gt; 120 \text{ kips} \text{ Controls} \text{ o.k.} )</td>
<td>( P_{xx} / \Omega_x = 85.2 \text{ kips} &gt; 80.0 \text{ kips} \text{ Controls} \text{ o.k.} )</td>
</tr>
<tr>
<td>( \phi_y P_{yy} = 221 \text{ kips} &gt; 120 \text{ kips} \text{ o.k.} )</td>
<td>( P_{yy} / \Omega_y = 147 \text{ kips} &gt; 80.0 \text{ kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below.

**Calculation Solution:**

**Geometric Properties:**

<table>
<thead>
<tr>
<th>WT7×34</th>
<th>( A_y = 9.99 \text{ in.}^2 )</th>
<th>( r_y = 1.81 \text{ in.} )</th>
<th>( r = 2.46 \text{ in.} )</th>
<th>( r_y = 3.19 \text{ in.} )</th>
<th>( J = 1.50 \text{ in.}^4 )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( H = 0.916 )</td>
<td>( d = 7.02 \text{ in.} )</td>
<td>( t_w = 0.415 \text{ in.} )</td>
<td>( b_y = 10.0 \text{ in.} )</td>
<td>( t_f = 0.720 \text{ in.} )</td>
</tr>
</tbody>
</table>

Manual Table 1-8
Check for slender elements

\[ \frac{d}{r_w} = \frac{7.02 \text{ in.}}{0.415 \text{ in.}} = 16.9 < 0.75 \sqrt{\frac{E}{F_y}} = 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 18.1 \]

therefore, the web is not slender.  

Table B4.1  
Case 8

\[ \frac{b_f}{2t_f} = \frac{10 \text{ in.}}{2(0.720 \text{ in.})} = 6.94 < 0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 13.5 \]

therefore, the flange is not slender.  

Table B4.1  
Case 3

There are no slender elements.

For compression members without slender elements, Specification Sections E3 and E4 apply. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional, and flexural-torsional buckling.

Check flexural buckling about the x-x axis

\[ \frac{KL}{r_x} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{1.81 \text{ in.}} = 133 \]

Section E3

\[ 4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 113 < 133 \]

therefore, Specification Equation E3-3 applies.

\[ F_x = \frac{E}{KL r} = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(133)^2} = 16.3 \text{ ksi} \]

Eqn. E3-4

\[ F_{cr} = 0.877 F_x = 0.877(16.3 \text{ ksi}) = 14.3 \text{ ksi} \]

controls  
Eqn. E3-3

Because the WT7\times34 section does not have any slender elements, Specification Section E4 will be applicable for torsional and flexural-torsional buckling. \( F_{cr} \) will be calculated using Specification Equation E4-2

Calculate \( F_{cry} \)

\( F_{cry} \) is taken as \( F_{cr} \) From Specification Section E3, where \( \frac{KL}{r} = \frac{KL}{r_x} \)

\[ \frac{KL}{r_x} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{2.46 \text{ in.}} = 97.6 < 113 \]

therefore, Eqn. E3-2 applies.

\[ F = \frac{\pi^2 E}{KL r} = \frac{\pi^2 (29,000 \text{ ksi})}{(97.6)^2} = 30.1 \text{ ksi} \]

Eqn. E3-4

\[ F_{cry} = F_{cr} = 0.658 F_x = 0.658 \left( \frac{30.0 \text{ ksi}}{50 \text{ ksi}} \right) = 24.9 \text{ ksi} \]

Eqn. E3-2
\[ F_{crx} = \frac{GJ}{A_f r_c^2} = \frac{(11,200 \text{ ksi})(1.50 \text{ in.}^4)}{(9.99 \text{ in.}^2)(3.19 \text{ in.})^2} = 165 \text{ ksi} \]  
Eqn. E4-3

\[ F_{cr} = \left(\frac{F_{crx} + F_{crz}}{2H}\right) \left[1 - \sqrt{1 - \frac{4F_{cr}F_{crx}H}{(F_{cr} + F_{crx})^2}}\right] \]  
Eqn. E4-2

\[ = \left(\frac{24.9 \text{ ksi} + 165 \text{ ksi} - 2(0.916)}{2(0.916)}\right) \left[1 - \sqrt{1 - \frac{4(24.9 \text{ ksi})(165 \text{ ksi})(0.916)}{(24.9 \text{ ksi} + 165 \text{ ksi})^2}}\right] \]

\[ = 24.5 \text{ ksi} \quad \text{does not control} \]

x-x axis flexural buckling governs, therefore

\[ P_n = F_{cr} A_f = (14.3 \text{ ksi})(9.99 \text{ in.}^2) = 143 \text{ kips} \]  
Eqn. E3-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi \frac{P_n}{\sigma} = 0.90(143 \text{ kips}) )</td>
<td>( \frac{P_n}{\Omega_c} = \frac{143 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>( = 128 \text{ kips} &gt; 120 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 85.4 \text{ kips} &gt; 80.0 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Section E1
Example E.8  Design of a WT Compression Member with Slender Elements

Given:
Select a WT-shape compression member with a length of 20 ft to support a dead load of 5 kips and live load of 15 kips in axial compression. The ends are pinned.

Because WT sections are cut from ASTM A992 W-shape beams, the material properties are:

Material Properties:
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 1.2(5.00 \text{ kips}) + 1.6(15.0 \text{ kips}) = 30.0 \text{ kips} )</td>
<td>( P_a = 5.00 \text{ kips} + 15.0 \text{ kips} = 20.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

Table Solution:

\( K = 1.0 \), therefore \( (KL)_x = (KL)_y = 20 \text{ ft} \).

Select the lightest member from Table 4-7 with sufficient available strength about both the \( x-x \) (upper portion of the table) and the \( y-y \) axis (lower portion of the table) to support the required strength.

Try a WT7×15

Determine the available strength in axial compression from Manual Table 4-7

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi, P_{as} = 66.6 \text{ kips} &gt; 30.0 \text{ kips} ) o.k.</td>
<td>( P_{as}/\Omega_x = 44.3 \text{ kips} &gt; 20.0 \text{ kips} ) o.k.</td>
</tr>
<tr>
<td>( \phi, P_{as} = 36.5 \text{ kips} &gt; 30.0 \text{ kips} ) controls o.k.</td>
<td>( P_{as}/\Omega_y = 24.3 \text{ kips} &gt; 20.0 \text{ kips} ) controls o.k.</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below.

Calculation Solution:

Geometric Properties:
WT7×15  \( A_g = 4.42 \text{ in.}^2 \)  \( r_x = 2.07 \text{ in.} \)  \( r_y = 1.49 \text{ in.} \)  \( \overline{r} = 2.90 \text{ in.} \)  \( J = 0.190 \text{ in.}^4 \)

\( H = 0.772 \)  \( Q_x = 0.609 \)  \( d = 6.92 \text{ in.} \)  \( t_w = 0.270 \text{ in.} \)  \( b_f = 6.73 \text{ in.} \)
Check for slender elements

\[
d = \frac{6.92 \text{ in.}}{0.270 \text{ in.}} = 25.6 > 0.75 \sqrt{\frac{E}{F_y}} = 0.75 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 18.1 \quad \text{therefore, the web is slender.}
\]

\[
\frac{b_y}{2t_y} = \frac{6.73 \text{ in.}}{2(0.385 \text{ in.})} = 8.74 < 0.56 \sqrt{\frac{E}{F_y}} = 0.56 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 13.4
\]

Therefore, the flange is not slender.

Because this WT7\times15 has a slender web, Specification Section E7 is applicable. The nominal compressive strength, \( P_n \), shall be determined based on the limit states of flexural, torsional, and flexural-torsional buckling.

Calculate the \( x-x \) axis critical elastic flexural buckling stress

\[
K_{xL} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{2.07 \text{ in.}} = 116
\]

\[
F_{x} = \frac{\pi^2 E}{(KL)^2} = \frac{\pi^2(29,000 \text{ ksi})}{(116)^2} = 21.3 \text{ ksi} \quad \text{does not control}
\]

Calculate the critical elastic torsional and flexural-torsional buckling stress

\[
K_{yL} = \frac{1.0(20.0 \text{ ft})(12 \text{ in./ft})}{1.49 \text{ in.}} = 161
\]

\[
F_{xy} = \frac{\pi^2 E}{(K_{yL})^2} = \frac{\pi^2(29,000 \text{ ksi})}{(161)^2} = 11.0 \text{ ksi}
\]

\[
F_{xy} = \left( \frac{\pi^2 E C_{w}}{(K_{yL})^2} + GJ \right) \frac{1}{A_y F_o} \quad \text{Omit term with \( C_{w} \) per User Note at end of Section E4}
\]

\[
F_{xy} = \frac{GJ}{A_y F_o} = \frac{11,200 \text{ ksi}(0.190 \text{ in.}^4)}{4.42 \text{ in.}^2(2.90 \text{ in.})^2} = 57.2 \text{ ksi}
\]

\[
F_e = \left( \frac{F_{xy} + F_{xy}}{2H} \right) \left[ 1 - \frac{4F_{xy}F_{xy}H}{(F_{xy} + F_{xy})^2} \right]
\]

Eqn. E4-5
\[ F_c = \left( \frac{11.0 \text{ ksi} + 57.2 \text{ ksi}}{2(0.772)} \right) \left[ 1 - \frac{4(11.0 \text{ ksi})(57.2 \text{ ksi})(0.772)}{(11.0 \text{ ksi} + 57.2 \text{ ksi})^2} \right] = 10.5 \text{ ksi} \]

The cross section is composed of only unstiffened compression elements. Therefore, \( Q_u = 1.0 \)

\[ Q = Q_u (Q_u = 1.0) = 0.609 \]

**Check limit for the applicable equation**

\[ 0.44 Q F_y = 0.44(0.609)(50 \text{ ksi}) = 13.4 \text{ ksi} > 10.5 \text{ ksi} \quad \text{therefore Eqn. E7-3 applies} \]

\[ F_{cc} = 0.877 F_c = 0.877(10.5 \text{ ksi}) = 9.21 \text{ ksi} \]

\[ P^u = F_{cc} A_g = (9.21 \text{ ksi})4.42 \text{ in.}^2 = 40.7 \text{ kips} \]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi_c = 0.90 & \Omega_c = 1.67 \\
\phi_c P^u = 0.90 (40.7 \text{ kips}) & P^u / \Omega_c = 40.7 \text{ kips}/1.67 \\
36.5 \text{ kips} > 30.0 \text{ kips} \textbf{ o.k.} & 24.3 \text{ kips} > 20.0 \text{ kips} \textbf{ o.k.} \\
\hline
\end{array}
\]
Example E.9  Design of a Rectangular HSS Compression Member without Slender Elements

Given:
Select a rectangular HSS compression member, with a length of 20 ft, to support a dead load of 85 kips and live load of 255 kips in axial compression. The base is fixed and the top is pinned.

Material Properties:
ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \quad F_u = 58 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2 \times 85.0 \text{ kips} + 1.6 \times 255 \text{ kips} = 510 \text{ kips} )</td>
<td>( P_u = 85.0 \text{ kips} + 255 \text{ kips} = 340 \text{ kips} )</td>
</tr>
</tbody>
</table>

Table Solution:

\( K = 0.8 \)  Commentary

\( (KL)_r = (KL)_y = 0.8 \times (20.0 \text{ ft}) = 16.0 \text{ ft} \)

Enter Manual Table 4-3 for rectangular sections or Table 4-4 for square sections.
Try an HSS12×10×\( \frac{1}{2} \)

Determine the available strength in axial compression

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \Phi \times P_u = 673 \text{ kips} &gt; 510 \text{ kips} )  ( \text{o.k.} )</td>
<td>( P_u / \Omega_y = 448 \text{ kips} &gt; 340 \text{ kips} )  ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below.

Calculation Solution:

Geometric Properties:
HSS12×10×\( \frac{1}{2} \)  \( A_g = 19.0 \text{ in.}^2 \)  \( r_x = 4.56 \text{ in.} \)  \( r_y = 3.96 \text{ in.} \)  \( t_{des} = 0.465 \text{ in.} \)
Check for slender elements

Note: if the corner radius is not known, \( b \) and \( h \) shall be taken as the outside dimension less three times the design wall thickness. This is generally a conservative assumption.

Calculate \( b/t \) of the most slender wall

\[
\frac{h}{t} = \frac{12.0 \text{ in.} - 3(0.465 \text{ in.})}{0.465 \text{ in.}} = 22.8 < 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2
\]

Therefore, the section does not contain slender elements.

Since \( r_y < r_x \) and \( (KL)_y = (KL)_x \), \( r_y \) will govern the available strength.

Determine the applicable equation

\[
\frac{K_yL}{r_y} = \frac{0.8(20.0 \text{ ft})(12 \text{ in./ft})}{3.96 \text{ in.}} = 48.5
\]

\[4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 118 > 48.5, \text{ therefore, use Specification Equation E3-2.}\]

\[
F_y = \frac{\pi^2 E}{(KL)^2} = \frac{\pi^2 (29,000) }{(48.5)^2} = 122 \text{ ksi}
\]

\[
F_{cr} = \left( 0.658 \frac{E}{F_y} \right) F_y = \left( 0.658 \frac{46 \text{ ksi}}{46 \text{ ksi}} \right) (46 \text{ ksi}) = 39.3 \text{ ksi}
\]

\[
P_c = F_{cr}A_b = (39.3 \text{ ksi}) (19.0 \text{ in.}^2) = 746 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c P_a = 0.90\ (746 \text{ kips}) )</td>
<td>( P_a / \Omega_c = \frac{746 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>= 673 \text{ kips} &gt; 510 \text{ kips} o.k.</td>
<td>= 448 \text{ kips} &gt; 340 \text{ kips} o.k.</td>
</tr>
</tbody>
</table>

Specification 

Section E1
Example E.10  Design of a Rectangular HSS Compression Member with Slender Elements

Given:
Select a rectangular HSS12×8 compression member with a length of 30 ft, to support an axial dead load of 26 kips and live load of 77 kips. The base is fixed, the top is pinned.

A column with slender elements has been selected to demonstrate the design of such member.

Material Properties:
ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2 (26.0 \text{ kips}) + 1.6 (77.0 \text{ kips}) = 154 \text{ kips} )</td>
<td>( P_u = 26.0 \text{ kips} + 77.0 \text{ kips} = 103 \text{ kips} )</td>
</tr>
</tbody>
</table>

Table Solution:

\( K = 0.8 \)  Therefore \((KL)_x = (KL)_y = 0.8(30.0 \text{ ft}) = 24.0 \text{ ft} \)

Enter Manual Table 4-3, in the HSS12×8 section and proceed across the page until the lightest section is found with an available strength that is equal to or greater than the required strength, in this case a HSS 12×8×x.

Determine the available strength in axial compression

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_u P_u = 155 \text{ kips} &gt; 154 \text{ kips} ) o.k.</td>
<td>( P_u / \Omega_v = 103 \text{ kips} \geq 103 \text{ kips} ) o.k.</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below, including adjustments for slender elements.

Calculation Solution:

Geometric Properties:
HSS12×8×\( \sqrt{16} \)  \( A_g = 6.76 \text{ in.}^2 \)  \( r_x = 4.56 \text{ in.} \)  \( r_y = 3.35 \text{ in.} \)  \( \frac{h}{t} = 66.0 \)  \( t_{des} = 0.174 \text{ in.} \)
Determine for slender elements

Calculate the limiting width-thickness ratios

\[ \lambda_e = 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2 < 43.0 \]  

therefore both the 10-in. and 12-in. walls are slender elements.

Note that for determining the width-thickness ratio, \( b \) is taken as the outside dimension minus three times the design wall thickness.

For the selected shape

\[ b = 8.00 \text{ in.} - 3(0.174 \text{ in.}) = 7.48 \text{ in.} \]

\[ h = 12.0 \text{ in.} - 3(0.174 \text{ in.}) = 11.5 \text{ in.} \]

For an HSS member with slender elements, the nominal compressive strength, \( P_n \), shall be determined based upon the limit states of flexural buckling. Torsional buckling will not govern unless the torsional unbraced length greatly exceeds the controlling flexural unbraced length.

**Compute effective area, \( A_{eff} \)**

\[ Q_a = \frac{A_{eff}}{A} \]

where

\( A_{eff} \) = summation of the effective areas of the cross section based on the reduced effective widths \( b_e \).

For flanges of square and rectangular slender element section of uniform thickness,

\[ b_e = 1.92t \sqrt{\frac{E}{f}} \left[ 1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{F_y}} \right] \leq b \]

where \( f = P_n / A_{eff} \), but can conservatively be taken as \( F_y \)

For the 8-in. walls,

\[ b_e = 1.92t \sqrt{\frac{E}{F_y}} \left[ 1 - \frac{0.38}{(b/t)} \sqrt{\frac{E}{F_y}} \right] \]

\[ = 1.92(0.174 \text{ in.}) \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \left[ 1 - \frac{0.38}{(43.0)} \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} \right] = 6.53 \text{ in.} \leq 7.48 \text{ in.} \]

Length that can not be used = \( b - b_e = 7.48 \text{ in.} - 6.53 \text{ in.} = 0.950 \text{ in.} \)

For the 12 inch walls.
\[
b_{c} = 1.92\sqrt{\frac{E}{F_{y}}} \left[ 1 - 0.38 \frac{E}{F_{y}} \right] = 1.92(0.174\text{ in.}) \sqrt{\frac{29,000\text{ ksi}}{46\text{ ksi}}} \left[ 1 - 0.38 \frac{29,000\text{ ksi}}{(66.0) \text{ ksi}} \right] = 7.18\text{ in.} \leq 11.5\text{ in.} \quad \text{Eqn. E-16}
\]

Length that can not be used = 11.5 in. – 7.18 in. = 4.32 in.

Therefore \( A_{\text{eff}} = 6.76\text{ in.}^2 - 2(0.174\text{ in.})(0.950\text{ in.}) - 2(0.174\text{ in.})(4.32\text{ in.}) = 4.93\text{ in.}^2 \)

Therefore \( Q = Q_{a} = \frac{A_{\text{eff}}}{A} = \frac{4.93\text{ in.}^2}{6.76\text{ in.}^2} = 0.729 \)

**Determine the appropriate equation for \( F_{c,r} \)**

\[
\frac{K_{L}}{r_{y}} = \frac{0.8(30.0\text{ ft})(12\text{ in./ft})}{3.35\text{ in.}} = 86.0 < 4.71 \sqrt{\frac{E}{QF_{y}}} = 4.71 \sqrt{\frac{29,000\text{ ksi}}{0.729(46\text{ ksi})}} = 139
\]

Therefore \( F_{c,r} = Q \left[ 0.658 \frac{QF_{y}}{F_{y}} \right] F_{y} \quad \text{Eqn. E7-2} \)

\[
F_{c} = \frac{\pi^{2}E}{(KL)^{2}} = \frac{\pi^{2}(29,000\text{ ksi})}{86.0^{2}} = 38.7\text{ ksi} \quad \text{Eqn. E3-4}
\]

\[
F_{c,r} = Q \left[ 0.658 \frac{QF_{y}}{F_{y}} \right] F_{y} = 0.729 \left[ 0.658 \frac{0.729(46\text{ ksi})}{38.7\text{ ksi}} \right] 46\text{ ksi} = 23.3\text{ ksi.} \quad \text{Eqn. E7-2}
\]

\[
P_{n} = F_{c,r}A_{g} = 23.3\text{ ksi}(6.76\text{ in.}^2) = 158\text{ kips} \quad \text{Eqn. 7-1}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_{e} = 0.90 )</td>
<td>( \Omega_{e} = 1.67 )</td>
</tr>
<tr>
<td>( \phi_{e}P_{n} = 0.90(158\text{ kips}) = 142\text{ kips} &lt; 154\text{ kips} )</td>
<td>( P_{n}/\Omega_{e} = \frac{158\text{ kips}}{1.67} = 94.7\text{ kips} &lt; 103\text{ kips} )</td>
</tr>
</tbody>
</table>

**See note below**

Note: A conservative initial assumption \( (f = F_{y}) \) was made in applying Specification Equation E7-18. A more exact solution is obtained by iterating from the Compute effective area, \( A_{\text{eff}} \) step using \( f = P_{n}/A_{\text{eff}} \) until the value of \( f \) converges. The HSS column strength tables in the Manual were calculated using this iterative procedure.
Example E.11  Design of a Pipe Compression Member

Given:
Select a Pipe compression member with a length 30 ft to support a dead load of 35 kips and live load of 105 kips in axial compression. The column is pin-connected at the ends in both axes and braced at the midpoint in the y-y direction.

Material Properties:
ASTM A53 Gr. B  \( F_y = 35 \text{ ksi} \quad F_u = 60 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_e = 1.2(35.0 \text{ kips}) + 1.6(105 \text{ kips}) = 210 \text{ kips} )</td>
<td>( P_u = 35.0 \text{ kips} + 105 \text{ kips} = 140 \text{ kips} )</td>
</tr>
</tbody>
</table>

Table Solution:

\( K = 1.0 \quad \therefore (KL)_x = 30.0 \text{ ft} \quad \text{and} \quad (KL)_y = 15.0 \text{ ft} \quad \text{Buckling about the x-x axis controls.} \)

Enter Manual Table 4-6 with a \( KL \) of 30 ft and proceed across the table until reaching the lightest section with sufficient available strength to support the required strength.

Try a 10 inch Standard Pipe. The available strength in axial compression is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_u = 215 \text{ kips} &gt; 210 \text{ kips} )</td>
<td>( P_u / \Omega = 143 \text{ kips} &gt; 140 \text{ kips} ) o.k.</td>
</tr>
</tbody>
</table>

The available strength can be easily determined by using the tables of the Manual. Available strength values can be verified by hand calculations, as shown below.

Calculation Solution:

Geometric Properties:
Pipe 10 Std. \( A = 11.1 \text{ in.}^2 \quad r = 3.68 \text{ in.} \)

All Steel Pipes shown in Manual Table 4-10 are compact at 35 ksi, so no local buckling check is required.

\[
\frac{KL}{r} = \frac{(30.0 \text{ ft})(12 \text{ in./ft})}{3.68 \text{ in.}} = 97.8
\]
\[ E = 4.71 \sqrt{\frac{F_y}{29,000 \text{ ksi}}} = 136 > 97.8 \]

\[ F_c = \frac{\pi^2 E}{(KL)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(97.8)^2} = 29.9 \text{ ksi} \]

\[ F_{cr} = \left( 0.658 \frac{F_c}{F_y} \right) F_y = \left( 0.658 \frac{35 \text{ ksi}}{29.9 \text{ ksi}} \right) (35 \text{ ksi}) = 21.4 \text{ ksi} \]

\[ P_n = F_{cr} A_y = (21.4 \text{ ksi})(11.1 \text{ in.}^2) = 238 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.90 )</td>
<td>( \Omega_c = 1.67 )</td>
</tr>
<tr>
<td>( \phi_c P_n = 0.90(238 \text{ kips}) )</td>
<td>( P_n / \Omega_c = \frac{238 \text{ kips}}{1.67} = 143 \text{ kips} &gt; 140 \text{ kips} )</td>
</tr>
<tr>
<td>= 215 kips &gt; 210 kips \text{ o.k.}</td>
<td>= 143 kips &gt; 140 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>

Note that the design procedure would be similar for a round HSS column, except that local buckling should also be checked.
Example E.12  Built-up I-Shaped Member with Different Flange Sizes

Given:
Compute the available strength of a built-up compression member with a length of 14 ft. The ends are pinned. The outside flange is PL\(\frac{2}{3}\times5\), the inside flange is PL\(\frac{2}{3}\times8\), and the web is PL\(\frac{2}{3}\times10\frac{1}{2}\). Material is ASTM A572 Grade 50.

Material Properties:
ASTM A572  \(F_y = 50\) ksi  \(F_u = 65\) ksi

Solution:
User note: There are no tables for special built-up shapes.

**Determine if the shape has any slender elements**

**Check outside flange slenderness**

Calculate \(k_c\)

\[
k_c = \frac{4}{\sqrt{\frac{h}{t_w}}} = \frac{4}{\sqrt{\frac{10.5\text{ in.}}{0.375\text{ in.}}}} = 0.756 \quad 0.35 \leq k_c \leq 0.76. \quad \text{o.k.}
\]

\[
\frac{b_t}{t} = \frac{2.50\text{ in.}}{0.75\text{ in.}} = 3.33
\]

\[
\lambda_r = 0.64 \sqrt{\frac{k_c E}{F_y}} = 0.64 \sqrt{\frac{0.756(29,000\text{ ksi})}{50\text{ ksi}}} = 13.4
\]

\[
\frac{b_t}{t} \leq \lambda_r \quad \text{therefore, the outside flange is not slender.}
\]

**Check inside flange slenderness**

\[
\frac{b_t}{t} = \frac{4.0\text{ in.}}{0.750\text{ in.}} = 5.33
\]

\[
\frac{b_t}{t} \leq \lambda_r \quad \text{therefore, the inside flange is not slender.}
\]
Check web slenderness

\[
\frac{h}{t} = \frac{10.5 \text{ in.}}{0.375 \text{ in.}} = 28.0
\]

\[
\lambda = 1.49 \sqrt{\frac{E}{F_y}} = 1.49 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 35.9
\]

\[
\frac{h}{t} < \lambda \quad \text{therefore, the web is not slender.}
\]

Calculate section properties (ignoring welds)

\[
A_w = b_f t_f + h t_w + b_{f2} t_{f2} = (8.00 \text{ in.})(0.750 \text{ in.}) + (10.50 \text{ in.})(0.375 \text{ in.}) + (5.00 \text{ in.})(0.75 \text{ in.})
\]

\[
= 13.7 \text{ in.}^2
\]

\[
\bar{y} = \frac{\sum A_y y}{\sum A_y} = \frac{(6.00 \text{ in.}^2)(11.6 \text{ in.}) + (3.94 \text{ in.}^2)(6.00 \text{ in.}) + (3.75 \text{ in.}^2)(0.375 \text{ in.})}{(6.00 \text{ in.}^2) + (3.94 \text{ in.}^2) + (3.75 \text{ in.}^2)} = 6.92 \text{ in.}
\]

Note that the neutral axis location is measured from the bottom of the outside flange.

\[
I_x = \left[ \left( \frac{8.0 \text{ in.}}{12} \right)^3 (0.75 \text{ in.})^2 + \left( \frac{8.0 \text{ in.}}{12} \right)^3 (4.71 \text{ in.})^2 \right] + \left[ \left( \frac{0.375 \text{ in.}}{12} \right)^3 (10.5 \text{ in.})^2 \right] + \left[ \left( \frac{5.0 \text{ in.}}{12} \right)^3 (0.750 \text{ in.})^2 \right] + \left[ \left( \frac{5.0 \text{ in.}}{12} \right)^3 (6.55 \text{ in.})^2 \right] = 334 \text{ in.}^4
\]

\[
r_x = \sqrt{\frac{I_x}{A_w}} = \sqrt{\frac{334 \text{ in.}^4}{13.7 \text{ in.}^2}} = 4.94 \text{ in.}
\]

\[
I_y = \left[ \left( \frac{0.75 \text{ in.}}{12} \right)^3 (8.0 \text{ in.})^2 \right] + \left[ \left( \frac{10.5 \text{ in.}}{12} \right)^3 (0.375 \text{ in.})^2 \right] + \left[ \left( \frac{0.750 \text{ in.}}{12} \right)^3 (5.0 \text{ in.})^2 \right] = 39.9 \text{ in.}^4
\]

\[
r_y = \sqrt{\frac{I_y}{A_w}} = \sqrt{\frac{39.9 \text{ in.}^4}{13.7 \text{ in.}^2}} = 1.71 \text{ in.}
\]

Calculate x-x axis flexural elastic critical buckling stress, \( F_e \)

\[
\frac{K L}{r_x} = \frac{1.0(14.0 \text{ ft})(12 \text{ in./ft})}{4.94 \text{ in.}} = 34.0
\]

Section E3
Calculate the flexural-torsional critical elastic buckling stress

\[
J = \sum \left( \frac{b^l t^l}{3} \right) = \left( \frac{8.00 \text{ in.}}{3} \right)(0.750 \text{ in.})^3 + \left( \frac{10.5 \text{ in.}}{3} \right)(0.375 \text{ in.})^3 + \left( \frac{5.00 \text{ in.}}{3} \right)(0.750 \text{ in.})^3 = 2.01 \text{ in.}^4
\]

\[
h_v = d - \frac{t_f}{2} = 12.0 \text{ in.} - \frac{0.750 \text{ in.}}{2} - \frac{0.750 \text{ in.}}{2} = 11.3 \text{ in.}
\]

\[
C_w = \frac{t_f h_v^2}{12} \left( \frac{h_1^3}{h_1^3 + h_2^3} \right) = \left( \frac{0.750 \text{ in.}}{12} \right)(11.25 \text{ in.})^2 \left( \frac{(8.00 \text{ in.})^3 (5.00 \text{ in.})^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3} \right) = 795 \text{ in.}^6
\]

Locate the shear center

Due to symmetry, both the centroid and the shear center lie on the y-axis. Therefore \( x_o = 0 \)

\[
e = h_v \left( \frac{b_1^3}{h_1^3 + h_2^3} \right) = 11.3 \text{ in.} \left( \frac{(8.00 \text{ in.})^3}{(8.00 \text{ in.})^3 + (5.00 \text{ in.})^3} \right) = 9.04 \text{ in.}
\]

Note that this shear center location is taken from the center of the outside flange. Therefore, add one-half the flange thickness to determine the shear center location measured from the bottom of the outside flange.

\[
e + \frac{t_f}{2} = 9.04 \text{ in.} + \frac{0.75 \text{ in.}}{2} = 9.42 \text{ in.}
\]

Therefore \( v_o = \left( e + \frac{t_f}{2} \right) - \bar{y} = 9.42 \text{ in.} - 6.92 \text{ in.} = 2.50 \text{ in.} \)

\[
\bar{r}_o^2 = x_o^2 + y_o^2 + \frac{J_y + I_y}{A_y} = 0 + (2.50 \text{ in.})^2 + \frac{334 \text{ in.}^4 + 39.9 \text{ in.}^4}{13.7 \text{ in.}^2} = 33.5 \text{ in.}^2
\]

\[
H = 1 - \frac{x_o^2 + y_o^2}{\bar{r}_o^2} = 1 - \frac{0 + (2.50 \text{ in.})^2}{33.5 \text{ in.}^2} = 0.813
\]

Since the ends are pinned, \( K = 1.0 \)

\[
\frac{K L}{r_y} = \frac{1.0(14.0 \text{ ft})(12.0 \text{ in.}/\text{ft})}{1.71 \text{ in.}} = 98.2
\]

\[
F_{xy} = \frac{\pi^2 E}{\left( \frac{K L}{r_y} \right)^2} = \frac{\pi^2 (29,000 \text{ ksi})}{(98.2)^2} = 29.7 \text{ ksi}
\]
\[ F_{cr} = \frac{\pi^2 EC_w + GJ}{(K,L)^2} \left( \frac{1}{A_b F_y^2} \right) \]

\[ = \frac{\pi^2 (29,000 \text{ ksi})(795 \text{ in}^4)}{(1.0)(14.0 \text{ ft})(12 \text{ in}/\text{ft})^2} + (11,200 \text{ ksi})(2.01 \text{ in}^4) \left( \frac{1}{(13.7 \text{ in}^2)(33.5 \text{ in}^2)} \right) = 66.6 \text{ ksi} \]

\[ F = \left( \frac{F_{cr} + F_{ct}}{2H} \right) \left[ 1 - \sqrt{1 - \frac{4F_{cr} F_{ct} H}{(F_{cr} + F_{ct})^2}} \right] \]

\[ = \left( \frac{29.7 \text{ ksi} + 66.6 \text{ ksi}}{2(0.813)} \right) \left[ 1 - \sqrt{1 - \frac{4(29.7 \text{ ksi})(66.6 \text{ ksi})(0.813)}{(29.7 \text{ ksi} + 66.6 \text{ ksi})^2}} \right] = 26.4 \text{ ksi} \]

**Torsional and flexural-torsional buckling governs**

0.44 \( F_y = 0.44(50 \text{ ksi}) = 22.0 \text{ ksi} < 26.4 \text{ ksi} \) therefore Equation E3-2 applies

\[ F_{cr} = \frac{P}{0.658 F_y} \quad F_y = \frac{50 \text{ ksi}}{0.658(26.4 \text{ ksi})} = 22.6 \text{ ksi} \]

\[ P_{cr} = F_{cr} A_p = (22.6 \text{ ksi})(13.7 \text{ in}^2) = 310 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi_p P = 0.90(310 \text{ kips}) = 279 \text{ kips} )</td>
<td>( P_a / \Omega = \frac{310 \text{ kips}}{1.67} = 186 \text{ kips} )</td>
</tr>
</tbody>
</table>

Section E1
INTRODUCTION

This Specification chapter contains provisions for calculating the flexural strength of members subject to simple bending about one principal axis. Included are specific provisions for I-shaped members, channels, HSS, tees, double angles, single angles, rectangular bars, rounds and unsymmetrical shapes. Also included is a section with proportioning requirements for beams and girders.

There are selection tables in the Manual for standard beams in the commonly available yield strengths. The section property tables for most cross sections provide information that can be used to conveniently identify noncompact and slender sections. LRFD and ASD information is presented side by side.

Most of the formulas from this chapter are illustrated by example below. The design and selection techniques illustrated in the examples for both LRFD and ASD designs are similar to past practices and will result in similar designs.

F1. GENERAL PROVISIONS

Selection and evaluation of all members is based on deflection requirements, and strength, which is determined based on the design flexural strength, $\phi b M_n$, or the allowable flexural strength, $M_n/\Omega_b$, where:

$M_n$ = the lowest nominal flexural strength based on the limit states of yielding, lateral torsional-buckling and local buckling, where applicable

$\phi b = 0.90$ (LRFD)  $\Omega_b = 1.67$ (ASD).

This design approach is followed in all examples.

The term $L_b$ is used throughout this chapter to describe the length between points which are either braced against lateral displacement of the compression flange or braced against twist of the cross section. Requirements for bracing systems and the required strength and stiffness at brace points are given in Specification Appendix 6.

The use of $Cb$ is illustrated in several examples below. Manual Table 3-1 provides tabulated $Cb$ values for many common situations.

F2. DOUBLY-SYMMETRIC COMPACT I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MAJOR AXIS

Section F2 applies to the design of compact beams and channels. As indicated in the User Note in Section F2 of the Specification, the vast majority of rolled I-shaped beams and channels fall into this category. The curve presented as a solid line in Figure F-1 below is a generic plot of the moment capacity, $M_n$, as a function of the unbraced length, $L_b$. The horizontal segment of the curve at the far left, between $L_b = 0$ ft and $L_p$, is the area where the strength is limited by flexural yielding. In this region, the nominal strength is taken as the full plastic moment strength of the section as given by Specification Equation F2-1. In the area of the curve at the far right, starting at $L_r$, the strength is limited by elastic buckling. The strength in this region is given by Specification Equation F2-3. Between these regions, within the linear region of the curve between $M_n = M_p$ at $L_p$ on the left, and $M_n = 0.7 M_p = 0.7F_y S_x$ at $L_r$ on the right, the strength is limited by inelastic buckling. The strength in this region is provided in Specification Equation F2-2.

The curve plotted as a heavy solid line represents the case where $Cb = 1.0$, while the heavy dashed line represents the case where $Cb$ exceeds 1.0. The nominal strengths calculated in both equations F2-2 and F2-3 are linearly proportional to $Cb$, but are limited to not more than $M_p$ as shown in the figure.
\[ M_a = M_p = F_y Z_y \]  

\[ M_a = C_b \left[ M_p - (M_p - 0.70 F_y S_y) \left( \frac{L_u - L_p}{L_u - L_p} \right) \right] \leq M_p \]  

\[ M_a = F_{cr} S_y \leq M_p \] where \( F_{cr} \) is evaluated as shown below  

\[ F_{cr} = \frac{C_r \pi^2 E}{L_o^2} \sqrt{1 + 0.078 \frac{J_c}{S_y h_y} \left( \frac{L_o}{r_o} \right)^2} \]  

The provisions of this section are illustrated in Example F.1 (W-shape beam) and Example F.2 (channel).

Plastic design provisions are given in Appendix 1. \( L_{pd} \), the maximum unbraced length for plastic design is less than \( L_p \).
F3. DOUBLY-SYMMETRIC I-SHAPED MEMBERS WITH COMPACT WEBS AND NONCOMPACT OR SLENDER FLANGES, BENT ABOUT THEIR MAJOR AXIS

The strength of shapes designed according to this section is limited by local buckling of the compression flange. Only a few standard wide flange shapes have noncompact flanges. For these sections, the strength reduction in with $F_y = 50$ ksi steel varies. The approximate percentages of $M_p$ about the strong axis that can be developed by noncompact members when braced such that $L_b \leq L_p$ are shown below:

<table>
<thead>
<tr>
<th>Section</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td>W21×48</td>
<td>99%</td>
</tr>
<tr>
<td>W14×99</td>
<td>99%</td>
</tr>
<tr>
<td>W14×90</td>
<td>96%</td>
</tr>
<tr>
<td>W12×65</td>
<td>98%</td>
</tr>
<tr>
<td>W10×12</td>
<td>99%</td>
</tr>
<tr>
<td>W8×31</td>
<td>99%</td>
</tr>
<tr>
<td>W8×10</td>
<td>99%</td>
</tr>
<tr>
<td>W6×15</td>
<td>94%</td>
</tr>
<tr>
<td>W6×8.5</td>
<td>97%</td>
</tr>
<tr>
<td>M4×6</td>
<td>85%</td>
</tr>
</tbody>
</table>

The strength curve for the flange local buckling limit state, shown in Figure F-2, is similar in nature to that of the lateral-torsional buckling curve. The horizontal axis parameter is $\lambda = \frac{b_f}{2t_f}$. The flat portion of the curve to the left of $\lambda_{pf}$ is the plastic yielding strength, $M_{p}$. The curved portion to the right of $\lambda_{pf}$ is the strength limited by elastic buckling of the flange. The linear transition between these two regions is the strength limited by inelastic flange buckling.

![Figure F-2 Flange Local Buckling Strength](image)

\[ M_n = M_p = F_y Z_n \]  
\[ M_n = \left[ M_p - (M_p - 0.7F_y S_y) \left( \frac{\lambda - \lambda_{pf}}{\lambda_{pf} - \lambda_{pf}} \right) \right] \]  
\[ M_n = \frac{0.9Ek_y S_y}{\lambda^2} \]

The strength reductions due to local flange buckling of the few standard rolled shapes with noncompact flanges are incorporated into the design tables in Chapter 3 of the Manual.

There are no standard I-shaped members with slender flanges. The noncompact flange provisions of this section are illustrated in Example F.3.
F4. OTHER I-SHAPED MEMBERS WITH COMPACT OR NONCOMPACT WEBS, BENT ABOUT THEIR MAJOR AXIS

This section of the Specification applies to doubly symmetric I-shaped members with noncompact webs and singly symmetric I-shaped members (those having different flanges) with compact or noncompact webs.

F5. DOUBLY-SYMMETRIC AND SINGLY-SYMMETRIC I-SHAPED MEMBERS WITH SLENDER WEBS BENT ABOUT THEIR MAJOR AXIS

This section applies to I-shaped members with slender webs, formerly designated as “plate girders”.

F6. I-SHAPED MEMBERS AND CHANNELS BENT ABOUT THEIR MINOR AXIS

I-shaped members and channels bent about their minor axis are not subject to lateral-torsional buckling. Rolled or built up shapes with noncompact or slender flanges, as determined by Specification Table B4.1, must be checked for local flange buckling using Equations F6-2 or F6-3 as applicable.

The vast majority of W-, M-, C-, and MC-shapes have compact flanges, and can therefore develop the full plastic moment, $M_p$ about the minor axis. The provisions of this section are illustrated in Example F.5.

F7. SQUARE AND RECTANGULAR HSS AND BOX-SHAPED MEMBERS

Square and rectangular HSS need only be checked for the limit states of yielding and local buckling. Although lateral-torsional buckling is theoretically possible for very long rectangular HSS bent about the strong axis, deflection will control the design as a practical matter.

The design and section property tables in the Manual were calculated using a design wall thickness of 93% of the nominal wall thickness. Strength reductions due to local buckling have been accounted for in the Manual design tables. The selection of rectangular or square HSS with compact flanges is illustrated in Example F.6. The provisions for rectangular or square HSS with noncompact flanges are illustrated in Example F.7. The provisions for HSS with slender flanges are illustrated in Example F.8.

F8. ROUND HSS AND PIPES

The definition of HSS encompasses both tube and pipe products. The lateral-torsion buckling limit state does not apply, but round HSS are subject to strength reductions from local buckling. Available strengths of round HSS and Pipe are listed in Manual Table 3-14 and 3-15. The tabulated properties and strengths of these shapes in the Manual are calculated using a design wall thickness of 93% of the nominal wall thickness. The design of a round HSS is illustrated in Example F.9.

F9. TEES AND DOUBLE ANGLES LOADED IN THE PLANE OF SYMMETRY

The specification provides a check for flange local buckling, which applies only when the flange is in compression due to flexure. This limit state will seldom govern. No explicit check for local buckling of the web is provided, but the lateral-torsional limit state equation converges to the local buckling limit state strength as the length approaches zero. Thus, this limit state must still be checked for members with very short or zero unbraced length when the tip of the stem is in flexural compression. As noted in the commentary, when the unbraced length is zero, the equation converges to $M_n = 0.424 \frac{EJ}{d}$. When the tip of the tee is in flexural tension and the beam is continuously braced, this limit state need not be checked. Attention should be given to end conditions of tees to avoid inadvertent fixed end moments which induce compression in the web unless this limit state is checked. The design of a WT-shape in bending is illustrated in Example F.10.

F10. SINGLE ANGLES

Section F10 permits the flexural design of single angles using either the principal axes or geometric axes ($x$-$x$ and $y$-$y$ axes). When designing single angles without continuous bracing using the geometric axis design provisions, $M_f$ must be multiplied by 0.80 for use in Equations F10-1, F10-2 and F10-3. The design of a single angle in bending is illustrated in Example F.11.
F11. **RECTANGULAR BARS AND ROUNDS**

There are no design tables in the Manual for these shapes. The local buckling limit state does not apply to any bars. With the exception of rectangular bars bent about the strong axis, solid square, rectangular and round bars are not subject to lateral-torsional buckling and are governed by the yielding limit state only. Rectangular bars bent about the strong axis are subject to lateral torsional buckling and are checked for this limit state with Equations F11-2 and F11-3 where applicable.

These provisions can be used to check plates and webs of tees in connections. A design example of a rectangular bar in bending is illustrated in **Example F.12**. A design example of a round bar in bending is illustrated in **Example F.13**.

F12. **UNSYMMETRICAL SHAPES**

Due to the wide range of possible unsymmetrical cross sections, specific lateral-torsional and local buckling provisions are not provided in this Specification section. A general template is provided, but appropriate literature investigation and engineering judgment are required for the application of this section. A Z-shaped section is designed in **Example F.14**.

F13. **PROPORTIONS FOR BEAMS AND GIRDERS**

This section of the Specification includes a limit state check for tensile rupture due to holes in the tension flange of beams, proportioning limits for I-shaped members, detail requirements for cover plates and connection requirements for beams connected side to side.
Example F.1-1a  
W-Shape Flexural Member Design in Strong-Axis Bending, Continuously Braced.

Given:
Select an ASTM A992 W-shape beam with a simple span of 35 feet. Limit the member to a maximum nominal depth of 18 in. Limit the live load deflection to \( L/360 \). The nominal loads are a uniform dead load of 0.45 kip/ft and a uniform live load of 0.75 kip/ft. Assume the beam is continuously braced.

![Diagram](image)

**Beam Loading & Bracing Diagram**

(full lateral support)

Solution:

Material Properties:
ASTM A992  
\( F_y = 50 \text{ ksi} \)  
\( F_u = 65 \text{ ksi} \)

Calculate the required flexural strength

\[
\begin{align*}
\omega_u &= 1.2(0.450 \text{ kip/ft}) + 1.6(0.750 \text{ kip/ft}) \\
&= 1.74 \text{ kip/ft} \\
M_u &= \frac{1.74 \text{ kip/ft} (35.0 \text{ ft})^2}{8} = 266 \text{ kip-ft} \\
\omega_a &= 0.450 \text{ kip/ft} + 0.750 \text{ kip/ft} \\
&= 1.20 \text{ kip/ft} \\
M_a &= \frac{1.20 \text{ kip/ft} (35.0 \text{ ft})^2}{8} = 184 \text{ kip-ft}
\end{align*}
\]

Calculate the required moment of inertia for live-load deflection criterion of \( L/360 \)

\[
\Delta_{max} = \frac{L}{360} = \frac{35.0 \text{ ft}(12 \text{ in./ft})}{360} = 1.17 \text{ in.}
\]

\[
I_{streq} = \frac{5wL^4}{384E\Delta_{max}} = \frac{5(0.750 \text{ kip/ft})(35.0 \text{ ft})^4(12 \text{ in./ft})^3}{384 (29,000 \text{ ksi})(1.17 \text{ in.})} = 748 \text{ in.}^4
\]

Select a W18×50 from Table 3-2

Per the User Note in Section F2, the section is compact. Since the beam is continuously braced and compact, only the yielding limit state applies.

| \( \phi_o M_o = \phi_o M_{pu} = 379 \text{ kip-ft} > 266 \text{ kip-ft} \) o.k. |
| \( \Omega_o = \frac{M_o}{M_{pu}} = 252 \text{ kip-ft} > 184 \text{ kip-ft} \) o.k. |
| \( I_o = 800 \text{ in.}^4 > 748 \text{ in.}^4 \) o.k. |

Manual
Table 3-2
Diagram 1
Example F.1-1b  W-Shape Flexural Member Design in Strong-Axis Bending, Continuously Braced.

Given:

Example F.1-1a can be easily solved by utilizing the tables of the AISC Steel Construction Manual. Alternatively, this problem can be solved by applying the requirements of the AISC Specification directly.

Solution:

Material Properties:
ASTM A992  \( F_y = 50 \text{ ksi} \quad F_u = 65 \text{ ksi} \)  

Geometric Properties:
W18×50  \( Z_x = 101 \text{ in.}^3 \)  

Required strength from Example F.1-1a

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 266 \text{ kip-ft} )</td>
<td>( M_u = 184 \text{ kip-ft} )</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the nominal flexural strength, \( M_n \)

Per the User Note in Section F2, the section is compact. Since the beam is continuously braced and compact, only the yielding limit state applies.

\[
M_n = M_p = F_y Z_x = 50 \text{ ksi}(101 \text{ in.}^3) = 5050 \text{ kip-in. or 421 kip-ft}
\]  

Eqn. F2-1

Calculate the available flexural strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \phi_b M_n = 0.90(421 \text{ kip-ft}) = 379 \text{ kip-ft} &gt; 266 \text{ kip-ft} \textbf{o.k.} )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n / \Omega_b = (421 \text{ kip-ft})/1.67 = 252 \text{ kip-ft} &gt; 184 \text{ kip-ft} \textbf{o.k.} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Section F1
Example F.1-2a  W-Shape Flexural Member Design in Strong-Axis Bending, Braced at Third Points

Given:
Verify the strength of the W18×50 beam selected in Example F.1-1a if the beam is braced at the ends and third points rather than continuously braced.

Solution:

Required flexural strength at midspan from Example F.1-1a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_a = 266$ kip-ft</td>
<td>$M_a = 184$ kip-ft</td>
</tr>
</tbody>
</table>

$L_o = \frac{35.0}{3} = 11.7$ ft

By inspection, the middle segment will govern. For a uniformly loaded beam braced at the ends and third points, $C_b = 1.01$ in the middle segment. Conservatively neglect this small adjustment in this case.

Obtain the available strength from Table 3-10

Enter Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_bM_a \approx 302$ kip-ft &gt; 266 kip-ft</td>
<td>$\Omega_b \approx 201$ kip-ft &gt; 184 kip-ft</td>
</tr>
</tbody>
</table>

Manual
Table 3-10
Example F.1-2b  W-Shape Flexural Member Design in Strong-Axis Bending, Braced at Third Points

Given:

Example F.1-2a was solved by utilizing the tables of the AISC Steel Construction Manual. Alternatively, this problem can be solved by applying the requirements of the AISC Specification directly.

Solution:

Material Properties:

- ASTM A992
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

Geometric Properties:

- W18×50
  - $S_x = 88.9$ in.$^3$

Required strength from Example F.1-2a

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u$ = 266 kip-ft</td>
<td></td>
<td>$M_a$ = 184 kip-ft</td>
</tr>
</tbody>
</table>

Calculate the nominal flexural strength, $M_n$

Calculate $C_b$

For the lateral-torsional buckling limit state, the nonuniform moment modification factor can be calculated using Specification Equation F1.1.

$$ C_b = \frac{12.5M_{\text{max}}}{2.5M_{\text{max}} + 3M_A + 4M_B + 3M_C} R_m \leq 3.0 $$

For the center segment of the beam, the required moments for Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: $M_{\text{max}} = 1.00$, $M_A = 0.972$, $M_B = 1.00$, $M_C = 0.972$.

$$ R_m = 1.0 \text{ for doubly-symmetric members} $$

$$ C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.972) + 4(1.00) + 3(0.972)} (1.0) = 1.01 $$

For the end-span beam segments, the required moments for Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: $M_{\text{max}} = 0.889$, $M_A = 0.306$, $M_B = 0.556$, and $M_C = 0.750$.

$$ C_b = \frac{12.5(0.889)}{2.5(0.889) + 3(0.306) + 4(0.556) + 3(0.750)} (1.0) = 1.46 $$
Thus, the center span, with the higher required strength and lower $C_b$, will govern.

$L_p = 5.83$ ft
$L_r = 17.0$ ft

Note: The more conservative formula for $L_r$ given in the User Note in Specification Section F2 can yield very conservative results.

For a compact beam with an unbraced length of $L_p < L_b \leq L_r$, the lesser of either the flexural yielding limit-state or the inelastic lateral-torsional buckling limit-state controls the nominal strength.

$M_p = 5050$ kip-in. (from Example F.1-2a)

$$M_a = C_b \left( M_p - (M_p - 0.7F_i S_i) \left( \frac{L_r - L_p}{L_r - L_p} \right) \right) \leq M_p$$

Eqn. F2-2

$$M_a = 1.01 \left[ 5050 \text{kip-in.} - \left( 5050 \text{kip-in.} - 0.7 \left( 50 \text{ksi} \right) \left( 88.9 \text{ in.}^3 \right) \left( \frac{11.7 \text{ ft} - 5.83 \text{ ft}}{17.0 \text{ ft} - 5.83 \text{ ft}} \right) \right) \right]$$

$\leq 5050 \text{kip-in.}$

Calculate the available flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_a = 0.90 \times 339 \text{ kip-ft}$</td>
<td>$M_a / \Omega_b = (339 \text{ kip-ft}) / 1.67$</td>
</tr>
<tr>
<td>= 305 kip-ft &gt; 266 kip-ft <strong>o.k.</strong></td>
<td>= 203 kip-ft &gt; 184 kip-ft <strong>o.k.</strong></td>
</tr>
</tbody>
</table>
Example F.1-3a.  W-Shape Flexural Member design in Strong-Axis Bending, Braced at Midspan

Given:

Verify the strength of the W18×50 beam selected in Example F.1-1a if the beam is braced at the ends and center point rather than continuously braced.

\[ w_0 = 0.45 \text{ kip/ft} \]
\[ w_1 = 0.75 \text{ kip/ft} \]

35 ft

Beam Loading & Bracing Diagram
(bracing at ends & midpoint)

Solution:

Required flexural strength at midspan from Example F.1-1a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_a = 266 \text{ kip-ft} )</td>
<td>( M_a = 184 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

\[ L_b = \frac{35.0 \text{ ft}}{2} = 17.5 \text{ ft} \]

For a uniformly loaded beam braced at the ends and at the center point, \( C_b = 1.30 \). There are several ways to make adjustments to Table 3-10 to account for \( C_b \) greater than 1.0.

Procedure A.

Available moments from the sloped and curved portions of the plots in from Manual Table 3-10 may be multiplied by \( C_b \), but may not exceed the value of the horizontal portion (\( \phi M_a \) for LRFD, \( M_a/\Omega \) for ASD).

Obtain the available strength of a W18×50 with an unbraced length of 17.5 ft from Manual Table 3-10

Enter Table 3-10 and find the intersection of the curve for the W18×50 with an unbraced length of 11.7 ft. Obtain the available strength from the appropriate vertical scale to the left.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi M_a \approx 222 \text{ kip-ft} )</td>
<td>( M_a/\Omega \approx 147 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( \phi M_p \approx 379 \text{ kip-ft} ) (upper limit on ( C_bM_a ))</td>
<td>( M_p/\Omega \approx 252 \text{ kip-ft} ) (upper limit on ( C_bM_a ))</td>
</tr>
<tr>
<td>Adjust for ( C_b )</td>
<td>Adjust for ( C_b )</td>
</tr>
<tr>
<td>(1.30)(222 kip-ft) = 288 kip-ft</td>
<td>(1.30)(147 kip-ft) = 191 kip-ft</td>
</tr>
</tbody>
</table>
Procedure B.

For preliminary selection, the required strength can be divided by \( C_b \) and directly compared to the strengths in Table 3-10. Members selected in this way must be checked to ensure that the required strength does not exceed the available plastic moment strength of the section.

**Calculate the adjusted required strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u' = \frac{266 \text{ kip-ft}}{1.3} = 205 \text{ kip-ft} )</td>
<td>( M_a' = \frac{184 \text{ kip-ft}}{1.3} = 142 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

**Obtain the available strength for a W18×50 with an unbraced length of 17.5 ft from Manual Table 3-10**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_u \approx 222 \text{ kip-ft} &gt; 205 \text{ kip-ft} \quad \text{o.k.} )</td>
<td>( M_u / \Omega_b \approx 147 \text{ kip-ft} &gt; 142 \text{ kip-ft} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>( \phi_b M_p \approx 379 \text{ kip-ft} &gt; 266 \text{ kips} \quad \text{o.k.} )</td>
<td>( M_p / \Omega_b \approx 252 \text{ kip-ft} &gt; 184 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>
Example F.1-3b.  W-Shape Flexural Member Design in Strong-Axis Bending, Braced at Midspan

Given:
Example F.1-3a was solved by utilizing the tables of the AISC Steel Construction Manual. Alternatively, this problem can be solved by applying the requirements of the AISC Specification directly.

Solution:

Geometric Properties:

\[ W18 \times 50 \quad r_s = 1.98 \text{ in.} \quad S_x = 88.9 \text{ in.}^3 \quad J = 1.24 \text{ in.}^4 \quad h_o = 17.4 \text{ in.} \]

Required strength from Example F.1-3a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 266 \text{ kip-ft} )</td>
<td>( M_u = 184 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Calculate the nominal flexural strength, \( M_n \)

Calculate \( C_b \)

\[
C_b = \frac{12.5 M_{\text{max}}}{2.5 M_{\text{max}} + 3 M_A + 4 M_B + 3 M_C} \quad \text{Eqn. F1-1}
\]

The required moments for Equation F1-1 can be calculated as a percentage of the maximum midspan moment as: \( M_{\text{max}} = 1.00, M_A = 0.438, M_B = 0.750, \) and \( M_C = 0.938. \)

\[
R_m = 1.0 \text{ for doubly-symmetric members}
\]

\[
C_b = \frac{12.5(1.00)}{2.5(1.00) + 3(0.438) + 4(0.750) + 3(0.938)}(1.0) = 1.30
\]

\( L_p = 5.83 \text{ ft} \)

\( L_r = 17.0 \text{ ft} \)

For a compact beam with an unbraced length \( L_b > L_r \), the limit state of elastic lateral-torsional buckling applies.

Calculate \( F_{cr} \) with \( L_b = 17.5 \text{ ft} \)

\[
F_{cr} = \frac{C_v \pi^2 E}{L_b^2} \left[ 1 + 0.078 \frac{J_c}{S_x h_o} \left( \frac{L_b}{r_s} \right)^2 \right] \quad \text{where} \quad c = 1.0 \text{ for doubly symmetric I-shapes} \quad \text{Eqn. F2-4}
\]

\[
F_{cr} = \frac{1.30 \pi^2 (29,000 \text{ ksi})}{(17.5 \text{ ft}(12 \text{ in./ft}))^2} \left[ 1 + 0.078 \frac{(1.24 \text{ in.}^4)(1.0)}{(88.9 \text{ in.}^3)(17.4 \text{ in.})} \left( \frac{17.5 \text{ ft}(12 \text{ in./ft})}{1.98 \text{ in.}} \right)^2 \right] = 43.2 \text{ ksi}
\]
\[ M_a = F_{cr} S_c \leq M_p \quad \text{Eqn. F2-3} \]

\[ M_a = 43.2 \text{ ksi}(88.9 \text{ in.}^2) = 3840 \text{ kip-in.} < 5050 \text{ kip-in.} \]
\[ M_a = 3840 \text{ kip-in.} \text{ or } 320 \text{ kip-ft} \]

*Calculate the available flexural strength*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90(320 \text{ kip-ft}) = 288 \text{ kip-ft} )</td>
<td>( M_a = \frac{320 \text{ kip-ft}}{1.67} = 192 \text{ kip-ft} )</td>
<td></td>
</tr>
<tr>
<td>288 kip-ft &gt; 266 kip-ft <strong>o.k.</strong></td>
<td>192 kip-ft &gt; 184 kip-ft <strong>o.k.</strong></td>
<td></td>
</tr>
</tbody>
</table>
Example F.2-1a  Compact Channel Flexural Member, Continuously Braced

Given:
Select an ASTM A36 channel to serve as a roof edge beam with a simple span of 25 ft. Limit the live load deflection to $L/360$. The nominal loads are a uniform dead load of 0.23 kip/ft and a uniform live load of 0.69 kip/ft. The beam is continuously braced.

Solution:

Material Properties:
ASTM A36  \( F_y = 36 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Calculate the required flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>$w_u = 1.2(0.230 \text{ kip/ft}) + 1.6(0.690 \text{ kip/ft}) = 1.38 \text{ kip/ft}$</td>
<td>$w_a = 0.230 \text{ kip/ft} + 0.690 \text{ kip/ft} = 0.920 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_u = \frac{1.38 \text{ kip/ft}(25.0 \text{ ft})^2}{8} = 108 \text{ kip-ft}$</td>
<td>$M_a = \frac{0.920 \text{ kip/ft}(25.0 \text{ ft})^2}{8} = 71.9 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Select a trial section

Per the User Note in Section F2, all ASTM A36 channels are compact. Because the beam is compact and continuously braced, the yielding limit state governs. Select C15×33.9 from Manual Table 3-8.

Check live load deflection

For C15×33.9, \( I_c = 315 \text{ in.}^4 \)

\[
\Delta_{max} = \frac{5wl^4}{384EI} = \frac{5(0.690 \text{ kip/ft})(25.0 \text{ ft})^3(12 \text{ in./ft})}{384 (29,000 \text{ ksi})(315 \text{ in.}^4)} = 0.664 \text{ in.} < 0.833 \text{ in.} \text{ o.k.}
\]
Example F.2-1b  Compact Channel Flexural Member, Continuously Braced

Given:
Example F.2-1a can be easily solved by utilizing the tables of the AISC *Steel Construction Manual*. Alternatively, this problem can be solved by applying the requirements of the AISC Specification directly.

Solution:

Material Properties:
- ASTM A36  
  \( F_y = 36 \text{ ksi} \)  
  \( F_u = 58 \text{ ksi} \)  

Geometric Properties:
- C15×33.9  
  \( Z_x = 50.8 \text{ in.}^3 \)  

Required strength from Example F.2-1a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_a = 108 \text{ kip-ft} )</td>
<td>( M_a = 71.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Calculate the nominal flexural strength, \( M_n \)

Per the User Note in Section F2, all ASTM A36 C- and MC-shapes are compact.

A channel that is continuously braced and compact is governed by the yielding limit state.

\[
M_n = M_p = F_y Z_x = 36 \text{ ksi}(50.8 \text{ in.}^3) = 1830 \text{ kip-in. or 152 kip-ft} \quad \text{Eqn. F2-1}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( \phi_b = 0.90 \) | \( \Omega_b = 1.67 \)  
  \( \phi_b M_a = 0.90(152 \text{ kip-ft}) \) | \( M_p/\Omega_b = 152 \text{ kip-ft}/1.67 \)  
  \( = 137 \text{ kip-ft} > 108 \text{ kip-ft} \quad \text{o.k.} \) | \( = 91.3 \text{ kip-ft} > 71.9 \text{ kip-ft} \quad \text{o.k.} \) 

| \( \Omega_b = 1.67 \) | \( M_p/\Omega_b = 152 \text{ kip-ft}/1.67 \) | \( = 91.3 \text{ kip-ft} > 71.9 \text{ kip-ft} \quad \text{o.k.} \) | Section F1
Example F.2-2a  Compact Channel Flexural Member
with Bracing at Ends and Fifth Points

Given:
Check the C15×33.9 beam selected in Example F.2-1a, assuming it is braced at the ends and the fifth points rather than continuously braced.

Solution:

Material Properties:
ASTM A36  \( F_y = 36 \) ksi  \( F_u = 58 \) ksi

The center segment will govern by inspection.

Required strength at midspan from Example F.2-1a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 108 ) kip-ft</td>
<td>( M_u = 71.9 ) kip-ft</td>
</tr>
</tbody>
</table>

With an almost uniform moment across the center segment, \( C_b = 1.0 \), so no adjustment is required.

\[ L_o = \frac{25.0 \text{ ft}}{5} = 5.00 \text{ ft} \]

Obtain the strength of the C15×33.9 with an unbraced length of 5.00 ft from Manual Table 3-11

Enter Table 3-11 and find the intersection of the curve for the C15×33.9 with an unbraced length of 5.00 ft. Obtain the available strength from the appropriate vertical scale to the left.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_u \approx 130 \text{ kip-ft} &gt; 108 \text{ kip-ft} )  o.k.</td>
<td>( M_u/\Omega_b \approx 87.0 \text{ kip-ft} &gt; 71.9 \text{ kip-ft} )  o.k.</td>
</tr>
</tbody>
</table>
Example F.2-2b  Compact Channel Flexural Member with Bracing at End and Fifth Points

Given:

Verify the results from Example F.2-2a by calculation using the provisions of the Specification.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A36</td>
<td>$F_y$</td>
<td>36 ksi</td>
</tr>
<tr>
<td></td>
<td>$F_u$</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Profile</th>
<th>Section Modulus</th>
</tr>
</thead>
<tbody>
<tr>
<td>C15×33.9</td>
<td>$S_x = 42.0 \text{ in.}^3$</td>
</tr>
</tbody>
</table>

Required strength from Example F.2-2a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$M_u = 108 \text{ kip-ft}$</td>
<td>$M_a = 71.9 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Calculate the nominal flexural strength, $M_n$

Per the User Note in Section F2, all ASTM A36 C- and MC-shapes are compact.

For the center segment of a uniformly loaded beam braced at the ends and the fifth points,

<table>
<thead>
<tr>
<th>Lengths</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_p$</td>
<td>3.75 ft</td>
</tr>
<tr>
<td>$L_r$</td>
<td>14.5 ft</td>
</tr>
</tbody>
</table>

For a compact channel with $L_p < L_b \leq L_r$, the lesser of the flexural yielding limit state or the inelastic lateral-torsional buckling limit-state controls the available flexural strength.

Lateral-torsional buckling limit state

From Example F.2-2a, $M_p = 1830 \text{ kip-in.}$

$$M_n = C_p \left[ M_p - (M_p - 0.7F_yS_x) \left( \frac{L_a - L_p}{L_a - L_p} \right) \right] \leq M_p$$

$$M_n = 1.0 \left[ 1830 \text{ kip-in.} - \left( 1830 \text{ kip-in.} - 0.7(36 \text{ ksi})(42.0 \text{ in.}^3) \right) \left( \frac{5.00 \text{ ft} - 3.75 \text{ ft}}{14.5 \text{ ft} - 3.75 \text{ ft}} \right) \right] \leq 1830 \text{ kip-in.}$$

$$M_n = 1740 \text{ kip-in.} \leq 1830 \text{ kip-in.} \quad \text{o.k.}$$

$M_n = 1740 \text{ kip-in.}$ or 145 kip-ft
**Calculate the available flexural strength**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
<td>$M_a = \frac{145 \text{ kip-ft}}{1.67} = 86.8 \text{ kip-ft}$</td>
</tr>
<tr>
<td>$\phi_b M_a = 0.90(145 \text{ kip-ft}) = 131 \text{ kip-ft}$</td>
<td>$\Omega_b$</td>
<td>$86.8 \text{ kip-ft} &gt; 71.9 \text{ kip-ft}$ <strong>o.k.</strong></td>
</tr>
<tr>
<td>$131 \text{ kip-ft} &gt; 108 \text{ kip-ft}$ <strong>o.k.</strong></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example F.3a  W-Shape Flexural Member with Noncompact Flanges in Strong-Axis Bending

Given:

Select an ASTM A992 W-shape beam with a simple span of 40 feet. The nominal loads are a uniform dead load of 0.05 kip/ft and two equal 18 kip concentrated live loads acting at the third points of the beam. The beam is continuously braced. Also calculate the deflection.

Note: A beam with noncompact flanges will be selected to demonstrate that the tabulated values of the *Steel Construction Manual* account for flange compactness.

Solution:

Material Properties:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A992</td>
<td>$F_y = 50$ ksi</td>
<td>$F_u = 65$ ksi</td>
</tr>
</tbody>
</table>

Calculate the required flexural strength at midspan

$$w_u = 1.2(0.0500 \text{ kip/ft}) = 0.0600 \text{ kip/ft}$$

$$P_u = 1.6(18.0 \text{ kips}) = 28.8 \text{ kips}$$

$$M_u = \frac{(0.0600 \text{ kip/ft})(40.0 \text{ ft}^2)}{8}$$

$$+ (28.8 \text{ kips})(\frac{40.0 \text{ ft}}{3})$$

$$= 396 \text{ kip-ft}$$

LRFD

$$w_u = 0.0500 \text{ kip/ft}$$

$$P_u = 18.0 \text{ kips}$$

$$M_u = \frac{(0.0500 \text{ kip/ft})(40.0 \text{ ft}^2)}{8}$$

$$+ (18.0 \text{ kips})(\frac{40.0 \text{ ft}}{3})$$

$$= 250 \text{ kip-ft}$$

ASD

Select the lightest section with the required strength from the bold entries in Manual Table 3-2

Select W21×48.

This beam has a noncompact compression flange at $F_y = 50$ ksi as indicated by footnote “f” in Manual Table 3-2. This is also footnoted in Manual Table 1-1.
Check the available strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b M_u = \phi_b M_p)</td>
<td>398 kip-ft &gt; 396 kip-ft (\text{O.K.})</td>
<td>(M_x = \frac{M_p}{\Omega_b} = 265) kip-ft &gt; 250 kip-ft (\text{O.K.})</td>
</tr>
</tbody>
</table>

Note: the value \(M_p\) in Table 3-2 include strength reductions due the noncompact nature of the shape.

Calculate deflection

\(I_s = 959\) in.\(^4\)

\[
\Delta_{\text{max}} = \frac{5wl^4}{384EI} + \frac{Pl^3}{28EI}
\]

\[
= \frac{5(0.0500\ \text{kip/ft})(40.0\ \text{ft})^4(12\ \text{in./ft})^3}{384(29,000\ \text{ksi})(959\ \text{in.}^4)} + \frac{18.0\ \text{kips}(40.0\ \text{ft})^3(12\ \text{in./ft})^3}{28(29,000\ \text{ksi})(959\ \text{in.}^4)}
\]

\(= 2.66\) in.

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than strength in beam design.
Example F.3b  W-Shape Flexural Member with Noncompact Flanges in Strong-Axis Bending

Given:

Verify the results from Example F.3a by calculation using the provisions of the Specification.

Solution:

Material Properties:
ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi

Geometric Properties:
$W21\times48$  $S_x = 93.0$ in.$^3$  $Z_x = 107$ in.$^3$

Check flange slenderness

$$\lambda = \frac{b_f}{2t_f} = 9.47$$

The limiting width-thickness ratios for the compression flange are:

$$\lambda_{pf} = 0.38 \frac{E}{F_y} = 0.38 \frac{29,000 \text{ ksi}}{50 \text{ ksi}} = 9.15$$

$$\lambda_{rf} = 1.0 \frac{E}{F_y} = 1.0 \frac{29,000 \text{ ksi}}{50 \text{ ksi}} = 24.1$$

$\lambda_{rf} > \lambda > \lambda_{pf}$; therefore, the compression flange is noncompact. This could also be determined from the footnote “f” in Manual Table 1-1.

Calculate the nominal flexural strength, $M_n$

Since the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the available strength is governed by Section F3

$$M_p = F_y Z_x = 50 \text{ ksi}(107 \text{ in.}^3) = 5350 \text{ kip-in. or 446 kip-ft.}$$

$$M_n = \left[ M_p - \left( M_p - 0.7 F_y S_x \left( \frac{\lambda - \lambda_{pf}}{\lambda_{rf} - \lambda_{pf}} \right) \right) \right]$$

Eqn. F3-1

$$M_n = \left[ 5350 \text{ kip-in.} - \left( 5350 \text{ kip-in.} - 0.7(50 \text{ ksi}) (93.0 \text{ in.}^3) \left( \frac{9.47 - 9.15}{24.1 - 9.15} \right) \right) \right]$$

$$= 5310 \text{ kip-in. or 442 kip-ft.}$$

Calculate the available flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_p = 0.90(442 \text{ kip-ft}) = 398 \text{ kip-ft} &gt; 396 \text{ kip-ft \ o.k.}$</td>
<td>$M_p \Omega_b = 442 \text{ kip-ft} / 1.67 = 265 \text{ kip-ft} &gt; 250 \text{ kip-ft \ o.k.}$</td>
</tr>
</tbody>
</table>

Note that these available strength values are identical to the tabulated values in Manual Table 3-2, which account for the non-compact flange.
Example F.4  W-shape Flexural Member, Selection by Moment of Inertia for Strong-Axis Bending

Given:
Select an ASTM A992 W-shape flexural member by the moment of inertia, to limit the live load deflection to 1 in. The span length is 30 ft. The nominal loads are a uniform dead load of 0.80 kip/ft and a uniform live load of 2 kip/ft. Assume the beam is continuously braced.

Solution:

Material Properties:
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Calculate the required flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.800 \text{ kip/ft}) + 1.6(2.00 \text{ kip/ft}) )  ( = 4.16 \text{ kip/ft} )</td>
<td>( w_u = 0.800 \text{ kip/ft} + 2.00 \text{ kip/ft} )  ( = 2.80 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{4.16 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 468 \text{ kip-ft} )</td>
<td>( M_d = \frac{2.80 \text{ kip/ft}(30.0 \text{ ft})^2}{8} = 315 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Calculate the minimum required moment of inertia

The maximum deflection, \( \Delta_{\text{max}} \), occurs at mid-span and is calculated as

\[
\Delta_{\text{max}} = \frac{5 w l^4}{384 E I}
\]

Rearranging and substituting \( \Delta_{\text{max}} = 1.00 \text{ in.} \)  

\[
I_{\text{min}} = \frac{5(2.00 \text{ kips/ft})(30.0 \text{ ft})^4(12\text{in./ft})^3}{384(29,000 \text{ ksi})(1.00 \text{ in.})} = 1,260 \text{ in.}^4
\]

Select the lightest section with the required moment of inertia from the bold entries in Manual Table 3-23

Select a W24×55

\( I_s = 1,350 \text{ in.}^4 > 1,260. \text{ in.}^4 \)  \( \text{O.K.} \)

Because the W24×55 is continuously braced and compact, its strength is governed by the yielding limit state and Section F2.1
Obtain the available strength from Manual Table 3-2

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_a = \phi_b M_{ps} = 503$ kip-ft</td>
<td>$\frac{M_p}{\Omega_b} = \frac{M_{ps}}{\Omega_b} = 334$ kip-ft</td>
</tr>
<tr>
<td>503 kip-ft &gt; 468 kip-ft  o.k.</td>
<td>334 kip-ft &gt; 315 kip-ft  o.k.</td>
</tr>
</tbody>
</table>
Example F.5  I-shaped Flexural Member in Minor-Axis Bending

Given:
Select an ASTM A992 W-shape beam loaded in its minor axis with a simple span of 15 ft. The nominal loads are a total uniform dead load of 0.667 kip/ft and a uniform live load of 2 kip/ft. Limit the live load deflection to \( L/240 \). Assume the beam is braced at the ends only.

Note: Although not a common design case, this example is being used to illustrate Specification Section F6 (I-shaped members and channels bent about their minor axis).

Solution:

Material Properties:
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Calculate the required flexural strength

\[
\begin{align*}
\text{LRFD} & & \text{ASD} \\
\omega_u &= 1.2(0.667 \text{ kip/ft}) + 1.6(2.00 \text{ kip/ft}) \\
&= 4.00 \text{ kip/ft} \\
M_u &= \frac{4.00 \text{ kip/ft}(15.0 \text{ ft})^2}{8} = 113 \text{ kip-ft} \\
\omega_l &= 0.667 \text{ kip/ft} + 2.00 \text{ kip/ft} \\
&= 2.67 \text{ kip/ft} \\
M_l &= \frac{2.67 \text{ kip/ft}(15.0 \text{ ft})^2}{8} = 75.0 \text{ kip-ft}
\end{align*}
\]

Calculate the minimum required moment of inertia

\[
\Delta_{\text{max}} = \frac{L}{240} = \frac{15.0 \text{ ft}(12 \text{ in./ft})}{240} = 0.750 \text{ in.}
\]

\[
I_{\text{req}} = \frac{5wl^4}{384E\Delta_{\text{max}}} = \frac{5(2.00 \text{ kip/ft})(15.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(0.750 \text{ in.})} = 105 \text{ in.}^4
\]

Select the lightest section from the bold entries in Manual Table 3-3, due to the likelihood that deflection will govern this design.
Try a \( W_{12} \times 58 \)

**Geometric Properties:**

\[
W_{12} \times 58 \quad S_y = 21.4 \text{ in}^3 \quad Z_y = 32.5 \text{ in}^3 \quad I_y = 107 \text{ in}^4
\]

\[I_y = 107 \text{ in}^4 > 105 \text{ in}^4 \quad \text{o.k.}
\]

Specification Section F6 applies. Since the \( W_{12} \times 58 \) has compact flanges per the User Note in this Section, the yielding limit state governs the design.

\[
M_n = M_p = F_y Z_y \leq 1.6 F_y S_y
\]

\[
= 50 \text{ ksi}(32.5 \text{ in}^3) \leq 1.6(50 \text{ ksi})(21.4 \text{ in}^3)
\]

\[
= 1630 \text{ kip-in.} \leq 1710 \text{ kip-in.} \quad \text{o.k.}
\]

\[
= 1630 \text{ kip-in.} \text{ or } 135 \text{ kip-ft}
\]

*Calculate the available flexural strength*

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90(135 \text{ kip-ft}) = 122 \text{ kip-ft} )</td>
<td>( M_n / \Omega_b = \frac{135 \text{ kip-ft}}{1.67} = 81.1 \text{ kip-ft} )</td>
<td>81.1 kip-ft &gt; 75.0 kip-ft \text{ o.k.}</td>
</tr>
<tr>
<td>122 kip-ft &gt; 113 kip-ft \text{ o.k.}</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example F.6  HSS Flexural Member with Compact Flange

Given:
Select a square ASTM A500 Gr. B HSS beam to span 7.5 feet. The nominal loads are a uniform dead load of 0.145 kip/ft and a uniform live load of 0.435 kip/ft. Limit the live load deflection to $L/240$. Assume the beam is continuously braced.

Solution:

Material Properties:
ASTM A500 Gr. B  $F_y = 46$ ksi  $F_u = 58$ ksi

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_u$</td>
<td>$1.2(0.145 \text{ kip/ft}) + 1.6(0.435 \text{ kip/ft})$</td>
<td>$0.870 \text{ kip/ft}$</td>
</tr>
<tr>
<td></td>
<td>$M_u = \frac{(0.870 \text{ kip/ft})(7.50 \text{ ft})^2}{8} = 6.12 \text{ kip-ft}$</td>
<td>$M_u = \frac{(0.580 \text{ kip/ft})(7.50 \text{ ft})^2}{8} = 4.08 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Calculate the minimum required moment of inertia

$\Delta_{\text{max}} = \frac{L}{240} = \frac{7.50 \text{ ft}(12\text{ in./ft})}{240} = 0.375 \text{ in.}$

$I_{\text{req}} = \frac{5wl^4}{384E\Delta_{\text{max}}}$

$= \frac{5(0.435 \text{ kip/ft})(7.50 \text{ ft})^3(12\text{ in./ft})^3}{384(29,000 \text{ ksi})(0.375 \text{ in.})}$

$= 2.85 \text{ in.}^4$

Select an HSS with a minimum $I_x$ of 2.85 in.$^4$, using Manual Table 1-12, having adequate available strength, using Manual Table 3-13.

Select HSS3×3×1/4

$I_x = 3.02 \text{ in.}^4 > 2.85 \text{ in.}^4$  o.k.

Obtain the required strength from Table 3-13

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_yM_u = 8.55 \text{ kip-ft} &gt; 6.12 \text{ kip-ft}$</td>
<td>$M_u / \Omega_o = 5.69 \text{ kip-ft} &gt; 4.08 \text{ kip-ft}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
Example F.7a  HSS Flexural Member with Noncompact Flange

Given:
Select a rectangular ASTM A500 Gr. B HSS beam with a span of 21 ft. The nominal loads include a uniform dead load of 0.15 kip/ft and a uniform live load of 0.40 kip/ft. Limit the live load deflection to \( L/240 \). Assume the beam is braced at the end points only. A noncompact member was selected here to illustrate the relative ease of selecting noncompact shapes from the Manual, as compared to designing a similar shape by applying the Specification requirements directly, as shown in Example F.7b.

Solution:

Material Properties:
ASTM A500 Gr. B \( F_y = 46 \text{ ksi} \) \( F_u = 58 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.150 \text{ kip/ft}) + 1.6(0.400 \text{ kip/ft}) = 0.820 \text{ kip/ft} )</td>
<td>( w_u = 0.150 \text{ kip/ft} + 0.400 \text{ kip/ft} = 0.550 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{0.820 \text{ kip/ft} \times \left(21.0 \text{ ft}\right)^2}{8} = 45.2 \text{ kip-ft} )</td>
<td>( M_u = \frac{0.550 \text{ kip/ft} \times \left(21.0 \text{ ft}\right)^2}{8} = 30.3 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Calculate the minimum moment of inertia

\[
\Delta_{\text{max}} = \frac{L}{240} = \frac{21.0 \text{ ft} \times (12 \text{ in./ft})}{240} = 1.05 \text{ in.}
\]

\[
\Delta_{\text{max}} = \frac{5wl^4}{384EI}
\]

Rearranging and substituting \( \Delta_{\text{max}} = 1.05 \text{ in.} \).

\[
I_{\text{min}} = \frac{5(0.400 \text{ kip/ft})(21.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(1.05 \text{ in.})} = 57.5 \text{ in.}^4
\]

Select a HSS with a minimum \( I_x \) of 57.5 in.\(^4\), using Manual Table 1-11, having adequate available strength, using Manual Table 3-12.

Select a HSS10×6×\( \frac{3}{16} \) oriented in the strong direction. This rectangular HSS section was purposely selected for illustration purposes because it has a noncompact flange as noted by footnote "f" in Manual Table 3-12.

\[
I_x = 74.6 \text{ in.}^4 > 57.5 \text{ in.}^4 \quad \text{o.k.}
\]
Obtain the required strength from Table 3-12

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_x = 57.0 \text{ kip-ft} &gt; 45.2 \text{ kip-ft} \ \text{ o.k.}$</td>
<td>$M_x / \Omega_b = 37.9 \text{ kip-ft} &gt; 30.3 \text{ kip-ft} \ \text{ o.k.}$</td>
</tr>
</tbody>
</table>

Table 1-11

Manual

Table 3-12
Example F.7b  HSS Flexural Member with Noncompact Flanges

Given:

Notice that in Example F.8b the required information was easily determined by consulting the tables of the Steel Construction Manual. The purpose of the following calculation is to demonstrate the use of the Specification equations to calculate the flexural strength of a HSS member with a noncompact compression flange.

Solution:

Material Properties:
ASTM A500 Gr. B  \( F_y = 46 \) ksi  \( F_u = 58 \) ksi

Geometric Properties:
HSS 10×6×\( \frac{3}{16} \)  \( Z_x = 18.0 \) in.\(^3\)  \( S_x = 14.9 \) in.\(^3\)

Check for flange compactness
\[ \lambda = \frac{b}{t} = 31.5 \]

The limiting ratio for a compact HSS flange in flexure is
\[ \lambda_p = 1.12 \sqrt{\frac{E}{F_y}} = 1.12 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 28.1 \]

Check flange slenderness

The limiting ratio for a slender HSS flange in flexure is
\[ \lambda_s = 1.40 \sqrt{\frac{E}{F_y}} = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2 \]

\[ \lambda_p < \lambda < \lambda_s \] therefore the flange is noncompact. For this situation, Specification Eqn. F7-2 applies

Check web slenderness

\[ \lambda = \frac{h}{t} = 54.5 \]

The limiting ratio for a compact HSS web in bending is
\[ \lambda_w = 2.42 \sqrt{\frac{E}{F_y}} = 2.42 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 60.8 > 54.5 \], therefore the web is compact.

For HSS with non-compact flanges and compact webs, Specification Section F7.2(b) applies.
\[ M_n = M_p - (M_p - F_y S) \left( \frac{3.57 b}{t} \sqrt{\frac{F_y}{E}} - 4.0 \right) \leq M_p \]

Eqn. F7-2

\[ M_p = F_y Z = 46 \text{ ksi}(18.0 \text{ in.}^3) = 828 \text{ kip-in.} \]

\[ M_n = (828 \text{ kip-in.}) - \left[ (828 \text{ kip-in.}) - (46 \text{ ksi})(14.9 \text{ in.}^3) \right] \left( 3.57 \frac{31.5}{29,000 \text{ ksi}} \sqrt{\frac{46 \text{ ksi}}{29,000 \text{ ksi}}} - 4.0 \right) \]

= 760 kip-in. or 63.3 kip-ft

Calculate the available flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi_b = 0.90 ]</td>
<td>[ \Omega_b = 1.67 ]</td>
<td></td>
</tr>
<tr>
<td>[ \phi_b M_n = 0.90(63.3 \text{ kip-ft}) = 57.0 \text{ kip-ft} ]</td>
<td>[ \frac{M_n}{\Omega_b} = \frac{63.3 \text{ kip-ft}}{1.67} = 37.9 \text{ kip-ft} ]</td>
<td></td>
</tr>
</tbody>
</table>
Example F.8a  HSS Flexural Member with Slender Flanges

Given:
Verify the strength of an ASTM A500 Gr. B HSS 8×8×x with a span of 21 ft. The nominal loads are a dead load of 0.125 kip/ft and a live load of 0.375 kip/ft. Limit the live load deflection to L/240.

Solution:

Material Properties:

ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \quad F_u = 58 \text{ ksi} \)

Calculate the required flexural strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u )</td>
<td>( 1.2(0.125 \text{ kip/ft}) + 1.6(0.375 \text{ kip/ft}) )</td>
<td>( 0.750 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u )</td>
<td>( \frac{(0.750 \text{ kip/ft})(21.0 \text{ ft})^2}{8} ) = 41.3 kip-ft</td>
<td>( M_a = \frac{(0.500 \text{ kip/ft})(21.0 \text{ ft})^2}{8} ) = 27.6 kip-ft</td>
</tr>
</tbody>
</table>

Obtain the flexural strength of the HSS 8×8×x from Manual Table 3-13

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_a )</td>
<td>45.3 kip-ft &gt; 41.3 kip-ft</td>
<td>o.k.</td>
</tr>
<tr>
<td>( M_a / \Omega_p )</td>
<td>30.1 kip-ft &gt; 27.6 kip-ft</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Check deflection

\[
\Delta_{max} = \frac{l}{240} = \frac{(21.0 \text{ ft})(12 \text{ in./ft})}{240} = 1.05 \text{ in.}
\]

\( I_x = 54.4 \text{ in.}^4 \)

\[
\Delta_{max} = \frac{5wl^4}{384EI} = \frac{5(0.375 \text{ kip/ft})^4(21.0 \text{ ft})^4(12 \text{ in./ft})^3}{384(29,000 \text{ ksi})(54.4 \text{ in.}^4)} = 1.04 \text{ in.} < 1.05 \text{ in.} \quad \text{o.k.}
\]
Example F.8b  HSS Flexural Member with Slender Flanges

Given:

In Example F.8a the available strengths were easily determined from the tables of the Steel Construction Manual. The purpose of the following calculation is to demonstrate the use of the Specification equations to calculate the flexural strength of a HSS member with slender flanges.

Solution:

Material Properties:

\[ F_y = 46 \text{ ksi} \quad F_u = 58 \text{ ksi} \]

Geometric Properties:

\[ I_x = 54.4 \text{ in.}^4 \quad Z_x = 15.7 \text{ in.}^3 \quad S_x = 13.6 \text{ in.}^3 \]

Check flange slenderness

The assumed outside radius of the corners of HSS shapes is 1.5t. The design thickness is used to check compactness. The limiting ratio for HSS flanges in bending is as follows:

The limiting ratio for a slender HSS flange in bending is:

\[ \lambda = \frac{b}{t} = 43.0 > \lambda_r, \text{ therefore flange is slender.} \]

Check for web compactness

The limiting ratio for a compact web in bending is:

\[ \lambda_p = \frac{h}{t} = 43.0 > \lambda_p, \text{ therefore the web is compact.} \]

For HSS sections with slender flanges and compact webs, Specification F7-2(c) applies.

\[ M_u = F_y S_{eff} \]

Where \( S_{eff} \) is the effective section modulus determined with the effective width of the compression flange taken as:
The ineffective width of the compression flange is:

7.48 in. – 6.53 in. = 0.950 in.

An exact calculation of the effective moment of inertia and section modulus could be performed taking into account the ineffective width of the compression flange and the resulting neutral axis shift. Alternatively, a simpler but slightly conservative calculation can be performed by removing the ineffective width symmetrically from both the top and bottom flanges.

\[ I_{\text{eff}} \approx \left( 0.950 \text{ in.} \times 0.174 \text{ in.} \right) 54.4 \text{ in.}^4 - 2 \left( 0.950 \text{ in.} \times (0.174 \text{ in.}) (3.91) + \frac{(0.950 \text{ in.}) (0.174 \text{ in.})^3}{12} \right) = 49.3 \text{ in.}^4 \]

The effective section modulus can now be calculated as follows:

\[ S_{\text{eff}} = \frac{I_{\text{eff}}}{d/2} = \frac{49.3 \text{ in.}^4}{(8.00 \text{ in.})/2} = 12.3 \text{ in.}^3 \]

**Calculate the nominal flexural strength, \( M_n \)**

\[ M_n = F_y S_{\text{eff}} = 46 \text{ ksi} (12.3 \text{ in.}^3) = 567 \text{ kip-in. or 47.3 kip-ft} \]

**Calculate the available flexural strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_n = 0.90 (47.3 \text{ kip-ft}) )</td>
<td>( M_n / \Omega_b = \frac{47.3 \text{ kip-ft}}{1.67} )</td>
</tr>
<tr>
<td>( = 42.5 \text{ kip-ft} &gt; 41.3 \text{ kip-ft} \ o.k. )</td>
<td>( = 28.3 \text{ kip-ft} &gt; 27.6 \text{ kip-ft} \ o.k. )</td>
</tr>
</tbody>
</table>

Note that the calculated available strengths are somewhat lower than those in Manual Table 3-13 due to the use of the conservative calculation of the approximate effective section modulus above.
Example F.9a  Pipe Flexural Member

Given:
Select an ASTM A53 grade B Pipe shape with a simple span of 16 ft. The nominal loads are a total uniform dead load of 0.32 kip/ft and a uniform live load of 0.96 kip/ft. Assume there is no deflection limit for this beam. The beam is braced only at the ends.

Solution:

Material Properties:
ASTM A500 Gr. B  \( F_y = 35 \text{ ksi} \)  \( F_u = 60 \text{ ksi} \)

Calculate the required flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u = 1.2(0.320 \text{ kip/ft}) + 1.6(0.960 \text{ kip/ft}) )</td>
<td>( w_a = 0.320 \text{ kip/ft} + 0.960 \text{ kip/ft} )</td>
</tr>
<tr>
<td>= 1.92 kip/ft</td>
<td>( = 1.28 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{1.92 \text{ kip/ft} (16.0 \text{ ft})^2}{8} )</td>
<td>( M_a = \frac{1.28 \text{ kip/ft} (16.0 \text{ ft})^2}{8} )</td>
</tr>
<tr>
<td>= 61.4 kip-ft</td>
<td>= 41.0 kip-ft</td>
</tr>
</tbody>
</table>

Select a member from Manual Table 3-15 having the required strength

Select Pipe 8 X-Strong.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi \beta M_u = 81.4 \text{ kip-ft} &gt; 61.4 \text{ kip-ft} ) o.k.</td>
<td>( M_u / \Omega_p = 54.1 \text{ kip-ft} &gt; 41.0 \text{ kip-ft} ) o.k.</td>
</tr>
</tbody>
</table>

Manual Table 3-15
Example F.9b  Pipe Flexural Member

Given:

The available strength in Example F.9a was easily determined using Manual Table 3-15. The following calculation demonstrates the calculation of the available strength by directly applying the requirements of the Specification.

Solution:

Material Properties:

\[ F_y = 35 \text{ ksi} \quad F_u = 60 \text{ ksi} \]

Geometric Properties:

Pipe 8 X-Strong \[ Z = 31.0 \text{ in}^3 \quad D = 8.63 \text{ in} \quad t = 0.465 \text{ in} \quad D/t = 18.5 \]

Manual Table 3-15  
Manual Table 3-15

Required flexural strength from Example F.9a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_u = 61.4 \text{ kip-ft} )</td>
<td>( M_u = 41.0 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Check compactness

For circular HSS in flexure, the limiting diameter to thickness ratio for a compact section is:

\[ \lambda_p = \frac{0.07E}{F_y} = \frac{0.07(29,000 \text{ ksi})}{35 \text{ ksi}} = 58.0 \]

\[ \lambda = \frac{D}{t} = 18.5 < \lambda_p \], therefore the section is compact and the limit state of flange local buckling does not apply.

By inspection, \( \frac{D}{t} < \frac{0.45E}{F_y} \), therefore Specification Section F8 applies.

Calculate the nominal flexural strength based on the flexural yielding limit state

\[ M_u = M_p = F_y Z = (35 \text{ ksi})(31.0 \text{ in}^3) = 1080 \text{ kip-in. or } 90.4 \text{ kip-ft} \]

Eqn. F8-1

Calculate the available flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90 (90.4 \text{ kip-ft}) )</td>
<td>( M_a / \Omega_b = \frac{90.4 \text{ kip-ft}}{1.67} = 54.1 \text{ kip-ft} &gt; 41.0 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( = 81.4 \text{ kip-ft} &gt; 61.4 \text{ kip-ft} ) o.k.</td>
<td></td>
</tr>
</tbody>
</table>
Example F.10  WT Shape Flexural Member

Given:
Select an ASTM A992 WT beam with a simple span of 6 ft. The toe of the stem of the WT is in tension. The nominal loads are a uniform dead load of 0.08 kip/ft and a uniform live load of 0.24 kip/ft. There is no deflection limit for this member. Assume full lateral support.

Solution:

Material properties:
ASTM A992  \( F_y = 50 \) ksi  \( F_u = 65 \) ksi

Calculate the required flexural strength

\[
\begin{align*}
wu &= 1.2(0.0800 \text{ kip/ft}) + 1.6(0.240 \text{ kip/ft}) \\
&= 0.480 \text{ kip/ft} \\
M_u &= \frac{0.480 \text{ kip/ft} \cdot (6.00 \text{ ft})^2}{8} = 2.16 \text{ kip-ft} \\
w_d &= 0.0800 \text{ kip/ft} + 0.240 \text{ kip/ft} \\
w_t &= 0.320 \text{ kip/ft} \\
M_d &= \frac{0.320 \text{ kip/ft} \cdot (6.00 \text{ ft})^2}{8} = 1.44 \text{ kip-ft}
\end{align*}
\]

Try WT 5×6

Geometric Properties:

\[
\begin{align*}
\text{WT} 5\times6 & \quad I_x = 4.35 \text{ in.}^4 \quad Z_x = 2.20 \text{ in.}^3 \quad S_x = 1.22 \text{ in.}^3 \quad b_f = 3.96 \text{ in.} \\
t_f &= 0.210 \text{ in.} \quad y = 1.36 \text{ in.} \quad S_w = \frac{I_x}{y_c} = \frac{4.35 \text{ in.}^4}{1.36 \text{ in.}} = 3.20 \text{ in.}^3
\end{align*}
\]

Calculate the nominal flexural strength, \( M_n \)

Flexural yielding limit state

\[
M_p = F_y Z_x \leq 1.6 M_y \quad \text{for stems in tension}
\]

\[
1.6M_y = 1.6F_y S_x = 1.6(50 \text{ ksi})(1.22 \text{ in.}^3) = 97.6 \text{ kip-in.}
\]


\[ M_p = F_cZ_c = (50 \text{ ksi})(2.20 \text{ in.}^3) = 110 \text{ kip-in.} > 97.6 \text{ kip-in.}, \text{ therefore use} \]

\[ M_p = 97.6 \text{ kip-in. or } 8.13 \text{ kip-ft} \]

\[ M_u = M_p = 8.13 \text{ kip-ft} \quad \text{controls} \quad \text{Eqn. F9-1} \]

\text{Lateral-torsional buckling limit state} \quad \text{Section F9.2}

Because the WT is fully braced and the stem is in tension, no check of the lateral-torsional buckling limit state is required. Note that if the stem is in compression, Equation F9-4 must be checked even for fully braced members, since the equation converges to the web local buckling limit state check at an unbraced length of zero. See Commentary Section F9.

\text{Flange local buckling limit state} \quad \text{Eqn. F9-7}

\text{Check flange compactness}

\[ \lambda = \frac{b_f}{2t} = \frac{3.96 \text{ in.}}{2(0.210 \text{ in.})} = 9.43 \]

\[ \lambda_p = 0.38 \sqrt{\frac{E}{F_y}} = 0.38 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 9.15 < 9.43; \text{ therefore the flange is not compact.} \quad \text{Table B4.1, Case 7} \]

\text{Check flange slenderness}

\[ \lambda_s = 1.0 \sqrt{\frac{E}{F_y}} = 1.0 \sqrt{\frac{29,000 \text{ ksi}}{50 \text{ ksi}}} = 24.1 > 9.43, \text{ therefore the flange is not slender.} \quad \text{Table B4.1, Case 7} \]

\text{Calculate critical flange local buckling stress}

For a Tee with a noncompact flange, the critical stress is:

\[ F_{cr} = \frac{F_y}{\sqrt{\frac{E}{F_y}}} \left(1.19-0.50 \left(\frac{b}{2t} \sqrt{\frac{E}{F_y}}\right)\right) = (50 \text{ ksi}) \left(1.19-0.50 \left(\frac{3.96}{2(0.210)}\right)\right) = 49.7 \text{ ksi} \quad \text{Eqn. F9-7} \]

\text{Calculate the nominal flexural strength}

\[ M_u = F_cS_w = 49.7 \text{ ksi}(3.20 \text{ in.}^3) = 159 \text{ kip-in.} \text{ or } 13.3 \text{ kip-ft} \quad \text{does not control} \quad \text{Eqn. F9-6} \]

\text{Calculate the available flexural strength}

\begin{tabular}{|c|c|}
\hline
LRFD & ASD \\
\hline
\[ \phi_b = 0.90 \] & \[ \Omega_b = 1.67 \] \\
\[ \phi_bM_a = 0.90(8.13 \text{ kip-ft}) \] & \[ M_u / \Omega_b = \frac{8.13 \text{ kip-ft}}{1.67} \] \\
\[ = 7.32 \text{ kip-ft} > 2.16 \text{ kip-ft} \quad \text{o.k.} \] & \[ = 4.87 \text{ kip-ft} > 1.44 \text{ kip-ft} \quad \text{o.k.} \] \\
\hline
\end{tabular} 

\text{Section F1}
Example F.11 Single Angle Flexural Member

Given:
Select an ASTM A36 single angle with a simple span of 6 ft. The vertical leg of the single angle is down and the toe is in tension. The nominal loads are a uniform dead load of 0.05 kip/ft and a uniform live load of 0.15 kip/ft. There is no deflection limit for this angle. Conservatively assume $C_b = 1.0$. Assume bending about the geometric $x$-$x$ axis and that there is no lateral-torsional restraint.

Solution:

Material Properties:
ASTM A36 $F_y = 36$ ksi $F_u = 58$ ksi

Manual Table 2-3

Calculate the required flexural strength

| LRFD | | ASD |
|------| |------|
| $w_u = 1.2(0.05 \text{ kip/ft}) + 1.6(0.15 \text{ kip/ft})$ | $w_u = 0.05 \text{ kip/ft} + 0.15 \text{ kip/ft}$ |
| $= 0.300 \text{ kip/ft}$ | $= 0.200 \text{ kip/ft}$ |
| $M_u = \frac{(0.300 \text{ kip/ft})(6 \text{ ft})^2}{8}$ | $M_u = \frac{(0.200 \text{ kip/ft})(6 \text{ ft})^2}{8}$ |
| $= 1.35 \text{ kip-ft}$ | $= 0.900 \text{ kip-ft}$ |

Try L4x4x\(\frac{1}{4}\)

Geometric Properties:
L4x4x\(\frac{1}{4}\) $S_x = 1.03 \text{ in}^3$ $S_z = 0.419 \text{ in}^3$

Manual Table 1-7

Calculate the nominal flexural strength, $M_n$

For all calculations, $M_y$ is taken as 0.80 times the yield moment calculated using the geometric section modulus

$$M_y = 0.80S_yF_y = 0.80(1.03 \text{ in}^3)(36 \text{ ksi}) = 29.7 \text{ kip-in.}$$

Section F10-2

Flexural yielding limit state

$$M_a = 1.5M_y = 1.5(29.7 \text{ kip-in.})$$
$$= 44.5 \text{ kip-in. or 3.71 kip-ft} \text{ controls}$$
Lateral-torsional buckling limit state

Determine $M_e$

For bending about one of the geometric axes of an equal-leg angle without continuous lateral-torsional restraint and with maximum tension at the toe, use Equation F10-4b.

$$M_e = \frac{0.66Eb^2t}{L^2} \left(1 + 0.78\left(\frac{Lt}{b^2}\right)^2 + 1\right)$$  \hspace{1cm} \text{Eqn. F10-4b}

\begin{align*}
M_e &= \frac{0.66(29,000 \text{ ksi})(4.00 \text{ in.})^4(0.250 \text{ in.})(1.0)}{(72.0 \text{ in.})^2} \left(1 + 0.78\left(\frac{(72.0 \text{ in.})(0.250 \text{ in.})}{(4.00 \text{ in.})^2}\right)^2 + 1\right) \\
&= 569 \text{ kip-in.} > 29.7 \text{ kip-in.} \text{ therefore, Equation F10-3 is applicable.}
\end{align*}

$$M_n = \begin{cases} 1.92 - 1.17 \left(\frac{M_e}{M_y}\right), M_y \leq 1.5M_y \\ 1.92 - 1.17 \left(\frac{29.7 \text{ kip-in.}}{569 \text{ kip-in.}}\right) 29.7 \text{ kip-in.} \leq 1.5(29.7 \text{ kip-in.}) \\
= 49.1 \text{ kip-in.} \leq 44.5 \text{ kip-in.}, \text{ therefore}
\end{cases}$$  \hspace{1cm} \text{Eqn. F10-3}

\begin{align*}
M_n &= 44.5 \text{ kip-in. or 3.71 kip-ft.}
\end{align*}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b = 0.90$</td>
<td>$\Omega_b = 1.67$</td>
</tr>
<tr>
<td>$\phi_bM_n = 0.90(3.71 \text{ kip-ft})$</td>
<td>$M_n / \Omega_b = \frac{3.71 \text{ kip-ft}}{1.67}$</td>
</tr>
<tr>
<td>$= 3.34 \text{ kip-ft} &gt; 1.35 \text{ kip-ft} \text{ o.k.}$</td>
<td>$= 2.22 \text{ kip-ft} &gt; 0.900 \text{ kip-ft} \text{ o.k.}$</td>
</tr>
</tbody>
</table>

Note: In this example it is assumed that the toe of the vertical leg of the single angle is in tension. If the toe of the outstanding leg is in compression, as in this example, the leg local buckling limit state must also be checked. The designer should also consider the possibility that restrained end conditions of a single angle member could unintentionally cause the outstanding leg to be in compression.
Example F.12  Rectangular Bar in Strong-Axis Bending

Given:
Select an ASTM A36 rectangular bar with a span of 12 ft. The bar is braced at the ends and at
the midpoint. Conservatively use $C_b = 1.0$. Limit the depth of the member to 5 in. The
nominal loads are a total uniform dead load of 0.44 kip/ft and a uniform live load of 1.32
kip/ft.

Solution:

Material Properties:
ASTM A36  \( F_y = 36 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

\[
\begin{array}{l}
w_u = 1.2(0.440 \text{ kip/ft}) + 1.6(1.32 \text{ kip/ft}) \\
= 2.64 \text{ kip/ft} \\
M_u = \frac{2.64 \text{ kip/ft}(12.0 \text{ ft})^2}{8} = 47.5 \text{ kip-ft} \\
w_a = 0.440 \text{ kip/ft} + 1.32 \text{ kip/ft} \\
= 1.76 \text{ kip/ft} \\
M_a = \frac{1.76 \text{ kip/ft}(12.0 \text{ ft})^2}{8} = 31.7 \text{ kip-ft}
\end{array}
\]

Try a 5 in.×3 in. bar.

Geometric Properties:
Rectangular bar 5×3  \( S_x = \frac{bd^2}{6} = \frac{(3.00 \text{ in.})(5.00 \text{ in.})^2}{6} = 12.5 \text{ in.}^3 \)
\( Z_x = \frac{bd^2}{4} = \frac{(3.00 \text{ in.})(5.00 \text{ in.})^2}{4} = 18.8 \text{ in.}^3 \)

Solution:

Calculate nominal flexural strength, \( M_n \)

Flexural yielding limit state

Check limit

\[
\frac{L_a d}{r^2} \leq \frac{0.08E}{F_y}
\]
\[
\frac{(72.0 \text{ in.})(5.00 \text{ in.})}{(3.00 \text{ in.})^2} \leq \frac{0.08(29,000 \text{ ksi})}{(36 \text{ ksi})}
\]

40.0 < 64.4, therefore the yielding limit state applies.

\[M_a = M_p = F_y Z \leq 1.6 M_y\]

Eqn. F11-1

\[1.6M_y = 1.6 F_y S_y = 1.6(36 \text{ ksi})(12.5 \text{ in.}^3) = 720 \text{ kip-in.}\]

\[M_p = F_y Z_s = (36 \text{ ksi})(18.8 \text{ in.}^3) = 675 \text{ kip-in.} < 720 \text{ kip-in.}\]

Use \(M_a = M_p = 675 \text{ kip-in. or 56.3 kip-ft}\)

**Lateral-torsional buckling limit state**

As calculated above, \(\frac{L_a d}{t^2} < \frac{0.08 E}{F_y}\), therefore the lateral-torsional buckling limit state does not apply.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi_b = 0.90)</td>
<td>(\Omega_b = 1.67)</td>
</tr>
<tr>
<td>(\phi_b M_a = 0.90(56.3 \text{ kip-ft}))</td>
<td>(M_a / \Omega_b = \frac{56.3 \text{ kip-ft}}{1.67})</td>
</tr>
<tr>
<td>= 50.6 kip-ft &gt; 47.5 kip-ft <strong>o.k.</strong></td>
<td>= 33.7 kip-ft &gt; 31.7 kip-ft <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Section F1
Example F.13  Round Bar in Bending

Given:
Select an ASTM A36 round bar with a span of 2.50 feet. The bar is unbraced. The material is ASTM A36. Assume $C_b = 1.0$. Limit the diameter to 2 in. The nominal loads are a concentrated dead load of 0.10 kips and a concentrated live load of 0.25 kips at the center. The weight of the bar is negligible.

Solution:

Material Properties:
ASTM A36  $F_y = 36$ ksi  $F_u = 58$ ksi

Manual Table 2-3

Calculate the required flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(0.100 \text{ kip}) + 1.6(0.250 \text{ kip})$</td>
<td>$P_u = 0.100 \text{ kip} + 0.250 \text{ kip}$</td>
</tr>
<tr>
<td>$M_u = \frac{(0.520 \text{ kip})(2.50 \text{ ft})}{4} = 0.325 \text{ kip-ft}$</td>
<td>$M_u = \frac{(0.350 \text{ kip})(2.50 \text{ ft})}{4} = 0.219 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Try 1 in. diameter rod.

Geometric Properties:
Round bar  \[ S_x = \frac{\pi d^3}{32} = \frac{\pi(1.00 \text{ in.})^3}{32} = 0.0982 \text{ in.}^3 \]
\[ Z_x = \frac{d^3}{6} = \frac{(1.00 \text{ in.})^3}{6} = 0.167 \text{ in.}^3 \]

Calculate the nominal flexural strength, $M_n$

Flexural yielding limit state

\[ M_n = M_p = F_y Z_x \leq 1.6 M_y \quad \text{Eqn. F11-1} \]

\[ 1.6M_y = 1.6(36 \text{ ksi})(0.0982 \text{ in.}^3) = 5.66 \text{ kip-in} \]

\[ F_y Z_x = 36 \text{ ksi}(0.167 \text{ in.}^3) = 6.00 \text{ kip-in} > 5.66 \text{ kip-in}. \]

Therefore, $M_n = 5.66$ kip-in. or 0.471 kip-ft
Lateral-torsional buckling limit state

This limit state need not be considered for rounds.

Calculate the available flexural strength

<table>
<thead>
<tr>
<th>$\phi_b = 0.90$</th>
<th>$\Omega_b = 1.67$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_b M_n = 0.90(0.471 \text{ kip-ft})$</td>
<td>$M_n / \Omega_b = \frac{0.471 \text{ kip-ft}}{1.67}$</td>
</tr>
<tr>
<td>$= 0.424 \text{ kip-ft} &gt; 0.325 \text{ kip-ft} \quad \text{O.K.}$</td>
<td>$= 0.282 \text{ kip-ft} &gt; 0.219 \text{ kip-ft} \quad \text{O.K.}$</td>
</tr>
</tbody>
</table>
Example F.14   Non-Symmetrical Z-shape in Strong-Axis Bending

Given:

Determine the available strength of the ASTM A36 Z-shape shown for a simple span of 18 ft. The Z-shape is braced at 6 ft on center. Assume a $C_b = 1.0$. The nominal loads are a uniform dead load of 0.025 kip/ft and a uniform live load of 0.10 kip/ft. The profile of the purlin is shown below.
Solution:

Material properties:

\[ Z \text{ Purlin} \quad F_y = 36 \text{ ksi} \quad F_u = 58 \text{ ksi} \]

Geometric Properties:

\[ t_w = t_f = 0.250 \text{ in.} \]

\[ A = (2.50 \text{ in.})(0.25 \text{ in.})(2) + (0.25 \text{ in.})(0.25 \text{ in.})(2) + (11.5 \text{ in.})(0.25 \text{ in.}) = 4.25 \text{ in.}^2 \]

\[ I_x = \left[ \frac{(0.25 \text{ in.})(0.25 \text{ in.})^3}{12} + (0.25 \text{ in.})^2 (5.62 \text{ in.})^2 \right] (2) \]

\[ + \left[ \frac{(2.50 \text{ in.})(0.25 \text{ in.})^3}{12} + (2.50 \text{ in.})(0.25 \text{ in.})(5.87 \text{ in.})^2 \right] (2) \]

\[ + \frac{(0.25 \text{ in.})(11.5 \text{ in.})^3}{12} \]

\[ = 78.8 \text{ in.}^4 \]

\[ \bar{y} = 6.00 \text{ in.} \]

\[ S_x = \frac{I_x}{\bar{y}} = \frac{78.8 \text{ in.}^4}{6.00 \text{ in.}} = 13.1 \text{ in.}^3 \]

\[ I_y = \left[ \frac{(0.25 \text{ in.})(0.25 \text{ in.})^3}{12} + (0.25 \text{ in.})^2 (2.25 \text{ in.})^2 \right] (2) \]

\[ + \left[ \frac{(0.25 \text{ in.})(2.50 \text{ in.})^3}{12} + (2.50 \text{ in.})(0.25 \text{ in.})(1.12 \text{ in.})^2 \right] (2) \]

\[ + \frac{(11.5 \text{ in.})(0.25 \text{ in.})^3}{12} \]

\[ = 2.88 \text{ in.}^4 \]

\[ r_y = \sqrt{\frac{I_y}{A}} = \sqrt{\frac{2.87 \text{ in.}^4}{4.25 \text{ in.}^2}} = 0.823 \text{ in.} \]

\[ r_w \approx \frac{b_f}{\sqrt{12 \left(1 + \frac{ht_w}{6b_f t_f}\right)}} = \frac{2.50 \text{ in.}}{\sqrt{12 \left(1 + \frac{(11.5 \text{ in.})(0.250 \text{ in.})}{6(2.50 \text{ in.})(0.250 \text{ in.})}\right)}} = 0.543 \text{ in.} \]

**Calculate the required flexural strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ w_w = 1.2(0.025 \text{ kip/ft}) + 1.6(0.10 \text{ kip/ft}) \]
  \[ = 0.190 \text{ kip/ft} \] | \[ w_a = 0.025 \text{ kip/ft} + 0.10 \text{ kip/ft} \]
  \[ = 0.125 \text{ kip/ft} \] |
| \[ M_w = \frac{(0.190 \text{ kip/ft})(18.0 \text{ ft})^2}{8} = 7.70 \text{ kip-ft} \] | \[ M_a = \frac{(0.125 \text{ kip/ft})(18.0 \text{ ft})^2}{8} = 5.06 \text{ kip-ft} \] |
Flexural yielding limit state

\[ F_n = F_y = 36 \text{ ksi} \quad \text{Eqn. F12-2} \]
\[ M_n = F_n S = 36 \text{ ksi}(13.1 \text{ in.}^3) = 473 \text{ kip-in. or } 39.4 \text{ kip-ft} \quad \text{Eqn. F12-1.} \]

Local buckling limit state

There are no specific local buckling provisions for Z-shapes in the Specification. Use provisions for rolled channels from Specification Table B4.1.

Check for flange slenderness

Conservatively neglecting the end return:

\[ \lambda = \frac{b}{t_f} = \frac{2.50 \text{ in.}}{0.250 \text{ in.}} = 10.0 \]

\[ \lambda_p = 0.38 \sqrt[3]{\frac{E}{F_y}} = 0.38 \sqrt[3]{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 10.8 > 10.0, \text{ therefore the flange is compact} \]

Table B4.1
Case 1

Check for web slenderness

\[ \lambda = \frac{h}{t_w} = \frac{11.5 \text{ in.}}{0.250 \text{ in.}} = 46.0 \]

\[ \lambda_p = 3.76 \sqrt[3]{\frac{E}{F_y}} = 3.76 \sqrt[3]{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 107 > 46.0, \text{ therefore the web is compact.} \]

Table B4.1
Case 9

Therefore, no limit state for local buckling applies

Lateral-torsional buckling limit state

Per the User Note in Section F12, take the critical lateral-torsional buckling stress as half that of the equivalent channel.

Calculate limiting unbraced lengths

For bracing at 6 ft on center, \( L_b = (6.00 \text{ ft})(12 \text{ in./ft}) = 72.0 \text{ in.} \)

\[ L_p = 1.76r \sqrt[3]{\frac{E}{F_y}} = 1.76(0.823 \text{ in.}) \sqrt[3]{\frac{29,000 \text{ ksi}}{36 \text{ ksi}}} = 41.1 \text{ in.} < 72.0 \text{ in.} \quad \text{Eqn. F2-5} \]

\[ L_v = 1.95 r_v \left( \frac{E}{0.7 F_v} \right) \sqrt[3]{\frac{Jc}{S \cdot h_b}} \sqrt[3]{1 + 6.76 \left( \frac{0.7 F_v S \cdot h_b}{E Jc} \right)^2} \quad \text{Eqn. F2-6} \]

Per the user note in Specification Section F2, the square root term in Specification Equation
F2-4 can conservatively be taken equal to one, therefore,

\[ L_e = \pi \left( \frac{E}{0.7 F_y} \right) = \pi \left( 0.543 \text{ in.} \right) \sqrt{\frac{29000 \text{ ksi}}{0.7(36 \text{ ksi})}} = 57.9 \text{ in.} < 72.0 \text{ in.} \]

Calculate one half of the critical lateral-torsional buckling stress of the equivalent channel

\[ L_b > L_{cr}, \text{ therefore,} \]

\[ F_{cr} = \left( 0.5 \right) \frac{C_e \pi^2 E}{\left( \frac{L_b}{r_{cr}} \right)^2} \sqrt{1 + 0.078 \left( \frac{J_c}{S, h_i} \right)} \left( \frac{L_b}{r_{cr}} \right)^2 \]

Conservatively taking the square root term as 1.0,

\[ F_{cr} = \left( 0.5 \right) \frac{C_e \pi^2 E}{\left( \frac{L_b}{r_{cr}} \right)^2} = \left( 0.5 \right) \frac{1.0 \pi^2 \left( 29,000 \text{ ksi} \right)}{\left( 72.0 \text{ in.} \right)^2} = 8.14 \text{ ksi} \]

\[ F_a = F_{cr} \leq F_y = 8.14 \text{ ksi} < 36 \text{ ksi} \quad \text{o.k.} \]

\[ M_a = F_a S \]

\[ = \left( 8.14 \text{ ksi} \right) \left( 13.1 \text{ in.}^3 \right) = 107 \text{ kip-in. or } 8.89 \text{ k-ft} \quad \text{controls} \]

Calculate the available strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section F1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_a = 0.90(8.89 \text{ kip-ft}) )</td>
<td>( M_a / \Omega_b = \frac{8.89 \text{ kip-ft}}{1.67} )</td>
<td></td>
</tr>
<tr>
<td>( = 8.00 \text{ kip-ft} &gt; 7.70 \text{ kip-ft o.k.} )</td>
<td>( = 5.32 \text{ kip-ft} &gt; 5.06 \text{ kip-ft o.k.} )</td>
<td></td>
</tr>
</tbody>
</table>
CHAPTER G
DESIGN OF MEMBERS FOR SHEAR

INTRODUCTION

This chapter covers webs of singly or doubly symmetric members subject to shear in the plane of the web, single angles, HSS sections, and shear in the weak direction of singly or doubly symmetric shapes.

Most of the formulas from this chapter are illustrated by example. Tables for all standard ASTM A992 W-shapes and ASTM A36 channels are included in the Manual. In the tables, where applicable, LRFD and ASD shear information is presented side-by-side for quick selection, design and verification.

LRFD shear strengths have been increased slightly over those in the previous LRFD Specification for members not subject to shear buckling. ASD strengths are essentially identical to those in the previous ASD Specification. LRFD and ASD will produce identical designs for the case where the live load effect is approximately three times the dead load effect.

G1. GENERAL PROVISIONS

The design shear strength, \( \phi V_n \), and the allowable shear strength, \( V_n/\Omega \), are determined as follows:

\[
V_n = \text{nominal shear strength based on shear yielding or shear buckling} \\
\phi_v = 0.90 \text{ (LRFD)} \\
\Omega_v = 1.67 \text{ (ASD)}.
\]

Exception: For all current ASTM A6, W, S, and HP shapes except W44×230, W40×149, W36×135, W33×118, W30×90, W24×55, W16×26, and W12×14 for \( F_y = 50 \text{ ksi} \):

\[
\phi_v = 1.00 \text{ (LRFD)} \\
\Omega_v = 1.50 \text{ (ASD)}.
\]

Section G2 does not utilize tension field action. Section G3 specifically addresses the use tension field action.

Strong axis shear values are tabulated for W-shapes in Manual Tables 3-2 and 3-6, for S-shapes in Manual Table 3-7, for C-shapes in Manual Table 3-8 and for MC-shapes in Manual Table 3-9. Weak axis shear values for W-shapes, S-shapes, C-shapes and MC-shapes and shear values for angles, rectangular HSS and box members, and round HSS are not tabulated.

G2. MEMBERS WITH UNSTIFFENED OR STIFFENED WEBS

As indicated in the User Note of this section, virtually all W, S and HP shapes are not subject to shear buckling and are also eligible for the more liberal safety and resistance factors, \( \phi_v = 1.00 \text{ (LRFD)} \) and \( \Omega_v = 1.50 \text{ (ASD)} \). This is presented in Example G.1 for a W-shape. A channel shear strength design is presented in Example G.2.

G3. TENSION FIELD ACTION

A built-up girder with a thin web and vertical stiffeners is presented in Example G.8.

G4. SINGLE ANGLES

Rolled angles are typically made from ASTM A36 steel. All single angles listed in the Manual have a \( C_v = 1.0 \). A single angle example is illustrated in Example G.3.
G5.  RECTANGULAR HSS AND BOX MEMBERS

The shear height, $h$, is taken as the clear distance between the radii. If the corner radii are unknown, the outside radius is taken as 3 times the design thickness. An HSS example is provide in Example G.4.

G6.  ROUND HSS

For all Round HSS and Pipes of ordinary length listed in the Manual, $F_{cr}$ can be taken as $0.6F_y$ in Specification Equation G6-1. A round HSS example is illustrated in Example G.5.

G7.  WEAK AXIS SHEAR IS SINGLE AND DOUBLY SYMMETRIC SHAPES

For a weak axis shear example see Example G.6 and Example G.7.

G8.  BEAMS AND GIRDERS WITH WEB OPENINGS

For a beam and girder with web openings example see AISC Design Guide 2.
**Example G.1a  W-Shape in Strong-Axis Shear.**

**Given:**
Verify the shear strength of a W24×62 ASTM A992 beam with end shears of 48 kips from dead load and 145 kips from live load.

**Solution:**

**Material Properties:**

<table>
<thead>
<tr>
<th>W24×62</th>
<th>ASTM A992</th>
<th>$F_y = 50$ ksi</th>
<th>$F_u = 65$ ksi</th>
<th>Manual Table 2-3</th>
</tr>
</thead>
</table>

**Geometric Properties:**

<table>
<thead>
<tr>
<th>W24×62</th>
<th>$d = 23.7$ in.</th>
<th>$t_w = 0.430$ in.</th>
<th>Manual Table 1-1</th>
</tr>
</thead>
</table>

**Calculate the required shear strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u = 1.2(48.0 \text{ kips}) + 1.6(145 \text{ kips}) = 290 \text{ kips}$</td>
<td>$V_u = 48.0 \text{ kips} + 145 \text{ kips} = 193 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Take the available shear strength from Manual Table 3-2**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi V_u = 306 \text{ kips}$</td>
<td>$V_u / \Omega_i = 204 \text{ kips}$</td>
</tr>
<tr>
<td>306 kips &gt; 290 kips  o.k.</td>
<td>204 kips &gt; 193 kips  o.k.</td>
</tr>
</tbody>
</table>
Example G.1b  W-Shape in Strong-Axis Shear.

The available shear strength, which can be easily determined by the tabulated values of the Steel Construction Manual, can be verified by directly applying the provisions of the Specification.

Except for very few sections, which are listed in the User Note, Specification Section G2.1(a) is applicable to the I-shaped beams published in the Manual when $F_y \leq 50$ ksi.

$C_v = 1.0$  

Calculate $A_w$

$A_w = d t_w = 23.7$ in. ($0.430$ in.) = $10.2$ in.$^2$

Calculate $V_n$

$V_n = 0.6 F_y A_w C_v = 0.6 (50$ ksi)(10.2 in.$^2$)(1.0) = $306$ kips  

**Calculate the available shear strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi v = 1.00$</td>
<td>$\Omega v = 1.50$</td>
</tr>
<tr>
<td>$\phi v V_n = 1.00 (306$ kips) = $306$ kips</td>
<td>$V_n/\Omega v = 306$ kips / $1.50 = 204$ kips</td>
</tr>
</tbody>
</table>
Example G.2a  C-Shape in Strong-Axis Shear.

Given:
Verify the shear strength of a C15×33.9 channel with end shears of 17.5 kips from dead load and 52.5 kips from live load.

Solution:

Material Properties:
C15×33.9  ASTM A36  \( F_y = 36 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  Manual Table 2-3

Geometric Properties:
C15×33.9  \( d = 15.0 \text{ in.} \)  \( t_w = 0.400 \text{ in.} \)  Manual Table 1-5

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 1.2(17.5 \text{ kips}) + 1.6(52.5 \text{ kips}) )</td>
<td>( V_u = 17.5 \text{ kips} + 52.5 \text{ kips} )</td>
</tr>
<tr>
<td>= 105 kips</td>
<td>= 70.0 kips</td>
</tr>
</tbody>
</table>

Take the available shear strength from Manual Table 3-8

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi V_u = 117 \text{ kips} )</td>
<td>( V_u/\Omega_i = 77.6 \text{ kips} )</td>
</tr>
<tr>
<td>117 kips &gt; 105 kips o.k.</td>
<td>77.6 kips &gt; 70.0 kips o.k.</td>
</tr>
</tbody>
</table>

Manual Table 3-8
Example G.2b  C-Shape in Strong-Axis Shear.

The available shear strength, which can be easily determined by the tabulated values of the Steel Construction Manual, can be verified by directly applying the provisions of the Specification. $C_v$ is 1.0 for all rolled channels when $F_y \leq 36$ ksi, and Specification Equation G2-1 is applicable.

$$C_v = 1.0$$  \hspace{1cm} \text{Eqn. G2-2}

*Calculate $A_w$*

$$A_w = 15.0 \text{ in.}(0.400 \text{ in.}) = 6.00 \text{ in.}^2$$

*Calculate $V_n$*

$$V_n = 0.6F_yA_wC_v = 0.6(36 \text{ ksi})(6.00 \text{ in.}^2)(1.0) = 130 \text{ kips}$$  \hspace{1cm} \text{Eqn. G2-1}

*Calculate the available shear strength*

The values of $\phi_v = 1.00$ (LRFD) and $\Omega_v = 1.50$ (ASD) do not apply to channels. The general values $\phi_v = 0.90$ (LRFD) and $\Omega_v = 1.67$ (ASD) must be used.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_vV_n = 0.90(130 \text{ kips}) = 117 \text{ kips}$</td>
<td>$V_n/\Omega_v = 130 \text{ kips} / 1.67 = 77.6 \text{ kips}$</td>
</tr>
</tbody>
</table>
Example G.3 Angle in Shear.

Given:
Verify the shear strength of a 5×3×¼ (LLV) ASTM A36 angle with end shears of 3.5 kips from dead load and 10.5 kips from live load.

Solution:

Material Properties:
L5×3×¼ angle ASTM A36 $F_y = 36$ ksi $F_u = 58$ ksi

Geometric Properties:
L5×3×¼ angle $d = 5.00$ in. $t_w = 0.250$ in.

Calculate the required shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u = 1.2(3.50 \text{ kips}) + 1.6(10.5 \text{ kips})$</td>
<td>$V_u = 3.50 \text{ kips} + 10.5 \text{ kips}$</td>
</tr>
<tr>
<td>= 21.0 kips</td>
<td>= 14.0 kips</td>
</tr>
</tbody>
</table>

Note: There are no tables for angles in shear, but the shear strength can be calculated as follows:

For angles in shear, use Specification Equation G2-1 with $C_v = 1.0$.

Calculate $A_w$

$A_w = dt_w = (5.00 \text{ in.})(0.250 \text{ in.}) = 1.25 \text{ in.}^2$

Calculate $V_n$

$V_n = 0.6F_yA_wC_v = 0.6(36 \text{ ksi})(1.25 \text{ in.}^2)(1.0) = 27.0 \text{ kips}$

Calculate the available shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_vV_u = 0.90(27.0 \text{ kips}) = 24.3 \text{ kips}$</td>
<td>$V_u\Omega_v = 27.0 \text{ kips} / 1.67 = 16.2 \text{ kips}$</td>
</tr>
<tr>
<td>24.3 kips &gt; 21.0 kips o.k.</td>
<td>16.2 kips &gt; 14.0 kips o.k.</td>
</tr>
</tbody>
</table>
Example G.4  Rectangular HSS in Shear.

Given:

Verify the shear strength of a HSS6×4×a ASTM A500 grade B member with end shears of 11 kips from dead load and 33 kips from live load. The beam is oriented with the shear parallel to the 6 in. dimension.

Solution:

Material Properties:
HSS6×4×a  ASTM A500B  \( F_y = 46 \) ksi  \( F_u = 58 \) ksi  

Geometric Properties:
HSS6×4×a  \( h = 6.00 \) in.  \( w = 4.00 \) in.  \( t = 0.349 \) in.  

Calculate the required shear strength

\[
\begin{align*}
V_u & = 1.2(11.0 \text{kips}) + 1.6(33.0 \text{kips}) \\
& = 66.0 \text{kips}
\end{align*}
\]

\[
\begin{align*}
V_a & = 11.0 \text{kips} + 33.0 \text{kips} \\
& = 44.0 \text{kips}
\end{align*}
\]

\( \phi_v = 0.90 \)

\[
\begin{align*}
\phi_v V_a & = 0.90(75.2 \text{kips}) = 67.7 \text{kips} \\
67.7 \text{kips} & > 66.0 \text{kips} \quad \text{o.k.}
\end{align*}
\]

User note: There are no Manual Tables for shear in HSS shapes, but the shear strength can be calculated as follows:

Calculate the nominal strength

For rectangular HSS in shear, use Section G2.1 with \( A_w = 2ht \) and \( k_v = 5 \).

If the exact radius is unknown, the radius is taken as 3 times the design thickness.

\[ \begin{align*}
h & = d - 2(3t_w) = 6.00 \text{ in.} - 2(3)(0.349 \text{ in.}) = 3.91 \text{ in.} \\
h/t_w & = 3.91 \text{ in.} / 0.349 \text{ in.} = 11.2
\end{align*} \]

\[
\begin{align*}
1.10 \sqrt{\frac{k_v}{E/F_y}} & = 1.10 \sqrt{\frac{5(29,000 \text{ ksi})}{46 \text{ ksi}}} = 61.8 \\
11.2 & \leq 61.8 \quad \text{Therefore } C_v = 1.0
\end{align*}
\]

Note: most standard HSS sections listed in the manual have \( C_v = 1.0 \) at \( F_y \leq 46 \text{ ksi} \).

Calculate \( A_w \)

\[
A_w = 2ht = 2(3.91 \text{ in.})(0.349 \text{ in.}) = 2.73 \text{ in.}^2
\]

Calculate \( V_n \)

\[
V_n = 0.6F_yA_wC_v = 0.6(46 \text{ ksi})(2.73 \text{ in.}^2)(1.0) = 75.2 \text{kips}
\]

Calculate the available shear strength

\[
\begin{align*}
\phi_v = 0.90 \\
\phi_v V_n & = 0.90(75.2 \text{kips}) = 67.7 \text{kips} \\
67.7 \text{kips} & > 66.0 \text{kips} \quad \text{o.k.}
\end{align*}
\]

\[
\begin{align*}
\Omega_v & = 1.67 \\
V_n/\Omega_v & = 75.2 \text{kips} / 1.67 = 45.0 \text{kips} \\
45.0 \text{kips} & > 44.0 \text{kips} \quad \text{o.k.}
\end{align*}
\]
Example G.5  Round HSS in Shear.

Given:

Verify the shear strength of a round HSS16.000×0.375 ASTM A500 grade B member spanning 32 feet with end shears of 30 kips from dead load and 90 kips from live load.

Solution:

Material Properties:

| HSS16.000×0.375 | ASTM A500 Gr.B | $F_y = 42$ ksi | $F_u = 58$ ksi | Manual Table 2-3 |

Geometric Properties:

| HSS16.000×0.375 | D = 16.0 in. | t = 0.349 in. | $A_g = 17.2$ in.$^2$ | Manual Table 1-13 |

Note: There are no Manual tables for Round HSS in shear, but the strength can be calculated as follows:

Calculate the required shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_u = 1.2(30.0 \text{ kips}) + 1.6(90.0 \text{ kips}) = 180 \text{ kips}$</td>
<td>$V_u = 30.0 \text{ kips} + 90.0 \text{ kips} = 120 \text{ kips}$</td>
</tr>
</tbody>
</table>

Calculate $F_{cr}$ as the smallest of:

$$F_{cr} = \frac{1.60E}{\sqrt{\frac{L_v}{D} \left(\frac{D}{t}\right)^3}}$$

take $L_v$ as half the span = 192 in.

$$= \frac{1.60(29,000 \text{ksi})}{\sqrt{\frac{192 \text{ in.}}{16.0 \text{ in.}} \left(\frac{16.0 \text{ in.}}{0.349 \text{ in.}}\right)^3}} = 112 \text{ ksi}$$

or

$$F_{cr} = \frac{0.78E}{(D/t)^2} = \frac{0.78(29,000 \text{ksi})}{(16.0 \text{ in.} / 0.349 \text{ in.})^2} = 73 \text{ ksi}$$

or

$$F_{cr} = 0.6F_y = 0.6(42 \text{ksi}) = 25.2 \text{ksi} \quad \text{controls}$$

Note: Equations G6-2a and G6-2b will not normally control for the sections published in the Manual except when high strength steel is used or the span is unusually long.

Calculate $V_n$

$$V_n = \frac{F_{cr}A_g}{2} = \frac{(25.2 \text{ ksi})(17.2 \text{ in.$^2$})}{2} = 217 \text{ kips}$$
Calculate the available shear strength  

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$\phi_v V_n = 0.90 (217 \text{ kips}) = 195 \text{ kips}$</td>
<td>$V_n / \Omega_v = 217 \text{ kips} / 1.67 = 130 \text{ kips}$</td>
</tr>
<tr>
<td>195 kips &gt; 190 kips  o.k.</td>
<td>130 kips &gt; 120 kips  o.k.</td>
</tr>
</tbody>
</table>
Example G.6 Doubly-Symmetric Shape in Weak-Axis Shear.

Given:
Verify the strength of a W21×48 ASTM A992 beam with end shears of 20 kips from dead load and 60 kips from live load in the weak direction.

Solution:

Material Properties:
W21×48 ASTM A992 \( F_y = 50 \text{ ksi} \) \( F_u = 65 \text{ ksi} \) Manual Table 2-3

Geometric Properties:
W21×48 \( b_f = 8.14 \text{ in.} \) \( t_f = 0.430 \text{ in.} \) Manual Table 1-1

Calculate the required shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips}) ) 120 kips</td>
<td>( V_u = 20.0 \text{ kips} + 60.0 \text{ kips} = 80.0 \text{ kips} )</td>
</tr>
</tbody>
</table>

For weak axis shear, use Equation G2-1 and Section G2.1(b) with \( A_w = b_ft_f \) for each flange and \( k_v = 1.2 \).

Calculate \( A_w \) (multiply by 2 for both shear resisting elements)

\( A_w = 2b_ft_f = 2(8.14 \text{ in.})(0.430 \text{ in.}) = 7.00 \text{ in}^2 \)

Calculate \( C_v \)

\( b_f/t_f = 8.14 \text{ in.} / 0.430 \text{ in.} = 18.9 \)

\[ 1.10\sqrt{k_v E/F_y} = 1.10\sqrt{1.2(29,000 \text{ ksi})/50 \text{ ksi}} = 29.0 > 18.9 \text{ therefore, } C_v = 1.0 \]

Note: For all ASTM A6 W, S, M, and HP shapes, when \( F_y \leq 50 \text{ ksi} \), \( C_v = 1.0 \).

Calculate \( V_n \)

\( V_n = 0.6F_yA_wC_v = 0.6(50 \text{ ksi})(7.00 \text{ in.}^2)(1.0) = 210 \text{ kips} \)

Calculate the available shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi V_n = 0.90(210 \text{ kips}) = 189 \text{ kips} ) 189 kips &gt; 120 kips</td>
<td>( \Omega V_n = 1.67 ) ( V_n/\Omega_n = 210 \text{ kips} / 1.67 = 126 \text{ kips} ) 126 kips &gt; 80.0 kips</td>
</tr>
</tbody>
</table>
Example G.7 Singly-Symmetric Shape in Weak-Axis Shear.

Given:
Verify the strength of a C9×20 ASTM A36 channel with end shears of 5 kips from dead load and 15 kips from live load in the weak direction.

Solution:

Material Properties:
C9×20 ASTM A36  
$F_y = 36$ ksi  $F_u = 58$ ksi  Manual Table 2-3

Geometric Properties:
C9×20  
$b_f = 2.65$ in.  $t_f = 0.413$ in.  Manual Table 1-5

Note: There are no Manual tables for this, but the strength can be calculated as follows:

Calculate the required shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| $V_u = 1.2(5.00 \text{ kips}) + 1.6(15.0 \text{ kips})$  
$= 30.0 \text{ kips}$ | $V_u = 5.00 \text{ kips} + 15.0 \text{ kips}$  
$= 20.0 \text{ kips}$ |

For weak axis shear, use Equation G2-1 and Section G2.1(b) with $A_w = b_f t_f$ for each flange and $k_v = 1.2$.

Calculate $A_w$ (multiply by 2 for both shear resisting elements)

$A_w = 2b_f t_f = 2(2.65 \text{ in.})(0.413 \text{ in.}) = 2.19 \text{ in.}^2$

Calculate $C_v$

$b_f / t_f = 2.65 \text{ in.} / 0.413 \text{ in.} = 6.42$

$1.10 \sqrt{k_v E / F_y} = 1.10 \sqrt{1.2(29,000 \text{ ksi})/36 \text{ ksi}} = 34.2 > 6.42$  Therefore, $C_v = 1.0$

Eqn. G2-3

Calculate $V_n$

$V_n = 0.6 F_y A_w C_v = 0.6(36 \text{ ksi})(2.19 \text{ in.}^2)(1.0) = 47.3 \text{ kips}$

Eqn. G2-1

Calculate the available shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| $\phi V_n = 0.90(47.3 \text{ kips}) = 42.6 \text{ kips}$  
$42.5 \text{ kips} > 30.0 \text{ kips}$ | $\Omega_v = 1.67$  
$V_n / \Omega_v = 47.3 \text{ kips} / 1.67 = 28.3 \text{ kips}$  
$28.3 \text{ kips} > 20.0 \text{ kips}$ |

o.k.  o.k.
Example G.8a  Built-up Plate Girder with Transverse Stiffeners

Given:

A built up ASTM A36 I-shaped girder spanning 56 ft. has a uniformly distributed dead load of 0.92 klf and a live load of 2.74 klf in the strong direction. The girder is 36 in. tall with 12 in. × 1½ in. flanges and a ½ in. web. Determine if the member has sufficient available shear strength to support the end shear, without and with tension field action. Use transverse stiffeners, as required.

User note: This built-up girder was purposely selected with a thin web in order to illustrate the design of transverse stiffeners. A more conventionally proportioned plate girder would have at least a ½ in. web and slightly smaller flanges.

Solution:

Material Properties:

Built-up girder  ASTM A36  $F_y = 36$ ksi  $F_u = 58$ ksi  Manual Table 2-3

Geometric Properties:

Built-up girder  $t_w = 0.313$ in.  $d = 36.0$ in.  $b_f = b_w = 12.0$ in.

Calculate the required shear strength at the support

LRFD  ASD

<table>
<thead>
<tr>
<th>$R_u$</th>
<th>$R_u = (0.92 	ext{ klf} + 2.74 	ext{ klf})(28.0 	ext{ ft})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>154 kips</td>
<td>$102$ kips</td>
</tr>
</tbody>
</table>

Determine if stiffeners are required

$A_w = dt_w = (36.0 	ext{ in.})(0.313 	ext{ in.}) = 11.3$ in.$^2$

$h/t_w = 33.0 	ext{ in.} / 0.313 	ext{ in.} = 105$

$105 < 260$ Therefore, $k_v = 5$  Section G2.1b

$1.37 \sqrt{k_v} E / F_y = 1.37 \sqrt{5}(29,000 	ext{ ksi})/(36 	ext{ ksi}) = 86.9$

$105 > 86.9$ therefore, use Specification Eqn. G2-5 to calculate $C_v$
\[ C_v = \frac{1.51 E k_w}{(h/t_w)^2 F_y} = \frac{1.51(29,000 \text{ ksi})(5)}{(105)^2(36 \text{ ksi})} = 0.547 \]  

Eqn. G2-5

Calculate \( V_n \)

\[ V_n = 0.6 F_y A_w C_v = 0.6(36 \text{ ksi})(11.3 \text{ in.}^2)(0.547) = 134 \text{ kips} \]  

Eqn. G2-1

Check the available shear strength without stiffeners

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 0.90 )</td>
<td>( \Omega_v = 1.67 )</td>
</tr>
<tr>
<td>( \phi_v V_n = 0.90(134 \text{ kips}) = 120 \text{ kips} )</td>
<td>( V_n / \Omega_v = 134 \text{ kips} / 1.67 = 80.0 \text{ kips} )</td>
</tr>
<tr>
<td>120 kips &lt; 154 kips</td>
<td>80.0 kips &lt; 102 kips</td>
</tr>
</tbody>
</table>

Therefore, stiffeners are required.

Section G1

Manual Tables 3-16a and 3-16b can be used to select stiffener spacings needed to develop the required stress in the web.

Limits on the Use of Tension Field Action:

Consideration of tension field action is not permitted if any of the following are true:

1. a) end panels in all members with transverse stiffeners
2. b) members when \( a/h \) exceeds 3.0 or \([260/(h/t_w)]^2\)
3. c) \( 2A_w/(A_{fc} + A_{ft}) > 2.5 \)
4. d) \( h/b_{fc} \) or \( h/b_{ft} > 6.0 \)

Select stiffener spacing for end panel

Tension field action is not permitted for end panels, therefore use Table 3-16a.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Use ( V_u = \phi_v V_n ) to determine the required stress in the web by dividing by the web area</td>
<td>Use ( V_u = V_u/\Omega_v ) to determine the required stress in the web by dividing by the web area</td>
</tr>
<tr>
<td>( \phi_v V_n / A_w = 154 \text{ kips} / 11.3 \text{ in.}^2 = 13.6 \text{ ksi} )</td>
<td>( V_u / \Omega_v A_w = 102 \text{ kips} / 11.3 \text{ in.}^2 = 9.03 \text{ ksi} )</td>
</tr>
</tbody>
</table>

Use Table 3-16a from the Manual to select the required stiffener ratio \( a/h \) based on the \( h/t \) ratio of the girder and the required stress. Interpolate and follow an available stress curve, \( \frac{\phi_v V_n}{A_w} = 13.6 \text{ ksi} \) for LRFD, \( \frac{V_u}{\Omega_v A_w} = 9.03 \text{ ksi} \) for ASD, until it intersects the horizontal line for a \( h/t_w \) value of 105. Project down from this intersection and take the maximum \( a/h \) value of 1.80 from the axis across the bottom. Since \( h = 33.0 \text{ in.} \), stiffeners are required at \((1.80)(33.0 \text{ in.}) = 59.4 \text{ in.} \) maximum. Therefore, use 59.0 in.

Select stiffener spacing for the second panel

Tension field action is allowed, but not required, since the second panel is not an end panel.

Section G3.1
Calculate the required shear strength at the start of the second panel, 59 in. from end

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 154(k - [1.2(0.92\text{klf}) + 1.6(2.74\text{klf})]) )</td>
<td>( V_o = 102k - (0.92\text{klf} + 2.74\text{klf}) )</td>
<td></td>
</tr>
<tr>
<td>(59.0 in. / 12 in./ft)</td>
<td>(59.0 in. / 12 in./ft)</td>
<td></td>
</tr>
<tr>
<td>= 127 kips</td>
<td>= 84.0 kips</td>
<td></td>
</tr>
</tbody>
</table>

Check the available shear strength without stiffeners

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_v = 0.90 )</td>
<td>( \Omega_v = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>\From previous calculation( \phi_v \cdot V_u = 0.90(134 \text{kips}) = 120 \text{kips} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>120 kips &lt; 127 kips</td>
<td>80.0 kips &lt; 84.0 kips</td>
<td></td>
</tr>
<tr>
<td>not o.k.</td>
<td>not o.k.</td>
<td></td>
</tr>
<tr>
<td>Therefore additional stiffeners are required.</td>
<td>Therefore additional stiffeners are required.</td>
<td></td>
</tr>
<tr>
<td>Use ( V_u - \phi_v \cdot V_u ) to determine the required stress in the web by dividing by the web area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{\phi_v \cdot V_u}{A_w} = \frac{V_u}{A_w} = 127 \text{kips} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{11.3 \text{in.}^2}{} = 11.2 \text{ksi} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use ( V_o = V_o \cdot \Omega_v ) to determine the required stress in the web by dividing by the web area</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{V_o}{\Omega_v \cdot A_w} = \frac{V_o}{A_w} = 84.0 \text{kips} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( \frac{11.3 \text{in.}^2}{} = 7.43 \text{ksi} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use Table 3-16b from the Manual to select the required stiffener \( a/h \) ratio based on the \( h/t \) ratio of the girder and the required stress. Interpolate and follow an available stress curve, \( \frac{\phi_v \cdot V_u}{A_w} = 11.2 \text{ksi} \) for LRFD, \( \frac{V_o}{\Omega_v \cdot A_w} = 7.43 \text{ksi} \) for ASD, until it intersects the horizontal line for a \( h/t \) value of 105. Because the available stress does not intersect the \( h/t \) value of 105, the maximum value of 3.0 for \( a/h \) may be used. Since \( h = 33.0 \text{ in.} \), an additional stiffener is required at (3.0)(33.0 in.) = 99.0 in. maximum from the previous one.

Select stiffener spacing for the second panel

Tension field action is allowed, but not required, since the next panel is not an end panel.

Calculate the required shear strength at the start of the third pane, 158 in. from end

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 154 \text{kips} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( - [1.2(0.920 \text{klf}) + 1.6(2.74 \text{klf})] ) (158.0 in. / 12 in./ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 81.7 kips</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( V_o = 102 \text{kips} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( - (0.920 \text{klf} + 2.74 \text{klf}) ) (158.0 in. / 12 in./ft)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>= 53.8 kips</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Check the available shear strength without stiffeners

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>From previous calculation</td>
<td>From previous calculation</td>
</tr>
<tr>
<td>$\phi_v V_n = 0.90(134 \text{ kips}) = 120 \text{ kips}$</td>
<td>$V_n/\Omega_v = 134 \text{ kips} / 1.67 = 80.0 \text{ kips}$</td>
</tr>
<tr>
<td>120 kips &gt; 81.7 kips</td>
<td>80.0 kips &gt; 53.8 kips</td>
</tr>
<tr>
<td>Therefore additional stiffeners are not required.</td>
<td>Therefore additional stiffeners are not required.</td>
</tr>
</tbody>
</table>

The four Available Shear Stress tables, Manual Tables 3-16a, 3-16b, 3-17a and 3-17b, are useful because they permit a direct solution for the required stiffener spacing. Alternatively, you can select a stiffener spacing and check the resulting strength, although this process is likely to be iterative. In the proof below, the stiffener spacings that were selected from the charts in the example above are used.
Example G.8b  Built-up Plate Girder with Transverse Stiffeners

The stiffener spacings from Example G.8a, which were easily determined from the tabulated values of the *Steel Construction Manual*, are verified below by directly applying the provisions of the Specification.

**Verify the shear strength of end panel**

Section G2.1b

\[
\frac{5}{(a/h)^2} = 5 + \frac{5}{(1.80)^2} = 6.54
\]

\[
\frac{h}{t_w} = 33.0 \text{ in.} / 0.313 \text{ in.} = 105
\]

**Check a/h limits**

\[
a/h = \frac{59 \text{ in.}}{33 \text{ in.}} = 1.80 \leq 3.0
\]

\[
a/h = 1.80 < \left[ \frac{260}{(h/t_w)} \right]^2 = \left[ \frac{260}{105} \right]^2 = 6.13
\]

Therefore, use \( k_v = 6.54 \).

Tension field action is not allowed since the panel is an end panel.

Since \( h/t_w > 1.37 \sqrt{k_v E/F_y} = 1.37 \sqrt{(6.54)(29,000 \text{ ksi})/(36 \text{ ksi})} = 99.5 \),

\[
C_v = \frac{1.51 E k_v}{(h/t_w)^2 F_y} = \frac{1.51(29,000 \text{ ksi})(6.54)}{(105)^2 36 \text{ ksi}} = 0.721
\]

**Calculate the available shear strength**

Section G2.1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi F_y = 0.90(176 \text{ kips}) = 159 \text{ kips} )</td>
<td>( V_n \Omega = (176 \text{ kips}) / 1.67 = 106 \text{ kips} )</td>
</tr>
<tr>
<td>159 kips &gt; 154 kips</td>
<td>106 kips &gt; 102 kips</td>
</tr>
</tbody>
</table>

**Verify the shear strength of the second panel**

\[
a/h \text{ for the second panel was 3.0}
\]

\[
\frac{5}{(a/h)^2} = 5 + \frac{5}{(3.0)^2} = 5.56
\]

**Check a/h limits**

\[
a/h = \frac{99 \text{ in.}}{33 \text{ in.}} = 3.00 \leq 3.0
\]

\[
a/h = 3.00 < \left[ \frac{260}{(h/t_w)} \right]^2 = \left[ \frac{260}{105} \right]^2 = 6.13
\]

Therefore use \( k_v = 5.56 \).
Since \( h/t_o > 1.37 \sqrt{k_x E / F_y} = 1.37 \sqrt{(5.56)(29,000 ksi)/(36 ksi)} = 91.7 \),

\[
C_v = \frac{1.51 k_x E \sqrt{h/t_o}}{(h/t_o)^2 F_y} = \frac{1.51(29,000 ksi)(5.56)}{(105)^2(36 ksi)} = 0.613
\]

Eqn. G2-5

Since \( h/t_o > 1.10 \sqrt{k_x E / F_y} = 1.10 \sqrt{5.56(29,000 ksi)/(36 ksi)} = 73.6 \), use Eqn. G3-2

\[
V_n = 0.6 F_v A_v \left[ C_v + \frac{1-C_v}{1.15 \sqrt{1+(a/h)^2}} \right]
\]

\[
= 0.6(36 ksi)(11.3 \text{ in.}^2) \left[ 0.613 + \frac{1-0.613}{1.15 \sqrt{1+(3.0)^2}} \right]
\]

\[ V_n = 176 \text{ kips} \]

Calculate the available shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section G1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega_v = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>( \phi V_n = 0.90(176 \text{ kips}) = 158 \text{ kips} )</td>
<td>( V_n \Omega_v = (176 \text{ kips}) / 1.67 = 105 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>156 kips &gt; 127 kips</td>
<td>105 kips &gt; 84.0 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
CHAPTER H
DESIGN OF MEMBERS FOR COMBINED FORCES AND TORSION

For all interaction equations in Specification Section H, the required forces and moments must include the results of a second-order analysis, as required by Section C of the Specification. This represents a significant change for ASD users, who are accustomed to using an interaction equation that includes a partial second-order amplification.
Example H.1a  W-shape Subjected to Combined Compression and Bending About Both Axes (braced frame).

Given:
Verify if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed below, obtained from a second order analysis that includes P-δ effects. The unbraced length is 14 ft and the member has pinned ends. \( KL_x = KL_y = L_b = 14.0 \) ft

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_a = 400 ) kips</td>
<td>( P_a = 267 ) kips</td>
</tr>
<tr>
<td>( M_{ax} = 250 ) kip-ft</td>
<td>( M_{ax} = 167 ) kip-ft</td>
</tr>
<tr>
<td>( M_{ay} = 80.0 ) kip-ft</td>
<td>( M_{ay} = 53.3 ) kip-ft</td>
</tr>
</tbody>
</table>

Solution:

Material Properties:
ASTM A992 \( F_y = 50 \) ksi \( F_u = 65 \) ksi

Try a W14×99

Take combined strength parameters from Manual Table 6-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( p = \frac{0.866}{10^3 \text{kips}} ) at 14.0 ft</td>
<td>( p = \frac{1.33}{10^3 \text{kips}} ) at 14.0 ft</td>
</tr>
<tr>
<td>( b_x = \frac{1.38}{10^3 \text{kip-ft}} ) at 14.0 ft</td>
<td>( b_x = \frac{2.08}{10^3 \text{kip-ft}} ) at 14.0 ft</td>
</tr>
<tr>
<td>( b_y = \frac{2.85}{10^3 \text{kip-ft}} )</td>
<td>( b_y = \frac{4.29}{10^3 \text{kip-ft}} )</td>
</tr>
</tbody>
</table>

Check limit for Equation H1-1a
\[
p \frac{p_a}{\phi P_a} = 0.866 \left( \frac{400 \text{kips}}{1,130 \text{kips}} \right) = 0.354
\]
Since \( \frac{p_a}{\phi P_a} = 0.2 \),
\[
p P_a + b_x M_{ax} + b_y M_{ay} \leq 1.0
\]
\[
= 0.866 \left( \frac{400 \text{kips}}{10^3 \text{kips}} \right) \left( 250 \text{kip-ft} \right)
+ \left( \frac{1.38}{10^3 \text{kip-ft}} \right) \left( 80.0 \text{kip-ft} \right)
+ \left( \frac{2.85}{10^3 \text{kip-ft}} \right) \left( 250 \text{kip-ft} \right)
= 0.346 + 0.345 + 0.228 = 0.927 \leq 1.0 \text{ o.k.}
\]

Check limit for Equation H1-1a
\[
p \frac{p_a}{\phi P_a} = \frac{267 \text{kips}}{751 \text{kips}} = 0.356
\]
Since \( \frac{p_a}{\phi P_a} = 0.2 \),
\[
p P_a + b_x M_{ax} + b_y M_{ay} \leq 1.0
\]
\[
= \frac{1.33}{10^3 \text{kips}} \left( 267 \text{kips} \right)
+ \left( \frac{2.08}{10^3 \text{kip-ft}} \right) \left( 167 \text{kip-ft} \right)
+ \left( \frac{4.29}{10^3 \text{kip-ft}} \right) \left( 53.3 \text{kip-ft} \right)
= 0.356 + 0.347 + 0.229 = 0.931 \leq 1.0 \text{ o.k.}
\]

Manual Table 6-1 simplifies the calculation of Specification Equations H1-1a and H1-1b. A direct application of these equations is shown in Example H.2.
Example H.1b  W-shape Column Subjected to Combined Compression and Bending Moment About Both Axes (braced frame)

Verify if an ASTM A992 W14×99 has sufficient available strength to support the axial forces and moments listed below, obtained from a second order analysis that includes second-order effects. The unbraced length is 14 ft and the member has pinned ends. $KL_x = KL_y = L_b = 14.0$ ft

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$ = 400 kips</td>
<td>$P_u$ = 267 kips</td>
<td></td>
</tr>
<tr>
<td>$M_{ax}$ = 250 kip-ft</td>
<td>$M_{ax}$ = 167 kip-ft</td>
<td></td>
</tr>
<tr>
<td>$M_{ay}$ = 80.0 kip-ft</td>
<td>$M_{ay}$ = 53.3 kip-ft</td>
<td></td>
</tr>
</tbody>
</table>

**Solution:**

**Material Properties:**

- ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi  
  - Manual  Table 2-3

Take the available axial and flexural strengths from the Manual Tables

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>at $KL_y = 14.0$ ft,</td>
<td>$P_c = \phi_c P_u = 1130$ kips</td>
<td></td>
</tr>
<tr>
<td>$P_c = \phi_c P_u$ = 1,130 kips</td>
<td></td>
<td></td>
</tr>
<tr>
<td>at $L_b = 14.0$ ft,</td>
<td>$M_{ax} = \phi M_{ax} = 642$ kip-ft</td>
<td></td>
</tr>
<tr>
<td>$M_{cy} = \phi M_{cy} = 311$ kip-ft</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\frac{P_u}{\phi_c P_u} = 0.354$</td>
<td>$\frac{P_u}{\phi_c P_u} = 0.356$</td>
<td></td>
</tr>
<tr>
<td>Since $\frac{P_u}{\phi_c P_u} &gt; 0.2$, use Eqn. H1.1a</td>
<td>Since $\frac{P_u}{\phi_c P_u} &gt; 0.2$, use Eqn. H1.1a</td>
<td></td>
</tr>
<tr>
<td>$\frac{P_c}{P_u} + \frac{8}{9} \left(\frac{M_{ax}}{M_{ax}} + \frac{M_{cy}}{M_{cy}}\right) \leq 1.0$</td>
<td>$\frac{P_c}{P_u} + \frac{8}{9} \left(\frac{M_{ax}}{M_{ax}} + \frac{M_{cy}}{M_{cy}}\right) \leq 1.0$</td>
<td></td>
</tr>
<tr>
<td>400 kips 8 \left(\frac{250}{642} \text{kip-ft} + \frac{80.0}{311} \text{kip-ft}\right)</td>
<td>267 kips 8 \left(\frac{167}{428} \text{kip-ft} + \frac{53.3}{207} \text{kip-ft}\right)</td>
<td></td>
</tr>
<tr>
<td>1130 kips 9 \left(\frac{311}{642} \text{kip-ft}\right)</td>
<td>751 kips 9 \left(\frac{311}{428} \text{kip-ft}\right)</td>
<td></td>
</tr>
<tr>
<td>= 0.354 + \frac{8}{9}(0.389 + 0.257) = 0.929 &lt; 1.0</td>
<td>= 0.356 + \frac{8}{9}(0.390 + 0.257) = 0.931 &lt; 1.0</td>
<td></td>
</tr>
</tbody>
</table>

o.k.  o.k.
Example H.2  W-Shape Column Subjected to Combined Compression and Bending Moment About Both Axes  
(by Specification Section H2)

Given:
Verify if an ASTM A992 W14×99 shown in Example H.1 has sufficient available strength, using Specification Section H2.1. This example is included primarily to illustrate the use of Specification Section H2.  \( KL_x = KL_y = L_b = 14.0 \text{ ft} \)

Solution:

Material Properties:  
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Geometric Properties:  
W14×99  \( A = 29.1 \text{ in.}^2 \)  \( S_x = 157 \text{ in.}^3 \)  \( S_y = 55.2 \text{ in.}^3 \)

Calculate the required flexural and axial stresses

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
\hline
P_a = 400 \text{ kips} & P_a = 267 \text{ kips} \\
M_{ax} = 250 \text{ kip-ft} & M_{ax} = 167 \text{ kip-ft} \\
M_{ay} = 80.0 \text{ kip-ft} & M_{ay} = 53.3 \text{ kip-ft} \\
\end{array}
\]

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
\hline
f_u = \frac{P_a}{A} & f_u = \frac{P_a}{A} \\
f_{bc} = \frac{M_{ax}}{S_x} & f_{bc} = \frac{M_{ax}}{S_x} \\
f_{bc} = \frac{M_{ay}}{S_y} & f_{bc} = \frac{M_{ay}}{S_y} \\
\end{array}
\]

Calculate the available flexural and axial stresses from the available strengths in Example H.1b

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
\hline
F_a = \phi_c F_{cr} & F_a = \phi_c F_{cr} \\
F_{bc} = \phi_c M_{ax} & F_{bc} = \phi_c M_{ax} \\
F_{bc} = \phi_c M_{ay} & F_{bc} = \phi_c M_{ay} \\
\end{array}
\]

As shown in the LRFD calculation of \( F_{bc} \) above, the available flexural stresses can exceed the yield stress in cases where the available strength is governed by yielding and the yielding strength is calculated using the plastic section modulus.
Calculate the combined stress ratio

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Eqn. H2-1</th>
</tr>
</thead>
</table>
| \[
\frac{f_a + f_{bw} + f_{bc}}{F_a + F_{bw} + F_{bc}} \leq 1.0
\] | \[
\frac{f_a + f_{bw} + f_{bc}}{F_a + F_{bw} + F_{bc}} \leq 1.0
\] | \[
13.7 \text{ ksi} + 19.1 \text{ ksi} + 17.4 \text{ ksi} = 0.998 < 1.0
\] | 9.18 ksi + 12.8 ksi + 11.6 ksi = 1.00 o.k. |
| \[
\frac{38.8 \text{ ksi} + 49.3 \text{ ksi}}{67.6 \text{ ksi}} = \text{ o.k.}
\] | |

A comparison of these results with those from Example H.1 shows that Equation H1-1a will produce less conservative results than Equation H2-1 when its use is permitted.

Note: this check is made at a point. The designer must therefore select which point along the length is critical, or check multiple points if the critical point cannot be readily determined.
Example H.3  W-Shape Subject to Combined Axial Tension and Flexure.

Given:
Select an ASTM A992 W-shape with a 14-in. nominal depth to carry nominal forces of 29 kips from dead load and 87 kips from live load in axial tension, as well as the following nominal moments:

\[ M_{xD} = 32.0 \text{ kip-ft} \quad M_{xL} = 96.0 \text{ kip-ft} \]
\[ M_{yD} = 11.3 \text{ kip-ft} \quad M_{yL} = 33.8 \text{ kip-ft} \]

The unbraced length is 30 ft and the ends are pinned. Assume the connections are made with no holes.

Solution:

Material Properties:
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_x = 1.2(29.0 \text{ kips}) + 1.6(87.0 \text{ kips}) ) = 174 kips</td>
<td>( P_u = 29.0 \text{ kips} + 87.0 \text{ kips} ) = 116 kips</td>
</tr>
<tr>
<td>( M_{ax} = 1.2(32.0 \text{ kip-ft}) + 1.6(96.0 \text{ kip-ft}) ) = 192 kip-ft</td>
<td>( M_{ay} = 1.2(32.0 \text{ kip-ft}) + 1.6(96.0 \text{ kip-ft}) ) = 128 kip-ft</td>
</tr>
<tr>
<td>( M_{ay} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft}) ) = 67.6 kip-ft</td>
<td>( M_{ay} = 1.2(11.3 \text{ kip-ft}) + 1.6(33.8 \text{ kip-ft}) ) = 45.1 kip-ft</td>
</tr>
</tbody>
</table>

Verify that a W14×82 has sufficient available strength

Geometric Properties:
W14×82  \( A = 24.0 \text{ in.}^2 \)  \( S_x = 123 \text{ in.}^3 \)  \( Z_x = 139 \text{ in.}^3 \)  \( S_y = 29.3 \text{ in.}^3 \)
\( Z_y = 44.8 \text{ in.}^3 \)  \( I_y = 148 \text{ in.}^4 \)  \( L_p = 8.76 \text{ ft} \)  \( L_r = 33.1 \text{ ft} \)

Calculate the nominal gross tensile strength

\[ P_n = F_yA_g = (50 \text{ ksi})(24.0 \text{ in.}^2) = 1200 \text{ kips} \]

Note that for a member with holes, the rupture strength of the member would also have to be computed using Specification Equation D2-2.

Calculate the nominal flexural strength for bending about the x-x axis

Yielding limit state

\[ M_{ax} = M_{py} = F_yZ_x = 50 \text{ ksi}(139 \text{ in.}^3) = 6950 \text{ kip-in} = 579 \text{ kip-ft} \]

Lateral-torsional buckling limit state

\[ L_0 = 30.0 \text{ ft} \]

Since \( L_p < L_0 \leq L_r \), Equation F2-2 applies
Calculate lateral-torsion buckling modification factor

From Manual Table 3-1, \( C_b = 1.14 \), without considering the beneficial effects of the tension force. However, \( C_b \) may be increased because the column is in axial tension.

\[
P_{ey} = \frac{\pi^2 EI_y}{L_b^2} = \frac{\pi^2 (29,000 \text{ ksi})(148 \text{ in.}^4)}{((30.0 \text{ ft})(12.0 \text{ in./ft}))^2} = 327 \text{ kips}
\]

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\sqrt{1 + \frac{P_n}{P_{ey}}} & \sqrt{1 + \frac{1.5P_n}{P_{ey}}} \\
\text{=} 1.24 & \text{=} 1.24 \\
\hline
\end{array}
\]

\( C_b = 1.24(1.14) = 1.41 \)

\[
M_n = C_b \left[ M_p - \left( M_p - 0.7F_yS_y \right) \frac{L_b - L_p}{L_p - L_p} \right] \leq M_p \quad \text{Eqn. F2-2}
\]

\[
M_n = 1.41 \left[ 6950 \text{ kip-in.} - \left( 6950 \text{ kip-in.} - 0.7(50 \text{ ksi})(123 \text{ in.}^3) \right) \frac{30.0 \text{ ft} - 8.76 \text{ ft}}{33.1 \text{ ft} - 8.76 \text{ ft}} \right]
\]

\[
= 6550 \text{ kip} < M_p \text{ therefore use:}
\]

\[
M_n = 6550 \text{ kip-in. or 545 kip-ft}
\]

controls

Local buckling limit state

Per Manual Table 1-1, the cross section is compact at \( F_y = 50 \text{ ksi} \); therefore, the local buckling limit state does not apply.

Calculate the nominal flexural strength for bending about the y-y axis

Yielding limit state

Since \( W14 \times 82 \) has compact flanges, only the limit state of yielding applies.

\[
M_{ny} = M_p = F_yZ_y \leq 1.6F_yS_y \quad \text{Eqn. F6-1}
\]

\[
= 50 \text{ ksi}(44.8 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(29.3 \text{ in.}^3)
\]

\[
= 2240 \text{ kip-in} < 2344 \text{ kip-in., therefore use:}
\]

\[
M_{ny} = 2240 \text{ kip-in. or 187 kip-ft}
\]
\[
\phi_b = \phi_t = 0.90
\]
\[
\phi_c = \phi_t = 0.90(1200 \text{ kips}) = 1080 \text{ kips}
\]
\[
M_{cx} = \phi_b M_{tc} = 0.90(545 \text{ kip-ft}) = 491 \text{ kip-ft}
\]
\[
M_{cy} = \phi_b M_{ty} = 0.90(187 \text{ kip-ft}) = 168 \text{ kip-ft}
\]
\[
\Omega_b = \Omega_t = 1.67
\]
\[
P_c = \phi_t P_n = 1200 \text{ kips} = 719 \text{ kips}
\]
\[
M_{cx} = M_{tc} / \Omega_t = 545 \text{ kip-ft} / 1.67 = 327 \text{ kip-ft}
\]
\[
M_{cy} = M_{ty} / \Omega_t = 187 \text{ kip-ft} / 1.67 = 112 \text{ kip-ft}
\]

Check limit for Equation H1-1a

\[
\frac{P_c}{\phi_t P_n} = \frac{174 \text{ kips}}{1080 \text{ kips}} = 0.161 < 0.2
\]

Therefore, Equation H1-1b applies

\[
\frac{P_c}{2P_c} + \left( \frac{M_{cx}}{M_{cx} + M_{cy}} \right) \leq 1.0
\]

Eqn. H1-1b

\[
\frac{174 \text{ kips} + 192 \text{ kip-ft}}{2(1080 \text{ kips}) + 491 \text{ kip-ft} + 168 \text{ kip-ft}} \leq 1.0
\]

0.874 < 1.0  \text{o.k.}

\[
\frac{116 \text{ kips} + 128 \text{ kip-ft}}{2(719 \text{ kips}) + 327 \text{ kip-ft} + 112 \text{ kip-ft}} \leq 1.0
\]

0.875 < 1.0  \text{o.k.}
Example H.4  W-Shape Subject to Combined Axial Compression and Flexure

**Given:** Select an ASTM A992 W-shape with a 10 in. nominal depth to carry nominal axial compression forces of 5 kips from dead load and 15 kips from live load. The unbraced length is 14 ft and the ends are pinned. The member also has the following nominal required moment strengths, not including second-order effects:

- \( M_{xD} = 15 \text{ kip-ft} \)
- \( M_{xL} = 45 \text{ kip-ft} \)
- \( M_{yD} = 2 \text{ kip-ft} \)
- \( M_{yL} = 6 \text{ kip-ft} \)

The member is not subject to sidesway.

**Solution:**

**Material Properties:**
ASTM A992  \( F_y = 50 \text{ ksi} \)  \( F_u = 65 \text{ ksi} \)

Calculate the required strength, not considering second-order effects

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} \\
P_u &= 1.2(5.00 \text{ kips}) + 1.6(15.0 \text{ kips}) = 30.0 \text{ kips} & P_u &= 5.00 \text{ kips} + 15.0 \text{ kips} = 20.0 \text{ kips} \\
M_{ux} &= 1.2(15.0 \text{ kip-ft}) + 1.6(45.0 \text{ kip-ft}) = 90.0 \text{ kip-ft} & M_{ux} &= 15.0 \text{ kip-ft} + 45.0 \text{ kip-ft} = 60.0 \text{ kip-ft} \\
M_{uy} &= 1.2(2.00 \text{ kip-ft}) + 1.6(6.00 \text{ kip-ft}) = 12.0 \text{ kip-ft} & M_{uy} &= 2.00 \text{ kip-ft} + 6.00 \text{ kip-ft} = 8.00 \text{ kip-ft}
\end{align*}
\]

Try a \( W_{10 \times 33} \)

**Geometric Properties:**

\( W_{10 \times 33} \quad A = 9.71 \text{ in.}^2 \quad S_x = 35.0 \text{ in.}^3 \quad Z_x = 38.8 \text{ in.}^3 \quad I_x = 171 \text{ in.}^4 \quad \text{Manual} \)

\( S_y = 9.20 \text{ in.}^3 \quad Z_y = 14.0 \text{ in.}^3 \quad I_y = 36.6 \text{ in.}^4 \quad \text{Table 1-1} \)

\( L_p = 6.85 \text{ ft} \quad L_r = 12.8 \text{ ft} \quad \text{Table 3-1} \)

**Calculate the available axial strength**

For a pinned-pinned condition, \( K = 1.0 \).

Since \( KL_x = KL_y = 14.0 \text{ ft} \) and \( r_x > r_y \), the y-y axis will govern.

\[
\begin{align*}
\text{LRFD} & \quad \text{ASD} & \quad \text{Manual} \\
P_c = \phi_c P_n = 253 \text{ kips} & \quad P_c = P_n/\Omega_c = 168 \text{ kips} & \quad \text{Table 4-1}
\end{align*}
\]
Calculate the required flexural strengths including second order amplification

Use “Amplified First-Order Elastic Analysis” procedure from Section C2.1b. Since the member is not subject to sidesway, only $P$-$\delta$ amplifiers need to be added.

$$B_i = \frac{C_m}{1 - \alpha P_i / P_{st}}$$ \hspace{1cm} \text{Eqn. C2-2}

$C_m = 1.0$

**X-X axis flexural magnifier**

$$P_{st} = \frac{\pi^2 EI}{(K_i L_s)^2} = \frac{\pi^2 (29,000 \text{ ksi})(171 \text{ in.}^4)}{((1.0)(14.0 \text{ ft})(12 \text{ in./ft}))^2} = 1730 \text{ kips}$$ \hspace{1cm} \text{Eqn. C2-5}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$B_i = \frac{1.0}{1 - 1.0(30.0 \text{ kips} / 1730 \text{ kips})} = 1.02$</td>
<td>$B_i = \frac{1.0}{1 - 1.6(20.0 \text{ kips} / 1730 \text{ kips})} = 1.02$ \hspace{1cm} \text{Eqn. C2-2}</td>
</tr>
<tr>
<td>$M_{ax} = 1.02(90.0 \text{ kip-ft}) = 91.8 \text{ kip-ft}$</td>
<td>$M_{ax} = 1.02(60.0 \text{ kip-ft}) = 61.2 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

**Y-Y axis flexural magnifier**

$$P_{st} = \frac{\pi^2 EI}{(K_i L_s)^2} = \frac{\pi^2 (29,000 \text{ ksi})(36.6 \text{ in.}^4)}{((1.0)(14.0 \text{ ft})(12 \text{ in./ft}))^2} = 371 \text{ kips}$$ \hspace{1cm} \text{Eqn. C2-5}

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha = 1.0$</td>
<td>$\alpha = 1.6$</td>
</tr>
<tr>
<td>$B_i = \frac{1.0}{1 - 1.0(30.0 \text{ kips} / 371 \text{ kips})} = 1.09$</td>
<td>$B_i = \frac{1.0}{1 - 1.6(20.0 \text{ kips} / 371 \text{ kips})} = 1.09$ \hspace{1cm} \text{Eqn. C2-2}</td>
</tr>
<tr>
<td>$M_{ay} = 1.09(12.0 \text{ kip-ft}) = 13.1 \text{ kip-ft}$</td>
<td>$M_{ay} = 1.09 (8.00 \text{ kip-ft}) = 8.76 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Calculate the nominal bending strength about the x-x axis

**Yielding limit state**

$$M_{ax} = M_p = F_y Z_x = 50 \text{ ksi}(38.8 \text{ in.}^3) = 1940 \text{ kip-in or 162 kip-ft}$$ \hspace{1cm} \text{Eqn. F2-1}

**Lateral-torsional buckling limit state**

Since $L_p < L_b < L_r$, Equation F2-2 applies

From Manual Table 3-1, $C_b = 1.14$ \hspace{1cm} \text{Manual Table 3-1}
\[ M_{ax} = C_s \left[ M_p \cdot 0.7F_yS_y \left( \frac{L_n - L_p}{L_n - L_p} \right) \right] \leq M_p \quad \text{Eqn. F2-2} \]

\[ M_{ax} = 1.14 \left[ 1940 \text{ kip}-\text{in.} \cdot \left( 1940 \text{ kip}-\text{in.} \cdot 0.7(50 \text{ ksi})(35.0 \text{ in.}^3) \right) \left( \frac{14.0 \text{ ft} - 6.85 \text{ ft}}{21.8 \text{ ft} - 6.85 \text{ ft}} \right) \right] \]

\[ = 1820 \text{ kip}-\text{in.} \leq 1940 \text{ kip}-\text{in.}, \text{ therefore use:} \]

\[ M_{ax} = 1820 \text{ kip}-\text{in.} \text{ or } 152 \text{ kip-ft} \]

**Local buckling limit state**

Per Manual Table 1-1, the member is compact for \( F_y = 50 \text{ ksi} \), so the local buckling limit state does not apply.

**Calculate the nominal bending strength about the y-y axis**

Since a W10×33 has compact flanges, only the yielding limit state applies.

\[ M_{ny} = M_p = F_yZ_y \leq 1.6F_yS_y \]

\[ = 50 \text{ ksi}(14.0 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(9.20 \text{ in.}^3) \]

\[ = 700 \text{ kip-in.} < 736 \text{ kip-in.}, \text{ therefore} \]

Use \( M_{ny} = 700 \text{ kip-in.} \text{ or } 58.3 \text{ kip-ft} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( M_{cx} = \phi_b M_{ax} = 0.90(152 \text{ kip-ft}) = 137 \text{ kip-ft} )</td>
<td>( M_{cx} = M_{ax}/\Omega_b = 152 \text{ kip-ft}/1.67 = 91.0 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( M_{cy} = \phi_b M_{ny} = 0.90(58.3 \text{ kip-ft}) = 52.5 \text{ kip-ft} )</td>
<td>( M_{cy} = M_{ny}/\Omega_b = 58.3 \text{ kip-ft}/1.67 = 34.9 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

**Check limit for Equation H1-1a**

\[ \frac{P_r}{2P_c} + \left( \frac{M_{ax}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \]

\[ = \frac{30.0 \text{ kips}}{2(253 \text{ kips})} + \left( \frac{91.8 \text{ kip-ft}}{137 \text{ kip-ft}} + \frac{13.1 \text{ kip-ft}}{52.5 \text{ kip-ft}} \right) \]

\[ = 0.0593 + 0.920 = 0.979 \leq 1.0 \quad \text{o.k.} \]

\[ \frac{P_r}{2P_c} + \left( \frac{M_{ax}}{M_{cx}} + \frac{M_{cy}}{M_{cy}} \right) \leq 1.0 \]

\[ = \frac{20.0 \text{ kips}}{2(168 \text{ kips})} + \left( \frac{61.2 \text{ kip-ft}}{91.0 \text{ kip-ft}} + \frac{8.76 \text{ kip-ft}}{34.9 \text{ kip-ft}} \right) \]

\[ = 0.0595 + 0.924 = 0.983 \leq 1.0 \quad \text{o.k.} \]
Example H.5a  Rectangular HSS Torsional Strength

Given:
Determine the available torsional strength of an ASTM A500 Gr. B HSS6×4×¼.

Solution:

Material Properties:
- ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  

Geometric properties:
- HSS6×4×¼  \( \frac{h}{t} = 22.8 \)  \( \frac{b}{t} = 14.2 \)  \( t = 0.233 \text{ in.} \)

Evaluate wall slenderness to determine the appropriate critical stress equation

\[
\frac{h}{t} > \frac{b}{t}, \text{ therefore, } \frac{h}{t} \text{ governs.}
\]

\[
\frac{h}{t} \leq 2.45 \sqrt{\frac{E}{F_y}}
\]

\[
22.8 \leq 2.45 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 61.5 \text{ therefore, use Eqn. H3-3}
\]

\[
F_{cr} = 0.6 F_y = 0.6(46 \text{ ksi}) = 27.6 \text{ ksi}
\]

Calculate the nominal torsional strength

\[
T_n = F_{cr} C = 27.6 \text{ ksi (10.1 in.\(^2\))} = 278 \text{ kip-in.}
\]

Calculate the available torsional strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_T = 0.90 )</td>
<td>( \Omega_T = 1.67 )</td>
</tr>
<tr>
<td>( \phi_T T_n = 0.90(278 \text{ kip-in.}) = 250 \text{ kip-in.} )</td>
<td>( T_n/\Omega_T = 278 \text{ kip-in.}/1.67 = 166 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>

Note: For more complete guidance on design for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*. 
Example H.5b  Round HSS Torsional Strength

Given:
Determine the available torsional strength of an ASTM A500 Gr. B HSS5.000×0.250 that is 14 ft long.

Solution:

Material Properties:
ASTM A500 Gr. B  \( F_y = 42 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Geometric properties:
HSS5.000×0.250  \( \frac{D}{t} = 21.5 \)  \( t = 0.233 \text{ in.} \)  \( D = 5.00 \text{ in.} \)

Calculate the available torsional strength

\( \phi_T = 0.90 \)
\( \phi_T T_n = 0.90(200 \text{ kip-in.}) = 180 \text{ kip-in.} \)
\( \Omega_T = 1.67 \)
\( T_n/\Omega_T = 200 \text{ kip-in./1.67} = 120 \text{ kip-in.} \)

Note: For more complete guidance on design for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*. 
Example H.5c  HSS Combined Torsional and Flexural Strength

Given:
Verify the strength of an ASTM A500 Gr. B HSS 6×4×1/4 loaded as shown. The beam is simply supported with torsionally fixed ends.

\[ w_D = 0.460 \text{ kip/ft applied 6 in. off centerline} \]
\[ w_L = 1.38 \text{ kip/ft applied 6 in. off centerline} \]

Solution:

Material Properties:
ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Geometric properties:
HSS 6×4×1/4  \[ \frac{h}{t} = 22.8 \]
\[ \frac{b}{t} = 14.2 \]
\[ t = 0.233 \text{ in.} \]
\[ Z_e = 8.53 \text{ in.}^3 \]

Calculate the required strengths

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( w_u )</td>
<td>( 1.2(0.460 \text{ kip/ft}) + 1.6(1.38 \text{ kip/ft}) ) ( = 2.76 \text{ kip/ft} )</td>
<td>( w_u = 0.460 \text{ kip/ft} + 1.38 \text{ kip/ft} ) ( = 1.84 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( V_r = V_u = \frac{w_l}{2} )</td>
<td>( = \frac{2.76 \text{ kip/ft}(8.00 \text{ ft})}{2} = 11.0 \text{ kips} )</td>
<td>( V_r = V_u = \frac{w_l}{2} ) ( = \frac{1.84 \text{ kip/ft}(8.00 \text{ ft})}{2} = 7.36 \text{ kips} )</td>
</tr>
<tr>
<td>( M_t = M_u = \frac{w_l l^2}{8} )</td>
<td>( = \frac{2.76 \text{ kip/ft}(8.00 \text{ ft})^2(12 \text{ in./ft})}{8} = 265 \text{ kip-in.} )</td>
<td>( M_r = M_u = \frac{w_l l^2}{8} ) ( = \frac{1.84 \text{ kip/ft}(8.00 \text{ ft})^2(12 \text{ in./ft})}{8} = 177 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( T_r = T_u = \frac{w_l le}{2} )</td>
<td>( = \frac{2.76 \text{ kip/ft}(8.00 \text{ ft})(6.00 \text{ in.})}{2} = 66.2 \text{ kip-in.} )</td>
<td>( T_r = T_u = \frac{w_l le}{2} ) ( = \frac{1.84 \text{ kip/ft}(8.00 \text{ ft})(6.00 \text{ in.})}{2} = 44.2 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>
Calculate the nominal shear strength

\[ h = 6.00 \text{ in.}^2 - 3(0.233 \text{ in.}) = 5.30 \text{ in.} \]

\[ A_w = 2ht = 2(5.30 \text{ in.})(0.233 \text{ in.}) = 2.47 \text{ in.}^2 \]

\[ k_v = 5 \]

Calculate the web shear coefficient

\[ \frac{h}{t_w} = 22.8 \leq 1.10 \sqrt{\frac{k_vE}{F_y}} = 1.10 \sqrt{\frac{5(29,000 \text{ksi})}{46 \text{ksi}}} = 61.7 \quad \text{therefore, } C_v = 1.0 \]

Eqn. G2-3

\[ V_s = 0.6F_A A_v C_v = 0.6(46 \text{ksi})(2.47 \text{ in.}^2)(1.0) = 68.2 \text{ kips} \]

Eqn. G2-1

Calculate the available shear strength

\[ \begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi_v = 0.90 & \Omega_v = 1.67 \\
V_v = \phi_v V_n = 0.90(68.2 \text{ kips}) = 61.4 \text{ kips} & V_v = V_n/\Omega_v = 68.2 \text{ kips}/1.67 = 40.8 \text{ kips} \\
\end{array} \]

Section G1

Calculate the nominal flexural strength

Section F7

Flexural yielding limit state

\[ M_n = M_p = F_y Z_c = 46 \text{ksi}(8.53 \text{ in.}^3) = 392 \text{ kip-in.} \]

Eqn. F7-1

Flange local buckling limit state

\[ \frac{b}{t} = 14.2 < 1.12 \sqrt{\frac{E}{F_y}} = 28.1 \quad \text{therefore the flange is compact and the flange local buckling limit state does not apply.} \]

Table B4.1 Case 12

Web local buckling limit state

\[ \frac{h}{t} = 22.8 < 2.42 \sqrt{\frac{E}{F_y}} = 60.8 \quad \text{therefore the the web is compact and the web local buckling limit state does not apply.} \]

Table B4.1 Case 13

Therefore \( M_n = 392 \text{ kip-in.}, \text{ controlled by the flexural yielding limit state.} \)

Calculate the available flexural strength

\[ \begin{array}{c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi_b = 0.90 & \Omega_b = 1.67 \\
M_c = \phi_b M_n = 0.90(392 \text{ kip-in.}) = 353 \text{ kip-in.} & M_c = M_n/\Omega_b = 392 \text{ kip-in./1.67} = 235 \text{ kip-in.} \\
\end{array} \]

Section F1

Take the available torsional strength from Example H.5a

\[ T_c = \phi_T T_n = 0.90(278 \text{ kip-in.}) = 250 \text{ kip-in.} \]

\[ T_c = T_n/\Omega_T = 278 \text{ kip-in./1.67} = 166 \text{ kip-in.} \]

Ex. H.5a
Check combined strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \left( \frac{P}{P_e} + \frac{M}{M_e} \right) + \left( \frac{V}{V_e} + \frac{T}{T_e} \right)^2 \leq 1.0 )</td>
<td>( \left( \frac{P}{P_e} + \frac{M}{M_e} \right) + \left( \frac{V}{V_e} + \frac{T}{T_e} \right)^2 \leq 1.0 )</td>
</tr>
<tr>
<td>( 0 + \frac{265 \text{ kip-in.}}{353 \text{ kip-in.}} \quad + \left( \frac{11.0 \text{ kips}}{61.4 \text{ kips}} + \frac{66.2 \text{ kip-in.}}{250 \text{ kip-in.}} \right)^2 )</td>
<td>( 0 + \frac{177 \text{ kip-in.}}{235 \text{ kip-in.}} \quad + \left( \frac{7.36 \text{ kips}}{40.8 \text{ kips}} + \frac{44.2 \text{ kip-in.}}{166 \text{ kip-in.}} \right)^2 )</td>
</tr>
<tr>
<td>=0.948 &lt; 1.0 <strong>o.k.</strong></td>
<td>= 0.953 &lt; 1.0 <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Eqn. H3-6
Example H.6  W-Shape Torsional Strength.

This design example is taken from AISC Design Guide 9 – Torsion Analysis of Structural Steel Members. As shown below, a W10x49 spans 15 ft (180 in.) and supports concentrated loads at midspan that act at a 6 in. eccentricity with respect to the shear center. Determine the stresses on the cross section and the adequacy of the section to support the loads.

Given:

\[ P_D = 2.50 \text{ kips applied 6 in. off centerline} \]
\[ P_L = 7.50 \text{ kips applied 6 in. off centerline} \]

\[ \begin{align*}
7.5 \text{ ft} & \quad 15 \text{ ft}
\end{align*} \]

Beam Loading Diagram

The end conditions are assumed to be flexurally and torsionally pinned. The eccentric load can be resolved into a torsional moment and a load applied through the shear center.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM A992</td>
<td>( F_y = 50 \text{ ksi} )</td>
</tr>
<tr>
<td>( F_u = 65 \text{ ksi} )</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>W10x49</td>
<td>( I_x = 272 \text{ in.}^4 )</td>
</tr>
<tr>
<td>( J = 1.39 \text{ in.}^4 )</td>
<td></td>
</tr>
<tr>
<td>( S_x = 54.6 \text{ in.}^3 )</td>
<td></td>
</tr>
<tr>
<td>( t_f = 0.560 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>( t_w = 0.340 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>( C_w = 2070 \text{ in.}^6 )</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>W10x49</td>
<td>( S_w = 33.0 \text{ in.}^4 )</td>
</tr>
<tr>
<td>( Q_f = 13.0 \text{ in.}^3 )</td>
<td></td>
</tr>
<tr>
<td>( Q_w = 30.2 \text{ in.}^3 )</td>
<td></td>
</tr>
</tbody>
</table>

Additional Torsional Properties:

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>W10x49</td>
<td>( W_{ns} = 23.6 \text{ in.}^2 )</td>
</tr>
<tr>
<td>( a = 62.1 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>( W_{ns} = 23.6 \text{ in.}^2 )</td>
<td></td>
</tr>
</tbody>
</table>

Calculate the required strength

<table>
<thead>
<tr>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td></td>
</tr>
<tr>
<td>( P_a = 1.2(2.50 \text{ kips}) + 1.6(7.50 \text{ kips}) )</td>
<td>( P_a = 2.50 \text{ kips} + 7.50 \text{ kips} )</td>
</tr>
<tr>
<td>= 15.0 kips</td>
<td>= 10.0 kips</td>
</tr>
<tr>
<td>( V_a = \frac{P_a}{2} = \frac{15.0 \text{ kips}}{2} = 7.50 \text{ kips} )</td>
<td>( V_a = \frac{P_a}{2} = \frac{10.0 \text{ kips}}{2} = 5.00 \text{ kips} )</td>
</tr>
<tr>
<td>( M_a = \frac{P_a I}{4} = \frac{15.0 \text{ kips}(15.0 \text{ ft})(12 \text{ in./ft})}{4} )</td>
<td>( M_a = \frac{P_a I}{4} = \frac{10.0 \text{ kips}(15.0 \text{ ft})(12 \text{ in./ft})}{4} )</td>
</tr>
<tr>
<td>= 675 kip-in.</td>
<td>= 450 kip-in.</td>
</tr>
<tr>
<td>( T_a = P_a e = 15.0 \text{ kips}(6.00 \text{ in.}) = 90.0 \text{ kip-in.} )</td>
<td>( T_a = P_a e = 10.0 \text{ kips}(6.00 \text{ in.}) = 60.0 \text{ kip-in.} )</td>
</tr>
</tbody>
</table>
**Calculate the normal and shear stresses from flexure**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\sigma_{bc} = \frac{M_u}{S_s} = \frac{675 \text{ kip-in.}}{54.6 \text{ in}^3} = 12.4 \text{ ksi}) (compression at top, tension at bottom)</td>
<td>(\sigma_{bc} = \frac{M_u}{S_s} = \frac{450 \text{ kip-in.}}{54.6 \text{ in}^3} = 8.24 \text{ ksi}) (compression at top, tension at bottom)</td>
</tr>
<tr>
<td>(\tau_{bc} = \frac{V_\ell}{I_s w} = \frac{7.5 \text{ kips (30.2 in.(^3)}}{272 \text{ in}^4 (0.340 \text{ in.})} = 2.45 \text{ ksi})</td>
<td>(\tau_{bc} = \frac{V_\ell}{I_s w} = \frac{5.00 \text{ kips (30.2 in.(^3)}}{272 \text{ in}^4 (0.340 \text{ in.})} = 1.63 \text{ ksi})</td>
</tr>
<tr>
<td>(\tau_{bl} = \frac{V_a f}{I_s f} = \frac{7.5 \text{ kips (13.0 in.(^3)}}{272 \text{ in}^4 (0.560 \text{ in.})} = 0.640 \text{ ksi})</td>
<td>(\tau_{bl} = \frac{V_a f}{I_s f} = \frac{5.00 \text{ kips (13.0 in.(^3)}}{272 \text{ in}^4 (0.560 \text{ in.})} = 0.427 \text{ ksi})</td>
</tr>
</tbody>
</table>

**Calculate torsional stresses**

The following functions are taken from AISC’s Design Guide 9 – Torsion Analysis of Structural Steel Members Appendix B, Case 3, with \(\alpha = 0.5\).

\[
l = \frac{180 \text{ in.}}{62.1 \text{ in.}} = 2.90
\]

At midspan \((z/l = 0.5)\)

Using the graphs for \(\theta, \theta \alpha, \theta'\) and \(\theta''\), select values

For \(\theta\); \(\theta \times \left(\frac{GJ}{T_r} \right) \left(\frac{1}{I} \right) = +0.09\) Solve for \(\theta = +0.09 \frac{T_l}{GJ}\)

For \(\theta\alpha\); \(\theta \alpha \times \left(\frac{GJ}{T_r} \right) a = -0.44\) Solve for \(\theta = -0.44 \frac{T_l}{GJa}\)

For \(\theta'\); \(\theta' \times \left(\frac{GJ}{T_r} \right) = 0\) Therefore \(\theta' = 0\)

For \(\theta''\); \(\theta'' \times \left(\frac{GJ}{T_r} \right) a^2 = -0.50\) Solve for \(\theta'' = -0.50 \frac{T_l}{GJa^2}\)

At the support \((z/l = 0)\)

For \(\theta\); \(\theta \times \left(\frac{GJ}{T_r} \right) \left(\frac{1}{I} \right) = 0\) Therefore \(\theta = 0\)

For \(\theta\alpha\); \(\theta \alpha \times \left(\frac{GJ}{T_r} \right) a = 0\) Therefore \(\theta \alpha = 0\)

For \(\theta'\); \(\theta' \times \left(\frac{GJ}{T_r} \right) = +0.28\) Solve for \(\theta' = +0.28 \frac{T_l}{GJ}\)

For \(\theta''\); \(\theta'' \times \left(\frac{GJ}{T_r} \right) a^2 = -0.22\) Solve for \(\theta'' = -0.22 \frac{T_l}{GJa^2}\)

In the above calculations note that the applied torque is negative with the sign convention used.
Calculate $\frac{T}{GJ}$ for use below

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{T_r}{GJ} = \frac{-90.0 \text{ kip-in.}}{(11,200 \text{ ksi})(1.39 \text{ in.}^4)}$</td>
<td>$\frac{T_w}{GJ} = \frac{-60.0 \text{ kip-in.}}{(11,200 \text{ ksi})(1.39 \text{ in.}^4)}$</td>
</tr>
<tr>
<td>= $-5.78 \times 10^{-3}$ \text{ rad/in.}</td>
<td>= $-3.85 \times 10^{-3}$ \text{ rad/in.}</td>
</tr>
</tbody>
</table>

Calculate the shear stresses due to pure torsion

$$\tau = G\theta'$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>At midspan</td>
<td>At midspan</td>
</tr>
<tr>
<td>$\theta' = 0$; $\tau_{ut} = 0$</td>
<td>$\theta' = 0$; $\tau_{ut} = 0$</td>
</tr>
<tr>
<td>At the support, for the web;</td>
<td>At the support, for the web;</td>
</tr>
<tr>
<td>$\tau_{ut} = 11,200 \text{ ksi}(0.340 \text{ in.})(0.28)(-5.78 \text{ rad})$</td>
<td>$\tau_{ut} = 11,200 \text{ ksi}(0.340 \text{ in.})(0.28)(-3.85 \text{ rad})$</td>
</tr>
<tr>
<td>= $-6.16 \text{ ksi}$</td>
<td>= $-4.11 \text{ ksi}$</td>
</tr>
<tr>
<td>At the support, for the flange;</td>
<td>At the support, for the flange;</td>
</tr>
<tr>
<td>$\tau_{ut} = 11,200 \text{ ksi}(0.560 \text{ in.})(0.28)(-5.78 \text{ rad})$</td>
<td>$\tau_{ut} = 11,200 \text{ ksi}(0.560 \text{ in.})(0.28)(-3.85 \text{ rad})$</td>
</tr>
<tr>
<td>= $-10.2 \text{ ksi}$</td>
<td>= $-6.76 \text{ ksi}$</td>
</tr>
</tbody>
</table>

Calculate the shear stresses due to warping

$$\tau = \frac{-ES_{ut}}{t_f} \theta'$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>At midspan</td>
<td>At midspan</td>
</tr>
<tr>
<td>$\theta' = 0$; $\tau_{uw} = 0$</td>
<td>$\theta' = 0$; $\tau_{uw} = 0$</td>
</tr>
<tr>
<td>At the support, for the web;</td>
<td>At the support, for the web;</td>
</tr>
<tr>
<td>$\tau_{uw} = -29,000 \text{ ksi}(33.0 \text{ in.}^4)(-0.50(-5.78 \text{ rad}))$</td>
<td>$\tau_{uw} = -29,000 \text{ ksi}(33.0 \text{ in.}^4)(-0.50(-3.85 \text{ rad}))$</td>
</tr>
<tr>
<td>= $-1.28 \text{ ksi}$</td>
<td>= $-0.853 \text{ ksi}$</td>
</tr>
<tr>
<td>At the support</td>
<td>At the support</td>
</tr>
<tr>
<td>$\tau_{uw} = -29,000 \text{ ksi}(33.0 \text{ in.}^4)(-0.22(-5.78 \text{ rad}))$</td>
<td>$\tau_{uw} = -29,000 \text{ ksi}(33.0 \text{ in.}^4)(-0.22(-3.85 \text{ rad}))$</td>
</tr>
<tr>
<td>= $-0.564 \text{ ksi}$</td>
<td>= $-0.375 \text{ ksi}$</td>
</tr>
</tbody>
</table>
Calculate the normal stresses due to warping

\[ \sigma_w = EW_{mn} \theta^t \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>At midspan</td>
<td>At midspan</td>
</tr>
<tr>
<td>[ \sigma_{uw} = 29,000 \text{ ksi}(23.6 \text{ in.}^2) \frac{-0.44(-5.78 \text{ rad})}{(62.1 \text{ in.})^{10^{3}}} ]</td>
<td>[ \sigma_{aw} = 29,000 \text{ ksi}(23.6 \text{ in.}^2) \frac{-0.44(-3.85 \text{ rad})}{(62.1 \text{ in.})^{10^{3}}} ]</td>
</tr>
<tr>
<td>= 28.0 ksi</td>
<td>= 18.7 ksi</td>
</tr>
<tr>
<td>At the support</td>
<td>At the support</td>
</tr>
<tr>
<td>Since ( \theta^t = 0, \sigma_{uw} = 0 )</td>
<td>Since ( \theta^t = 0, \sigma_{aw} = 0 )</td>
</tr>
</tbody>
</table>

Design Guide 9
Eqn. 4.3a

Calculate the combined stresses

The summarized stresses are as follows:

<table>
<thead>
<tr>
<th>Location</th>
<th>Normal Stresses</th>
<th>Shear Stresses</th>
<th>Normal Stresses</th>
<th>Shear Stresses</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( \sigma_{uw} )</td>
<td>( \sigma_{ub} )</td>
<td>( f_{uw} )</td>
<td>( t_{uw} )</td>
</tr>
<tr>
<td>Midspan</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flange</td>
<td>( \pm 28.1 )</td>
<td>( \pm 12.4 )</td>
<td>0</td>
<td>-1.28</td>
</tr>
<tr>
<td>Web</td>
<td>( \pm 40.4 )</td>
<td>0</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>Support</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Flange</td>
<td>0</td>
<td>0</td>
<td>-10.2</td>
<td>-0.564</td>
</tr>
<tr>
<td>Web</td>
<td>( \pm 40.4 )</td>
<td>0</td>
<td>( \pm 6.16 )</td>
<td>( \pm 2.45 )</td>
</tr>
<tr>
<td>Maximum</td>
<td>( \pm 40.4 )</td>
<td></td>
<td>( \pm 11.4 )</td>
<td></td>
</tr>
</tbody>
</table>

The maximum normal stress due to flexure and torsion occurs at the edge of the flange at midspan and is equal to 40.4 ksi.

The maximum shear stress due to flexure and torsion occurs in the middle of the flange at the support and is equal to 7.56 ksi.

Calculate the available strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_T = 0.90 )</td>
<td>( \Omega_T = 1.67 )</td>
</tr>
</tbody>
</table>

**Normal Stresses**

**Yielding limit state**

\( F_u = F_y = 50 \text{ ksi} \) controls

**Lateral-torsional buckling limit state**

\( C_h = 1.32 \)
Compute \( F_n \) using values from Table 3-10. with \( L_b = 15.0 \) ft and \( C_b = 1.0 \)

\[ \phi_b M_n = 206 \text{ kip-ft} \]

\[ F_n = F_{cr} = C_b \frac{\phi_b M_n}{\phi_b S_x} \]

\[ = 1.32 \frac{206 \text{ kip-ft}}{0.90 \left( \frac{54.6 \text{ in.}^3}{\text{ft}} \right)} \left( \frac{12 \text{ in.}}{\text{ft}} \right) \]

\[ = 66.4 \text{ ksi} > 50 \text{ ksi \ does \ not \ control} \]

\[ \phi_T F_n = 0.9(50 \text{ ksi}) = 45.0 \text{ ksi} > 40.4 \text{ ksi \ o.k.} \]

*Shear yielding limit state*

\[ F_n = 0.6 F_y \]

*Design shear strength*

\[ \phi_T F_n = 0.90(0.6)(50 \text{ ksi}) \]

\[ = 27.0 \text{ ksi} > 11.4 \text{ ksi \ o.k.} \]

Compute \( F_n \) using values from Table 3-10. with \( L_b = 15.0 \) ft and \( C_b = 1.0 \)

\[ M_n / \Omega_b = 138 \text{ kip-ft} \]

\[ F_n = F_{cr} = C_b \frac{M_n / \Omega_b}{S_y} \]

\[ = 1.32 \left( \frac{138 \text{ kip-ft}}{54.6 \text{ in.}^3} \right) \left( \frac{12 \text{ in.}}{\text{ft}} \right) \]

\[ = 66.9 \text{ ksi} > 50 \text{ ksi \ does \ not \ control} \]

\[ F_n / \Omega_T = 50 \text{ ksi} / 1.67 = 30.0 \text{ ksi} > 26.9 \text{ ksi \ o.k.} \]

*Shear yielding limit state*

\[ F_n = 0.6 F_y \]

*Allowable shear strength*

\[ F_n / \Omega_T = (0.6)(50 \text{ ksi}) / 1.67 \]

\[ = 18.0 \text{ ksi} > 7.56 \text{ ksi \ o.k.} \]

**Calculate the maximum rotation at service load**

The maximum rotation occurs at midspan. The service-load torque is:

\[ T = P e = - (2.50 \text{ kips} + 7.50 \text{ kips})(6.00 \text{ in.}) = -60.0 \text{ kip-in.} \]

The maximum rotation is:

\[ \theta = +0.9 \frac{T l}{G J} = \frac{0.09 \left( -60.0 \text{ kip-in.} \right)(180 \text{ in.})}{11,200 \text{ ksi} \left( 1.39 \text{ in.}^4 \right)} = -0.624 \text{ rads} = -3.58 \text{ degrees} \]
CHAPTER I
DESIGN OF COMPOSITE MEMBERS

I1. GENERAL PROVISIONS
The available strength of composite sections may be calculated by one of two methods - the plastic stress distribution method or the strain-compatibility method. The composite design tables in the Steel Construction Manual are based on the plastic stress distribution method.

I2. AXIAL MEMBERS
Generally, the available compressive strength of a composite member is based on a summation of the strengths of all of the components of the column. The Specification contains several requirements to ensure that the steel and concrete components work together.

For tension members, the concrete tensile strength is ignored and only the strength of the steel member and properly connected reinforcing is permitted to be used in the calculation of available tensile strength.

Because of concerns about the deformation compatibility of steel and concrete in resisting shear, either the steel or the reinforced concrete, but not both, are permitted to be used in the calculation of available shear strength. Whether the composite column is an encased column or a filled column, it is important to consider the load path within the composite member, and to provide shear transfer mechanisms and appropriate top and bottom details.

The design of encased composite compression and tension members is presented in Examples I-3 and I-4. There are no tables in the Manual for the design of these members.

The design of filled composite compression and tension members is presented in Examples I-2 and I-5. The Manual includes tables for the design of filled composite members in compression.

I3. FLEXURAL MEMBERS
Because a plastic stress distribution is used for both LRFD and ASD designs, the flexural strength of composite beams is generally greater than that of former ASD designs. Shear connectors, in many cases, have lower horizontal shear strength than was permitted in past LRFD specifications. Designers are encouraged to read the discussion on this subject in Commentary Chapter I. The design of a typical composite beam member is illustrated in Example I-1.

I4. COMBINED AXIAL FORCE AND FLEXURE
Design for combined axial force and flexure may be accomplished in any of the three methods outlined in the Commentary. Example I-7 illustrates the plastic-distribution method.

To assist in developing this curve, a series of equations is provided in Figure I-1. These equations define selected points on the interaction curve, without consideration of slenderness effects. Figures I-1a through I-1d outline specific cases, and the applicability of the equations to a cross-section that differs should be carefully considered. As an example, the equations in Figure I-1a are appropriate for the case of side bars located at the centerline, but not for other side bar locations. In contrast, these equations are appropriate for any amount of reinforcing at the extreme reinforcing bar location. In Figure I-1b, the equations are appropriate only for the case of 4 reinforcing bars at the corners of the encased section. When design cases deviate from those presented the appropriate interaction equations can be derived from first principles.
PLASTIC CAPACITIES FOR RECTANGULAR, ENCASED W-SHAPES BENT ABOUT THE X-X AXIS

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Point</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0.85(f'<em>{c})  (F_y)  (F</em>{yr})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(A)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(B)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(C)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(D)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

For below the flange \(h_n \leq \frac{d}{2} - t_f\):

\[
P_B = 0
\]

\[
M_D = h_n F_{yr} - Z_{sm} \left(0.85 f'_{c} \right)
\]

\[
Z_{sm} = h_n^2 - Z_{sm}
\]

For \(h_n\) within the flange \(\frac{d}{2} - t_f < h_n \leq \frac{d}{2}\):

\[
h_n = 0.85 f'_{c} \left[A_y + A_{y,pr} - 2 F_{yr} A_{y,pr} \right] - 2 F_{yr} A_{y,pr}
\]

\[
Z_{sm} = Z_y - h_n \left(\frac{d}{2} - h_n\right) \left(\frac{d}{2} + h_n\right)
\]

For \(h_n\) above the flange \(h_n > \frac{d}{2}\):

\[
h_n = 0.85 f'_{c} \left[A_y + A_{y,pr} - 2 F_{yr} A_{y,pr} \right] - 2 F_{yr} A_{y,pr}
\]

\[
Z_{sm} = Z_{sm} = \text{full x-axis plastic section modulus of steel shape}
\]

\[P_A = A_y F_{y} + A_{y,pr} F_{yr} + 0.85 f'_{c} A_c
\]

\[M_A = 0
\]

\[A_y = \text{area of steel shape}
\]

\[A_{y,pr} = \text{area of all continuous reinforcing bars}
\]

\[A_c = h_n h_2 - A_y - A_{y,pr}
\]

\[P_C = 0.85 f'_{c} A_c
\]

\[M_C = M_B
\]

\[P_D = 0
\]

\[M_D = Z_{sm} F_{yr} - Z_{sm} \left(0.85 f'_{c} \right)
\]

\[Z_{sm} = h_n^2 - Z_{sm}
\]

\[Z_x = \left(A_{y,pr} - A_{y,pr} \right) \left(\frac{h_n}{2} - \frac{d}{2}\right)
\]

\[Z_y = \frac{1}{4} \left(2 Z_x + Z_{sm} \right)
\]

\[Z_{sm} = Z_{sm} = \text{full x-axis plastic section modulus of steel shape}
\]

Figure I-1a. W-Shapes, Strong-Axis Anchor Points.
PLASTIC CAPACITIES FOR RECTANGULAR, ENCASED W-SHAPE BENT ABOUT THE Y-Y AXIS

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Point</th>
<th>Defining Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td></td>
<td></td>
<td>$P_A = A_s F_y + A_{rV} F_{rV} + 0.85 f'_c A_s$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_A = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_s = \text{area of steel shape}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_{rV} = \text{area of continuous reinforcing bars}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$A_{c} = h_1 h_2 - A_s - A_{rV}$</td>
</tr>
<tr>
<td>E</td>
<td></td>
<td></td>
<td>$P_E = A_s F_y + \left( 0.85 f'<em>c \left( A_s - \frac{h_1}{2} \left( h_2 - h_f \right) + \frac{A</em>{c}}{2} \right) \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_E = M_D - Z_{sE} F_{rV} - \sqrt{2} Z_{cE} \left( 0.85 f'_c \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{sE} = Z_{s} = \text{full y-axis plastic section modulus of steel shape}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{cE} = \frac{h_1 h_2^2}{4} - Z_{sE}$</td>
</tr>
<tr>
<td>B</td>
<td></td>
<td></td>
<td>$P_B = 0$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_B = M_D - Z_{sE} F_{rV} - \sqrt{2} Z_{cE} \left( 0.85 f'_c \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{sE} = \frac{h_1 h_2^2}{4} - Z_{s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$h_{b} = \frac{b_f}{2}$ if $h_{b} \leq \frac{b_f}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$h_{b} = \frac{0.85 f'<em>c \left( A_s + A</em>{rV} - 2 t_f b_f - 2 F_{rE} \left( A_s - 2 t_f b_f \right) \right)}{2 \left( 4 t_f F_{rE} + \left( h_1 - 2 t_f \right) 0.85 f'<em>c \right)}$ if $h</em>{b} &gt; \frac{b_f}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{sE} = Z_s - 2 t_f \left( \frac{b_f}{2} + h_{b} \right) \left( \frac{h_1}{2} - h_{b} \right)$</td>
</tr>
<tr>
<td>C</td>
<td></td>
<td></td>
<td>$P_C = 0.85 f'_c A_s$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_C = M_B$</td>
</tr>
<tr>
<td>D</td>
<td></td>
<td></td>
<td>$P_D = \frac{0.85 f'_c}{2}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$M_D = Z_{sE} F_{rV} + Z_{cE} \left( 0.85 f'_c \right)$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{s} = \frac{h_1 h_2^2}{4} - Z_{s}$</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>$Z_{c} = \frac{h_1 h_2^2}{4} - Z_{s}$</td>
</tr>
</tbody>
</table>

Figure I-1b. W-Shapes, Weak-Axis Anchor Points
**PLASTIC CAPACITIES FOR COMPOSITE, FILLED HSS BENT ABOUT THE X-X OR Y-Y AXIS**

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Point</th>
<th>Defining Equations</th>
</tr>
</thead>
</table>
| A       | ![Diagram A]         | A     | \[
P_A = A_s f_y + A_s \left(0.85 f_y\right)\]
|         |                     |   | \[
M_A = 0\]
|         |                     |   | \[
A_s = \text{area of steel shape}\]
|         |                     |   | \[
A_c = h_b h_2 - 0.858 r^2\]
| C       | ![Diagram C]         | C     | \[
P_C = A_c \left(0.85 f_c\right)\]
|         |                     |   | \[
M_C = M_B\]
| D       | ![Diagram D]         | D     | \[
P_D = \frac{0.85 f_c A_c}{2}\]
|         |                     |   | \[
M_D = Z_c f_y + \frac{\sqrt{2}}{2} \left(0.85 f_c\right)\]
|         |                     |   | \[
Z_c = \text{full y-axis plastic section modulus of steel shape}\]
|         |                     |   | \[
Z_c = \frac{h_b h_2}{4} - 0.192 r^2\]
| B       | ![Diagram B]         | B     | \[
P_B = 0\]
|         |                     |   | \[
M_B = M_D - Z_{cm} f_y - \frac{\sqrt{2}}{2} Z_{cm} \left(0.85 f_c\right)\]
|         |                     |   | \[
Z_{cm} = 2 t_c h_{h_c}^2\]
|         |                     |   | \[
Z_{cm} = h_{h_c}^2\]
|         |                     |   | \[
\frac{h_{h_c}}{2} = \frac{0.85 f_c A_c}{2 \left(0.85 f_c h_b + 4 t_c f_y\right)}\]

Figure I-1c. Filled Rectangular or Square HSS, Strong- or Weak Axis Anchor Points
### PLASTIC CAPACITIES FOR COMPOSITE, FILLED ROUND HSS BENT ABOUT ANY AXIS

<table>
<thead>
<tr>
<th>Section</th>
<th>Stress Distribution</th>
<th>Point</th>
<th>Defining Equations</th>
</tr>
</thead>
</table>
| A       | ![Diagram A]         |       | \( P_A = A_s f_c y + 0.85 f_c y A_c \) *  
|         |                     |       | \( M_A = 0 \)  
|         |                     |       | \( A_s = \pi (d t - c^2) \)  
|         |                     |       | \( A_c = \frac{\pi d^2}{4} \) |
| C       | ![Diagram C]         |       | \( P_C = 0.85 f_c y A_c \)  
|         |                     |       | \( M_C = M_B \)  
| D       | ![Diagram D]         |       | \( P_D = \frac{0.85 f_c y A_c}{2} \)  
|         |                     |       | \( M_D = Z_s f_c y + \frac{\pi}{2} Z_c (0.85 f_c y) \)  
|         |                     |       | \( Z_s = \) plastic section modulus of steel shape = \( \frac{d^3}{6} - Z_c \)  
|         |                     |       | \( Z_c = \frac{b^3}{6} \)  
| B       | ![Diagram B]         |       | \( P_B = 0 \)  
|         |                     |       | \( M_B = Z_{dB} f_c y + \frac{\pi}{2} Z_{dB} (0.85 f_c y) \)  
|         |                     |       | \( Z_{dB} = \frac{d^3}{6} \sin^3 \left( \frac{\theta}{2} \right) Z_{dB} \)  
|         |                     |       | \( Z_{dB} = \frac{b^3}{6} \sin^3 \left( \frac{\theta}{2} \right) \)  
|         |                     |       | \( \theta = \frac{0.0260 K_s^2 - 2 K_s}{0.0848 K_s} \)  
|         |                     |       | \( \theta = \left( \frac{0.0260 K_s^2 + 2 K_s}{0.0848 K_s} \right)^{\frac{1}{2}} \)  
|         |                     |       | \( K_s = f_c y \theta^2 \)  
|         |                     |       | \( K_s = f_c y \left( \frac{d t - c^2}{2} \right) \)  
|         |                     |       | \( h_n = \frac{b \sin \left( \frac{\pi - \theta}{2} \right)}{2} \) (not used, for reference only) |

* \( P_A = A_s f_c y + 0.95 f_c y A_c \) is permitted to be used when the composite column is loaded only in axial compression.

---

**Figure I-1d. Filled Round HSS Anchor Points**
Example I-1  Composite Beam Design

Given:

A series of 45-ft. span composite beams at 10 ft. o/c are carrying the loads shown below. The beams are ASTM A992 and are unshored. The concrete has $f'_c = 4$ ksi. Design a typical floor beam with 3 in. 18 gage composite deck, and 4½ in. normal weight concrete above the deck, for fire protection and mass. Select an appropriate beam and determine the required number of shear studs.

Solution:

Material Properties:

Concrete: $f'_c = 4$ ksi

Beam

<table>
<thead>
<tr>
<th>Property</th>
<th>$F_y$</th>
<th>$F_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
</tbody>
</table>

Manual Table 2-3

Loads:

Dead load:

- Slab: $0.075$ kip/ft$^2$
- Beam weight: $0.008$ kip/ft$^2$ (assumed)
- Miscellaneous: $0.010$ kip/ft$^2$ (ceiling etc.)

Live load:

- Non-reduced: $0.10$ kips/ft$^2$

Since each beam is spaced at 10 ft. o.c.

Total dead load: $0.093$ kip/ft$^2$(10 ft.) = 0.93 kips/ft.

Total live load: $0.10$ kip/ft$^2$(10 ft.) = 1.00 kips/ft.

Construction dead load (unshored): $0.083$ kip/ft$^2$(10 ft) = 0.83 kips/ft

Construction live load (unshored): $0.020$ kip/ft$^2$(10 ft) = 0.20 kips/ft
Determine the required flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_a = 1.2(0.93 \text{kip/ft}) + 1.6(1.0 \text{kip/ft})$</td>
<td>$w_a = 0.93 \text{kip/ft} + 1.0 \text{kip/ft}$</td>
</tr>
<tr>
<td>$= 2.72 \text{kip/ft}$</td>
<td>$= 1.93 \text{kip/ft}$</td>
</tr>
<tr>
<td>$M_a = \frac{2.72 \text{kip/ft}(45 \text{ft})^2}{8}$</td>
<td>$M_a = \frac{1.93 \text{kip/ft}(45 \text{ft})^2}{8}$</td>
</tr>
<tr>
<td>$= 687 \text{kip-ft.}$</td>
<td>$= 489 \text{kip-ft.}$</td>
</tr>
</tbody>
</table>

Use Tables 3-19, 3-20 and 3-21 from the Manual to select an appropriate member

Determine $b_{eff}$

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

1. one-eighth of the beam span, center to center of supports
   $\frac{45 \text{ ft}}{8} = 5.625 \text{ ft} = 6.0 \text{ ft}$
2. one-half the distance to center-line of the adjacent beam
   $\frac{10 \text{ ft}}{2} = 5.0 \text{ ft}$ Controls
3. the distance to the edge of the slab
   Not applicable

Calculate the moment arm for the concrete force measured from the top of the steel shape, $Y_2$.

Assume $a = 1.0 \text{ in.}$ (Some assumption must be made to start the design process. An assumption of 1.0 in. has proven to be a reasonable starting point in many design problems.)

$Y_2 = t_{slab} - a/2 = 7.5 - 0.5 = 7.0 \text{ in.}$

Enter Manual Table 3-19 with the required strength and $Y_2=7.0 \text{ in.}$ Select a beam and neutral axis location that indicates sufficient available strength.

Select a W21×50 as a trial beam.

When PNA location 5 (BFL), this composite shape has an available strength of:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{b}M_a = 770 \text{kip-ft} &gt; 687 \text{kip-ft}$</td>
<td>$M_a/\Omega_b = 512 \text{kip-ft} &gt; 489 \text{kip-ft}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
Check the beam deflections and available strength

Check the deflection of the beam under construction, considering only the weight of concrete as contributing to the construction dead load.

Limit deflection to a maximum of 2.5 in. to facilitate concrete placement.

\[ I_{req} = \frac{5}{384} \frac{w_{nd} l^4}{E \Delta} = \frac{5(0.83 \text{ kip/ft})(45 \text{ ft})^4(1728 \text{ in.}^3/\text{ft}^4)}{384(29,000 \text{ ksi})(2.5 \text{ in.})} = 1,060 \text{ in.}^4 \]

From Manual Table 3-20, a W21×50 has \( I_e = 984 \text{ in.}^4 \), therefore this member does not satisfy the deflection criteria under construction.

Using Manual Table 3-20, revise the trial member selection to a W21×55, which has \( I_e = 1140 \text{ in.}^4 \), as noted in parenthesis below the shape designation.

Check selected member strength as an un-shored beam under construction loads assuming adequate lateral bracing through the deck attachment to the beam flange.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculate the required strength</strong></td>
<td><strong>Calculate the required strength</strong></td>
</tr>
<tr>
<td>1.4 ( DL = 1.4 ) (0.83 kips/ft) = 1.16 kips/ft</td>
<td>1.03 kips/ft</td>
</tr>
<tr>
<td>1.2( DL+1.6LL = 1.2 ) (0.83) + 1.6(0.20) = 1.32 klf</td>
<td>1.32 klf</td>
</tr>
<tr>
<td>( M_u(\text{unshored}) = \frac{1.31 \text{ kip}/\text{ft}(45 \text{ ft})^2}{8} = 331 \text{ kip-ft} )</td>
<td>260 kip-ft</td>
</tr>
</tbody>
</table>

The design strength for a W21×55 is 473 kip-ft > 331 kip-ft **o.k.**

The allowable strength for a W21×55 is 314 kip-ft > 260 kip-ft **o.k.**

For a W21×55 with \( Y_2=7.0 \) in, the member has sufficient available strength when the PNA is at location 6 and \( \sum Q_n = 292 \) kips.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_b M_u = 767 \text{ kip-ft} &gt; 687 \text{ kip-ft} ) <strong>o.k.</strong></td>
<td>( M_u/\Omega_b = 510 \text{ kip-ft} &gt; 489 \text{ kip-ft} ) <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Check \( a \)

\[ a = \frac{\sum Q_n}{0.85 f'_{c} b} = \frac{292 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft.})(12 \text{ in./ft.})} = 0.716 \text{ in.} \]

0.716 in. < 1.0 in. assumed **o.k.**

Check live load deflection

\[ \Delta_{LL} < l/360 = ((45 \text{ ft.})(12 \text{ in./ft.}))/360 = 1.5 \text{ in.} \]

A lower bound moment of inertia for composite beams is tabulated in Manual Table 3-20.
For a W21×55 with γ2=7.0 and the PNA at location 6, \( I_{L,B} = 2440 \text{ in.}^4 \)

\[
\Delta_{LL} = \frac{5}{384} \frac{w_{PNA}l^4}{EI_{L,B}} = \frac{5(1.0 \text{kip/ft})(45 \text{ ft})^4 (1728 \text{ in.}^3/\text{ft}^4)}{384(29,000 \text{ksi})(2440 \text{ in.}^4)} = 1.30 \text{ in.}
\]

1.30 in. < 1.5 in. \textbf{o.k.}

**Determine if the beam has sufficient available shear strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = \frac{45\text{ft}}{2} (2.72\text{kip/ft}) = 61.2 \text{kips} )</td>
<td>( V_u = \frac{45\text{ft}}{2} (1.93\text{kip/ft}) = 43.4 \text{kips} )</td>
</tr>
<tr>
<td>( \phi V_u = 234 \text{kips} &gt; 61.2 \text{kips} \textbf{o.k.} )</td>
<td>( \frac{V_u}{\Omega} = 156 \text{kips} &gt; 43.4 \text{kips} \textbf{o.k.} )</td>
</tr>
</tbody>
</table>

**Determine the required number of shear stud connectors**

Using perpendicular deck with one \( \frac{3}{4}\)-in. diameter weak stud per rib in normal weight 4 ksi concrete. \( Q_n = 17.2 \text{kips/stud} \)

\[
\frac{\sum Q_n}{Q_n} = \frac{292 \text{kips}}{17.2 \text{kips}} = 17, \text{ on each side of the beam.}
\]

Total number of shear connectors; use 2(17) = 34 shear connectors.

**Check the spacing of shear connectors**

Since each flute is 12 in., use one stud every flute, starting at each support, and proceed for 17 studs on each end of the span.

\( 6d_{\text{stud}} < 12 \text{ in.} < 8t_{\text{slab}} \), therefore, the shear stud spacing requirements are met.

The studs are to be 5 in. long, so that they will extend a minimum of 1½ in. into slab.
Example I-2  Filled Composite Column in Axial Compression

**Given:**

Determine if a 14-ft long HSS10×6×3/8 ASTM A500 grade B column filled with \( f'_c = 5 \text{ ksi} \) normal weight concrete can support a dead load of 56 kips and a live load of 168 kips in axial compression. The column is pinned at both ends and the concrete at the base bears directly on the base plate. At the top, the load is transferred to the concrete in direct bearing.

**Solution:**

*Calculate the required compressive strength*

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(56 \text{ kips}) + 1.6(168 \text{ kips}) )</td>
<td>( P_u = 56 \text{ kips} + 168 \text{ kips} )</td>
<td>( P_u = 56 \text{ kips} + 168 \text{ kips} )</td>
</tr>
<tr>
<td></td>
<td>( = 336 \text{ kips} )</td>
<td>( = 224 \text{ kips} )</td>
</tr>
</tbody>
</table>

The available strength in axial compression can be determined directly from the Manual at \( KL = 14 \) ft as:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_u = 353 \text{ kips} )</td>
<td>( P_u / \Omega_c = 236 \text{ kips} )</td>
<td>Manual Table 4-14</td>
</tr>
<tr>
<td>353 kips &gt; 336 kips</td>
<td>o.k.</td>
<td>236 kips &gt; 224 kips</td>
</tr>
</tbody>
</table>

**Supporting Calculations**

The available strength of this filled composite section can be most easily determined by using Table 4-14 of the Manual. Alternatively, the available strength can be determined by direct application of the Specification requirements, as illustrated below.

**Material Properties:**

HSS10×6×3/8 \( F_y = 46 \text{ ksi} \) \( F_u = 58 \text{ ksi} \) \nConcrete \( f'_c = 5 \text{ ksi} \) \( E_c = w^{1.5} \sqrt{f'_c} = (145)^{1.5} \sqrt{5} = 3,900 \text{ksi} \)
**Geometric Properties:**

HSS10×6×3/8  \( t = 0.375 \text{ in.} \)  \( b = 10.0 \text{ in.} \)  \( h = 6.0 \text{ in.} \)

Concrete:

The concrete area is calculated as follows

\[
r = 2t = 2(0.375 \text{ in.}) = 0.75 \text{ in.} \text{ (outside radius)}
\]

\[
b_f = b - 2r = 10 - 2(0.75) = 8.50 \text{ in.}
\]

\[
h_f = h - 2r = 6 - 2(0.75) = 4.50 \text{ in.}
\]

\[
A_c = bh_f + \pi (r-t)^2 + 2bh_f(r-t) = (8.50 \text{ in.})(4.50 \text{ in.}) + \pi (0.375 \text{ in.})^2 + 2(8.50 \text{ in.})(0.375 \text{ in.}) + 2(4.50 \text{ in.})(0.375 \text{ in.}) = 48.4 \text{ in.}^2
\]

\[
I_c = \frac{bh_f^3}{12} + \frac{2(b_f)(h_f)^3}{12} + 2(r-t) \left( \frac{\pi}{8} \cdot \frac{8}{9\pi} \right) + 2 \left( \frac{\pi (r-t)^2}{2} \right) \left( \frac{h_f}{2} + \frac{4(r-t)}{3\pi} \right)
\]

\[
= 111 \text{ in.}^4
\]

For this shape, buckling will take place about the weak axis, thus

\[
h_1 = 6 - 2(0.75) = 5.25 \text{ in.}
\]

\[
b_1 = 10 - 4(0.375) = 8.5 \text{ in.}
\]

\[
h_2 = 6 - 4(0.375) = 4.5 \text{ in.}
\]

\[
b_2 = 0.375 \text{ in.}
\]

\[
(r-t) = 0.75 - 0.375 = 0.375 \text{ in.}
\]

\[
I_c = \frac{(8.5)(5.25)^3}{12} + \frac{2(0.375)(4.50)^3}{12} + 2(0.375)^4 \left( \frac{\pi}{8} \cdot \frac{8}{9\pi} \right) + 2 \left( \frac{\pi (0.375)^2}{2} \right) \left( \frac{4.5}{2} + \frac{4(0.375)}{3\pi} \right)^2
\]

\[
= 111 \text{ in.}^4
\]

**Manual Table 1-11**

HSS10×6×3/8:  \( A_s = 10.4 \text{ in.}^2 \)  \( I_s = 61.8 \text{ in.}^4 \)  \( h/t = 25.7 \)

Limitations:

1) Normal weight concrete 10 ksi \( \leq f'c \leq 3 \text{ ksi} \) \( f'c = 5 \text{ ksi} \) \text{ o.k.}

2) Not Applicable.

3) The cross-sectional area of the steel HSS shall comprise at least one percent of the total composite cross section.

\[
10.4 \text{ in.}^2 \geq (0.01)(48.6 \text{ in.}^2 + 10.4 \text{ in.}^2) = 0.590 \text{ in.}^2 \text{ o.k.}
\]

4) The maximum b/t ratio for a rectangular HSS used as a composite column shall be equal to \( 2.26 \sqrt{E/F_y} \).

\[
b/t = 25.7 \leq 2.26 \sqrt{E/F_y} = 2.26 \sqrt{29,000 \text{ksi}/46\text{ksi}} = 56.7 \text{ o.k.}
\]

User note: For all rectangular HSS sections found in the Manual the b/t ratios do not exceed \( 2.26 \sqrt{E/F_y} \).

5) Not Applicable.
Calculate the available compressive strength

\[ C_2 = 0.85 \text{ for rectangular sections} \]

\[ P_o = A_s f_y + A_{cr} f_y + C_2 A_c f'_c \]
\[ = (10.4 \text{ in.}^2)(46 \text{ ksi}) + 0.85(48.4 \text{ in.}^2)(5 \text{ ksi}) = 684 \text{ kips} \]

\[ C_3 = 0.6 + 2 \left( \frac{A_i}{A_i + A_j} \right) = 0.6 + 2 \left( \frac{10.4 \text{ in.}^2}{48.4 \text{ in.}^2 + 10.4 \text{ in.}^2} \right) = 0.954 \geq 0.90 \]

Therefore use 0.90

\[ EI_{eff} = E_i I_i + E_c I_c + C_3 E_c I_c \]
\[ = (29,000 \text{ ksi})(61.8 \text{ in.}^4) + (0.90)(3,900 \text{ ksi})(111 \text{ in.}^4) \]
\[ = 2,180,000 \text{ kip-in.}^2 \]

User note: \( K \) value is from Chapter C and for this case \( K = 1.0 \).

\[ P_e = \pi^2 (EI_{eff})/(KL)^2 = \pi^2 (2,180,000 \text{ kip-in.}^2) / ((1.0)(14\text{ ft})(12\text{ in./ft}))^2 = 762 \text{ kips} \]

\[ \frac{P_e}{P_e} = 684 \text{ kips} / 762 \text{ kips} = 0.898 \]

0.898 \leq 2.25 Therefore use Eqn. 12-2 to solve \( P_n \)

\[ P_n = P_o \left[ 0.658 \left( \frac{P_e}{P_e} \right) \right] = (684 \text{ kips}) \left[ 0.658(0.898) \right] = 470 \text{ kips} \]

\( \phi_c = 0.75 \)

\( \Omega_c = 2.00 \)

\( \phi_c P_n = 0.75(470\text{kips}) = 353 \text{ kips} \)

\( P_n/\Omega_c = 470 \text{ kips} / 2.00 = 235 \text{ kips} \)

353 kips > 336 kips \( \text{o.k.} \)

235 kips > 224 kips \( \text{o.k.} \)
Example I-3  Encased Composite Column in Axial Compression

Given:

Determine if a 14 ft tall W10×45 steel section encased in a 24 in.×24in. concrete column with $f'_c = 5$ ksi, is adequate to support a dead load of 350 kips and a live load of 1050 kips in axial compression. The concrete section has 8-#8 longitudinal reinforcing bars and #4 transverse ties @ 12in. o/c., The column is pinned at both ends and the load is applied directly to the concrete encasement.

Solution:

Material Properties:

Column W10×45  $F_y = 50$ ksi  $F_u = 65$ ksi  Manual Table 2-3

Concrete  $f'_c = 5$ ksi  $E_c = 3,900$ ksi (145pcf concrete)

Reinforcement  $F_{yst} = 60$ ksi

Geometric Properties:

W10×45:  Manual Table 1-1

$A_s = 13.3$ in.$^2$

$I_s = 53.4$ in.$^4$

Reinforcing steel:

$A_r = 6.32$ in.$^2$ (the area of 1-#8 bar is 0.79 in.$^2$, per ACI)

$I_r = \frac{\pi r^4}{4} + Ad^2 = 8 \frac{\pi(0.50)^4}{4} + 6(0.79)(9.5)^2 = 428$ in.$^4$

Concrete:

$A_c = A_{cg} - A_s - A_r = 576$ in.$^2$ - 13.3 in.$^2$ - 6.32 in.$^2$ = 556 in.$^2$

$I_c = I_{cg} - I_s - I_r = 27,200$ in.$^4$

Note: The weak axis moment of inertia is used as part of a slenderness check.
Limitations:

1) Normal weight concrete 10 ksi ≥ $f'_c$ ≥ 3 ksi  \[ f'_c = 5 \text{ ksi} \] \text{o.k.} \hspace{1cm} \text{Section I1.2}

2) $F_{ysl} \leq 75 \text{ ksi}$ \hspace{1cm} $F_{ysl} = 60 \text{ ksi}$ \text{o.k.}

3) The cross-sectional area of the steel core shall comprise at least one percent of the total composite cross section.

\[ 13.3 \text{ in.}^2 > (0.01)(576 \text{ in.}^2) = 5.76 \text{ in.}^2 \] \text{o.k.} \hspace{1cm} \text{Section I2.1a}

4) Concrete encasement of the steel core shall be reinforced with continuous longitudinal bars and lateral ties or spirals. The minimum transverse reinforcement shall be at least 0.009 in.$^2$ of tie spacing.

\[ 0.20 \text{ in.}^2/12 \text{ in.} = 0.0167 \text{ in.}^2/\text{in.} > 0.009 \text{ in.}^2/\text{in.} \] \text{o.k.}

5) The minimum reinforcement ratio for continuous longitudinal reinforcing, $\rho_{sr}$, shall be 0.004.

\[ \rho_{sr} = \frac{A_{sr}}{A_g} = \frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.011 > 0.004 \] \text{o.k.} \hspace{1cm} \text{Eqn. I2-1}

**Calculate the total required compressive strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u = 1.2(350 \text{ kips}) + 1.6(1050 \text{ kips})$ = 2100 kips</td>
<td>$P_u = 350 \text{ kips} + 1050 \text{ kips}$ = 1400 kips</td>
</tr>
</tbody>
</table>

**Calculate the available compressive strength**

\[ P_a = A_sF_y + A_{sr}F_{yr} + 0.85A_c f'_c \\ = (13.3 \text{ in.}^2)(50 \text{ ksi}) + (6.32 \text{ in.}^2)(60 \text{ ksi}) + 0.85(556 \text{ in.}^2)(5 \text{ ksi}) = 3410 \text{ kips} \] \hspace{1cm} \text{Eqn. I2-2}

\[ C_i = 0.1 + 2 \left( \frac{A_s}{A_g + A_s} \right) = 0.1 + 2 \left( \frac{13.3 \text{ in.}^2}{556 \text{ in.}^2 + 13.3 \text{ in.}^2} \right) = 0.15 \] \hspace{1cm} \text{Eqn. I2-5}

\[ EI_{eff} = E I_s + 0.5E_s I_{sr} + C_i E I_c \\ = (29,000 \text{ ksi})(53.4 \text{ in.}^4) + 0.5(29,000 \text{ ksi})(428 \text{ in.}^4) + (0.15)(3,900 \text{ in.}^4)(27,200) \\ = 23,700,000 \text{ kip-in.}^2 \] \hspace{1cm} \text{Eqn. I2-4}

User note: $K$ value is from Chapter C and for this case $K = 1.0$.

\[ P_u = \pi^2(EI_{eff})/(KL)^2 = \pi^2(23,700,000 \text{ kip-in.}^2) / (1(1.0)(14\text{in.})(12\text{in./ft}))^2 = 8,290 \text{ kips} \] \hspace{1cm} \text{Eqn. I2-3}

\[ \frac{P_a}{P_u} = \frac{3,410 \text{ kips}}{8,290 \text{ kips}} = 0.411 \] \hspace{1cm} \text{Section I2.1b}

\[ 0.411 < 2.25 \] Therefore use Eqn. I2-2 to solve $P_u$
\[ P_n = P_o \left[ 0.658 \left( \frac{P_o}{P_i} \right) \right] = (3,410 \text{ kips}) \left[ 0.658^{(0.411)} \right] = 2870 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.75 )</td>
<td>( \Omega_c = 2.00 )</td>
</tr>
<tr>
<td>( \phi_c P_n = 0.75(2870\text{ kips}) = 2150 \text{ kips} )</td>
<td>( P_n/\Omega_c = 2870 \text{ kips} / 2.00 =1440 \text{ kips} )</td>
</tr>
<tr>
<td>2150 kips &gt; 2100 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>1440 kips &gt; 1400 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Because the entire load in the column was applied directly to the concrete, accommodations must be made to transfer an appropriate portion of the axial force to the steel column. This force is transferred as a shear force at the interface between the two materials.

Determine the number and spacing of ½-in. diameter headed shear studs to transfer the axial force.

**Solution:**

**Material Properties:**
- Conc. \( f'_c = 5 \text{ ksi} \) \( E_c = 4070 \text{ ksi} \)
- Shear Studs \( F_u = 65 \text{ ksi} \)

**Geometric Properties:**
- W10×45 \( A_d = 13.3 \text{ in.}^2 \) \( A_{flange} = 4.97 \text{ in.}^2 \) \( A_{web} = 3.36 \text{ in.}^2 \) \( d_d = 10.1 \text{ in.} \)
- Shear Studs \( A_{sc} = 0.196 \text{ in.}^2 \)
- Conc. \( d_c = 24 \text{ in.} \)

**Calculate the shear force to be transferred**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = \frac{P_n}{\phi_c} = \frac{2,100}{0.75} = 2,800 \text{ kips} )</td>
<td>( V = P_n/\Omega_c = 1,400(2) = 2,800 \text{ kips} )</td>
</tr>
</tbody>
</table>

\( V' = V(A_d F_u / P_o) = 2800 \text{ kips } ((13.3 \text{ in.}^2)(50 \text{ ksi}) / (3410 \text{ kips})) = 546 \text{ kips} \)

**Calculate the nominal strength of one ½ in. diameter shear stud connector**
\[ Q_n = 0.5 A_{nc} \sqrt{f'_c E_c} \leq A_{nc} F_u \]

\[ 0.5 A_{nc} \sqrt{f'_c E_c} = 0.5 (0.196 \text{ in.}^2) \sqrt{(5 \text{ksi})(3900 \text{ksi})} = 13.7 \text{ kips} \]

\[ A_{nc} F_u = (0.196 \text{ in.}^2)(65 \text{ ksi}) = 12.7 \text{ kips} \]

Therefore use 12.7 kips.

*Calculate the number of shear studs required to transfer the total force, \( V' \)*

\[ \frac{V'}{Q_n} = \frac{546 \text{ kips}}{12.7 \text{ kips}} = 43 \]

An even number of studs are required to be placed symmetrically on two faces. Therefore use 22 studs minimum per flange

*Determine the spacing for the shear studs*

The maximum stud spacing is 16 in.

The available column length is 14ft (12 in./ft) = 168 in. and the maximum spacing is

\[ = 168 \text{ in.} / (22+1) = 7.3 \text{ in.} \]

Therefore, on the flanges, use single studs @ 7 in.

Stud placement is to start 10.5 in. from one end.

*Determine the length of the studs for the flanges;*

\[ \left( \frac{d_x - d_w}{2} \right) - 3 \text{ in.} = \left( \frac{24 \text{ in.} - 10.1 \text{ in.}}{2} \right) - 3 \text{ in.} = 3.95 \text{ in.} \]

Therefore use 3 ½ in. in length for the flanges.

Note: The subtraction of 3 in. is to ensure sufficient cover.

Summary: use \( \frac{1}{2} \)-in. diameter shear stud connectors as shown on each flange, spaced @ 7 in.
Example I-4  Encased Composite Column in Axial Tension

Given:

Determine if the composite column in Example I-3 can support a dead load compression of 150 kips and a wind load tension of 645 kips. The column is pinned at both ends. The steel W-shape and the reinforcing are attached at each end in order to transfer any tensile force.

Solution:

Material Properties:

- **Column W10×45**
  - $F_y = 50$ ksi
  - $F_u = 65$ ksi

- **Concrete**
  - $f'c = 5$ ksi
  - $Ec = 3,900$ ksi

- **Reinforcement**
  - $Fy_{st} = 60$ ksi

Geometric Properties:

- **W10×45**
  - $A_t = 13.3$ in.$^2$
  - $I_y = 53.4$ in.$^4$

- **Reinforcing steel**
  - $A_{yr} = 6.32$ in.$^2$
  - $I_{yr} = 428$ in.$^4$

- **Concrete**
  - $A_c = 556$ in.$^2$
  - $I_c = 27,200$ in.$^4$
Limitations:

1) Normal weight concrete $f'_{c} \geq 3$ ksi; $f'_{c} = 5$ ksi \textbf{o.k.} \hspace{1cm} \text{Section I1.2}

2) $F_{yst} \leq 75$ ksi; \hspace{0.5cm} $F_{yst} = 60$ ksi \textbf{o.k.}

3) The cross-sectional area of the steel core shall comprise at least one percent of the total composite cross section.

$$13.3 \text{ in.}^2 > (0.01)(576 \text{ in.}^2) = 5.76 \text{in.}^2 \textbf{o.k.}$$

4) Concrete encasement of steel core shall be reinforced with continuous longitudinal bars and lateral ties or spirals. The minimum transverse reinforcement shall be at least 0.009 in.$^2$ of tie spacing.

$$0.20 \text{ in.}^2/12 \text{ in.} = 0.0167 \text{ in.}^2/\text{in.} > 0.009 \text{ in.}^2/\text{in.} \textbf{o.k.}$$

5) The minimum reinforcement ratio for continuous longitudinal reinforcing, $\rho_{sr}$, shall be 0.004.

$$\rho_{sr} = \frac{A_{sr}}{A_{g}} = \frac{6.32 \text{in.}^2}{576 \text{in.}^2} = 0.011 > 0.004 \textbf{o.k.} \hspace{1cm} \text{Eqn. I2-1}$$

\begin{tabular}{|c|c|}
\hline
\textbf{LRFD} & \textbf{ASD} \\
\hline
$P_{u} = 0.9(-150 \text{ kips}) + 1.6(645 \text{ kips})$ & $P_{a} = 0.6(-150 \text{ kips}) + 645 \text{ kips}$ \\
$= 897 \text{ kips}$ & $= 555 \text{ kips}$ \\
\hline
\end{tabular}

\textit{Calculate the required tensile strength}

\begin{tabular}{|c|c|}
\hline
\textbf{LRFD} & \textbf{ASD} \\
\hline
$\phi_{t} = 0.90$ & $\Omega_{t} = 1.67$ \\
$\phi_{t}P_{u} = 0.90(1040 \text{ kips}) = 940 \text{ kips}$ & $P_{a}/\Omega_{t} = 1040 \text{ kips} / 1.67 = 625 \text{ kips}$ \\
$940 \text{ kips} > 897 \text{ kips} \textbf{o.k.}$ & $625 \text{ kips} > 555 \text{ kips} \textbf{o.k.}$ \\
\hline
\end{tabular}

\textit{Calculate the available tensile strength}

$$P_{n} = P_{t} = A_{s}F_{y} + A_{sr}F_{yr}$$

$$= (13.3 \text{ in.}^2)(50 \text{ ksi}) + (6.32 \text{ in.}^2)(60\text{ksi}) = 1040 \text{ kips}$$

\begin{tabular}{|c|c|}
\hline
\textbf{LRFD} & \textbf{ASD} \\
\hline
\text{Section I2.1c}  \\
\end{tabular}
Example I-5  Filled Composite Column in Axial Tension

Given:

Determine if the filled composite column in Example I-2 is adequate to support a dead load compression of 70 kips and a wind load tension of 300 kips. The column is pinned at both ends and all of the load is transferred by the base and cap plates.

Solution:

Material Properties:
HSS10×6×¾  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Concrete  \( f'_c = 5 \text{ ksi} \)  \( E_c = 4,070 \text{ ksi} \)

Geometric Properties:
HSS10×6×¾:  \( A = 10.4 \text{ in.}^2 \)

Calculate the required tensile strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 0.9(-70 \text{ kips}) + 1.6(300 \text{ kips}) )</td>
<td>= 417 kips</td>
<td>( P_a = 0.6(-70 \text{ kips}) + 300 \text{ kips} )</td>
</tr>
</tbody>
</table>

Determine the available tensile strength

\( P_a = P_o = A,F_y \)
\[ = (10.4 \text{ in.}^2)(46 \text{ ksi}) = 478 \text{ kips} \]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td></td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_o = 0.90(478 \text{ kips}) = 430 \text{ kips} )</td>
<td></td>
<td>( P_o/\Omega = 478 \text{ kips} / 1.67 = 286 \text{ kips} )</td>
</tr>
<tr>
<td>430 kips &gt; 417 kips</td>
<td>o.k.</td>
<td>286 kips &gt;258 kips</td>
</tr>
</tbody>
</table>

User note: The concrete is not considered to contribute to the available tensile strength, therefore no shear transfer between the encasing steel and the concrete fill is required for this load case.
Example I-6  Filled Composite Member Design for Shear.

Given:

This composite member ASTM A500 grade B with 5ksi normal weight concrete has a dead load end shear of 20 kips and a live load end shear of 60 kips. Verify that this end shear can be safely carried by both LRFD and ASD analysis.

User note: Shear strength shall be calculated based on either the shear strength of the steel section alone as specified in Specification Chapter G or the shear strength of the reinforced concrete portion alone.

Solution:

Material Properties:
ASTM A500 Gr. B  \( F_y = 46 \text{ksi} \)  \( F_u = 58 \text{ksi} \)

Geometric Properties:
HSS 10×6×\( \frac{3}{8} \)  \( d = 10 \text{ in.} \)  \( t_w = 0.349 \text{ in.} \)

<table>
<thead>
<tr>
<th>Calculate the required shear strength</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_u = 1.2(20 \text{ kips}) + 1.6(60 \text{ kips}) )</td>
<td>120 kips</td>
<td>20 kips + 60 kips = 80 kips</td>
</tr>
</tbody>
</table>

Calculate the available shear strength
The available shear area for rectangular HSS and box members is \( 2ht \), where \( h \) is the outside dimension minus the top and bottom radii. If the exact corner radius is not known, \( h \) shall be taken as the outside dimension minus three times the design wall thickness.

User note: \( k_V = 5.0 \) for all sizes of HSS rectangular or box members listed in the Manual. Also all HSS listed in the manual have a \( C_v = 1.0 \).

Calculate \( h \)
\[ h = d - (3t_w) = 10 \text{ in.} - (3)(0.349 \text{ in.}) = 8.95 \text{ in.} \]

Calculate \( A_w \)
\[ A_w = 2ht_w = 2(8.95 \text{ in.})(0.349 \text{ in.}) = 6.25 \text{ in}^2 \]

Calculate \( V_n \)
\[ V_n = 0.6F_yA_wC_v = 0.6(46 \text{ ksi})(6.25 \text{ in}^2)(1.0) = 173 \text{ kips} \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_v = 0.90$</td>
<td>$\Omega_v = 1.67$</td>
</tr>
<tr>
<td>$V_a = \phi_v V_n = 0.90(173 \text{ kips}) = 156 \text{ kips}$</td>
<td>$V_a = V_n/\Omega_v = 173 \text{ kips} / 1.67 = 104 \text{ kips}$</td>
</tr>
<tr>
<td>156 kips $\geq$ 120 kips o.k.</td>
<td>104 kips $\geq$ 80 kips o.k.</td>
</tr>
</tbody>
</table>

User note: For illustration of $C_v = 1.0$ refer to Chapter G, Example G.4.
Example I-7  Combined Axial and Flexural Strength

Given:

A 14 ft long composite column consists of a W10×45 steel section encased in a 24 in.×24in. concrete section. The concrete is reinforced with 8-#8 longitudinal bars and #4 transverse ties @ 12 in. o.c., as illustrated below. Determine if the member has sufficient available strength to support an axial dead load of 260 kips and an axial live load of 780 kips in compression, as well as a dead load moment of 80 k-ft and a live load moment of 240 k-ft. The load is applied directly to the concrete encasement.

Solution:

Material Properties:

W10×45:
ASTM A992
\[ F_y = 50 \text{ ksi} \quad F_u = 65 \text{ ksi} \]

Concrete
\[ f' = 5 \text{ ksi} \quad E_c = 3,900 \text{ ksi} \]

Reinforcement
\[ F_{y_t} = 60 \text{ ksi} \]

Geometric Properties:

W10×45:
\[ A_s = 13.3 \text{ in.}^2 \quad I_s = 53.4 \text{ in.}^4 \quad Z = 54.9 \text{ in}^3 \quad d = 10.1 \text{ in.} \quad b_f = 8.02 \text{ in.} \quad t_w = 0.35 \text{ in.} \]

Reinforcing steel:
\[ A_{sy} = 6.32 \text{ in.}^2 \quad A_{stras} = 1.59 \text{ in.}^2 \quad I_{sy} = 428 \text{ in.}^4 \]

Concrete:
\[ A_c = 556 \text{ in.}^2 \quad I_c = 27,200 \text{ in.}^4 \]
Limitations:

1) Normal weight concrete $10 \text{ ksi} \geq f'_{c} \geq 3 \text{ ksi} \quad f'_{c} = 5 \text{ ksi} \quad \text{OK}$  

2) $F_{yst} \leq 75 \text{ ksi} \quad F_{yst} = 60 \text{ ksi} \quad \text{o.k.}$

3) The cross-sectional area of the steel core shall comprise at least one percent of the total composite cross section.

$$13.3 \text{ in.}^2 > (0.01)(576 \text{ in.}^2) = 5.76 \text{ in.}^2 \quad \text{o.k.}$$

4) Concrete encasement of steel core shall be reinforced with continuous longitudinal bars and lateral ties or spirals. The minimum transverse reinforcement shall be at least $0.009 \text{ in.}^2$ per inch of tie spacing.

$$0.20 \text{ in.}^2/12 \text{ in.} = 0.0167 \text{ in.}^2/\text{in.} > 0.009 \text{ in.}^2/\text{in.} \quad \text{o.k.}$$

5) The minimum reinforcement ratio for continuous longitudinal reinforcing, $\rho_{sr}$, shall be $0.004$.

$$\rho_{sr} = \frac{A_{sr}}{A_{g}} = \frac{6.32 \text{ in.}^2}{576 \text{ in.}^2} = 0.011 > 0.004 \quad \text{o.k.} \quad \text{Eqn. I2-1}$$

Calculate the required strengths

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>$1.2(260 \text{ kips}) + 1.6(780 \text{ kips})$</td>
<td>$P_a = 260 \text{ kips} + 780 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 1560 \text{ kips}$</td>
<td>$= 1040 \text{ kips}$</td>
</tr>
<tr>
<td>$M_a$</td>
<td>$1.2(80 \text{ k-ft}) + 1.6(240 \text{ k-ft})$</td>
<td>$M_a = 80 \text{ k-ft} + 240 \text{ k-ft}$</td>
</tr>
<tr>
<td></td>
<td>$= 480 \text{ k-ft}$</td>
<td>$= 320 \text{ k-ft}$</td>
</tr>
</tbody>
</table>

The available strength of the composite section subjected to combined axial and flexural loads is determined by constructing an interaction curve. The curve is generated by calculating the available strength of the section at a series of points on the interaction curve and reducing the strength for slenderness effects and multiplying by the resistance factor for LRFD or dividing by the safety factor for ASD. The defining equations of the interaction curve were given earlier in Figure I-1a, and will be used to construct the curve illustrated in Commentary Figure C4-3, and repeated here.
Point A \((M = 0)\)

**Determine the available compressive strength and moment strength**

\[
P_o = A_s f_y + A_r f_y' + 0.85 A_c f'_c
\]

\[
= (13.3 \text{ in.}^2)(50 \text{ ksi}) + (6.32 \text{ in.}^2)(60 \text{ksi}) + 0.85(556 \text{ in.}^2)(5\text{ksi}) = 3,410 \text{kips}
\]

Eqn. I2-4

\[
C_I = 0.1 + 2 \left( \frac{A_r}{A_c} \right) = 0.1 + 2 \left( \frac{13.3 \text{in.}^2}{556 \text{in.}^2 + 13.3 \text{in.}^2} \right) = 0.15
\]

Eqn. I2-7

\[
EI_{\text{eff}} = EI_s + 0.5 EI_r + C_I Ec
\]

\[
= (29,000 \text{ksi})(53.4 \text{ in.}^4) + 0.5(29,000 \text{ksi})(428 \text{ in.}^4) + (0.15)(3,900 \text{ in.}^4)(27,200)
\]

\[
= 23,700,000 \text{ kip-in.}^2
\]

Eqn. I2-6

User note: \(K\) value is from Chapter C and for this case \(K = 1.0\).

\[
P_e = \frac{\pi^2(EI_{\text{eff}})}{(KL)^2} = \frac{\pi^2(23,700,000 \text{ kip-in.}^2)}{((1.0)(14 \text{ ft})(12\text{in./ft}))^2} = 8,290 \text{kips}
\]

Eqn. I2-5

\[
\frac{P_o}{P_e} = \frac{3,410 \text{kips}}{8,290 \text{kips}} = 0.411
\]

\[0.411 \leq 2.25 \text{ Therefore use Eqn. I2-2 to determine } P_n\]

\[
P_{nd} = P_o \left[ 0.658 \left( \frac{P_o}{P_e} \right)^{3} \right] = (3410 \text{kips}) \left[ 0.658^{(0.411)} \right] = 2870 \text{kips}
\]

Eqn. I2-2
Point D \( P_D = \frac{A (0.85 f'_c)}{2} \)

\[ P_D = \frac{A (0.85 f'_c)}{2} \]

\[ P_D = \frac{(556 \text{ in}^2)(0.85)(5.0 \text{ ksi})}{2} \]

\[ P_D = 1180 \text{ kips} \]

\( Z_y = \text{full x-axis plastic modulus of steel shape} \)

\[ Z_y = 54.9 \text{ in}^3 \]

\[ Z_r = \left( A_{yr} - A_{yr} \right) \left( \frac{h_y}{2} - c \right) = \left( 6.32 \text{ in}^2 - 1.59 \text{ in}^2 \right) \left( \frac{24 \text{ in}}{2} - 2 \frac{1}{2} \text{ in} \right) = 44.9 \text{ in}^3 \]

\[ Z_c = \frac{h_c h_y^2}{4} - Z_y - Z_r = \frac{(24 \text{ in})(24 \text{ in})^2}{4} - 54.9 \text{ in}^3 - 44.9 \text{ in}^3 = 3360 \text{ in}^3 \]

\[ M_D = Z_y F_y + Z_c f_y + \frac{1}{2} Z_c (0.85 f'_c) \]

\[ M_D = \left( 54.9 \text{ in}^3 \right)(50 \text{ ksi}) + \left( 44.9 \text{ in}^3 \right)(60.0 \text{ ksi}) + \frac{1}{2} \left( 3360 \text{ in}^3 \right)(0.85)(5.0 \text{ ksi}) \]

\[ = 12,600 \text{ k-in} \]

\[ M_{nd} = \frac{12,600 \text{ k-in}}{12 \text{ in/ft}} = 1050 \text{ k-ft} \]

\[ P_{nd} = P_o \left[ 0.658 \left( \frac{P}{P_o} \right) \right] = (1,180 \text{ kips}) \left[ 0.658 \left( 0.411 \right) \right] = 994 \text{ kips} \]
Point B ($P_B = 0$)

For $h_n$ in the flange \( \left( \frac{d}{2} - t_f \right) < h_n \leq \frac{d}{2} \)

\[
h_n = \frac{0.85 f_c \left( A_c + A_s - d b_f + A_{sbr} \right) - 2 F_y \left( A_s - d b_f \right) - 2 F_y A_{sbr}}{2 \left[ 0.85 f_c \left[ h_i - b_f \right] + 2 F_y b_f \right]}
\]

\[
h_n = \frac{0.85 \left( 5.0 \text{ ksi} \right) \left( 556 \text{ in.}^2 + 13.3 \text{ in.}^2 - 10.1(8.02) + 1.59 \right) - 2 \left( 50 \text{ ksi} \right) \left( 13.3 \text{ in.}^2 - 10.1(8.02) \right) - 2(60)(1.59)}{2 \left[ (0.85)(5.0 \text{ ksi}) \left( 24 \text{ in.-8.02} \right) + 2(50)(8.02) \right]}
\]

\[
h_n = 4.98 \text{ in.}
\]

\[
Z_{sn} = Z_s - b_f \left( \frac{d}{2} - h_n \right) \left( \frac{d}{2} + h_n \right)
\]

\[
Z_{sn} = 54.9 \text{ in}^3 - 8.02 \left( \frac{10.1}{2} - 4.98 \right) \left( \frac{10.1}{2} + 4.98 \right) = 49.3 \text{ in}^3
\]

\[
Z_{cn} = h_i h_n^2 - Z_{sn} = (24 \text{ in.})(4.98 \text{ in.})^2 - 49.3 \text{ in}^3
\]

\[
Z_{cn} = 546 \text{ in}^3
\]

\[
M_B = M_D - Z_{sn} F_y - \frac{1}{2} Z_{cn} \left( 0.85 f_c \right)
\]

\[
M_B = (12,600 \text{ k-in.}) - (49.3 \text{ in}^3)(50.0 \text{ ksi})
\]

\[
- \frac{1}{2} \left( 546 \text{ in}^3 \right)(0.85)(5.0 \text{ ksi}) = 8,970 \text{ k-in.}
\]

\[
M_{nB} = \frac{8,970 \text{ k-in.}}{12 \sqrt{A}} = 748 \text{ k-ft}
\]

Point C ($M_C = M_B; P_C = 0.85 f_c A_c$)

\[
P_C = A_c \left( 0.85 f_c \right)
\]

\[
P_C = \left( 556 \text{ in}^2 \right)(0.85)(5.0 \text{ ksi}) = 2360 \text{ kips}
\]

\[
M_{nC} = M_{nB} = 748 \text{ k-ft}
\]

\[
P_{nC} = P_C \left[ 0.658 \left( \frac{P}{P^*} \right) \right] = 2360 \left[ 0.658^{0.411} \right] = 1990 \text{ kips}
\]
Summary of nominal strength including length effects

\[ P_{a,d} = 2870 \text{ kips} \]
\[ M_{aA} = 0 \]
\[ P_{aB} = 0 \]
\[ M_{aB} = 748 \text{ k-ft} \]
\[ P_{aC} = 1990 \text{ kips} \]
\[ M_{aC} = 748 \text{ k-ft} \]
\[ P_{aD} = 994 \text{ kips} \]
\[ M_{aD} = 1050 \text{ k-ft} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section I4</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_a = 0.75 )  ( \phi_b = 0.90 )</td>
<td>( \Omega_c = 2.00 )  ( \Omega_b = 1.67 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_a P_{a,d} = 2150 \text{ kips} )</td>
<td>( P_{a,d} / \Omega_c = 1440 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_{aA} = 0 )</td>
<td>( M_{aA} / \Omega_b = 0 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b P_{aB} = 0 )</td>
<td>( P_{aB} / \Omega_c = 0 )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_{aB} = 673 \text{ k-ft} )</td>
<td>( M_{aB} / \Omega_b = 448 \text{ k-ft} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b P_{aC} = 1490 \text{ kips} )</td>
<td>( P_{aC} / \Omega_c = 995 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_{aC} = 673 \text{ k-ft} )</td>
<td>( M_{aC} / \Omega_b = 448 \text{ k-ft} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b P_{aD} = 746 \text{ kips} )</td>
<td>( P_{aD} / \Omega_c = 497 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( \phi_b M_{aD} = 945 \text{ k-ft} )</td>
<td>( M_{aD} / \Omega_b = 629 \text{ k-ft} )</td>
<td></td>
</tr>
</tbody>
</table>

**Interaction**

\( P_u > P_c \)

1560 kips > 1490 kips

thus

\[
\frac{P_u - P_c}{P_a - P_c} + \frac{M_{cy}}{M_{cy}} \leq 1.0
\]

\[
\frac{1560 - 1490}{2150 - 1490} + \frac{480}{673} = 0.82 > 1.0
\]

Therefore, this column is adequate for the specified loading

**Interaction**

\( P_u > P_c \)

1040 kips > 995 kips

thus

\[
\frac{P_u - P_c}{P_a - P_c} + \frac{M_{cy}}{M_{cy}} \leq 1.0
\]

\[
\frac{1040 - 995}{1440 - 995} + \frac{320}{448} = 0.82 > 1.0
\]

Therefore, this column is adequate for the specified loading

The shear connector requirements for this encased composite column are similar to that of Example I-2
INTRODUCTION

Chapter J of the Specification addresses the design and checking of connections. The chapter’s primary focus is the design of welded and bolted connections. Design requirements for fillers, splices, column bases, concentrated forces, anchors rods, and other threaded parts are also covered. Special requirements for connections subject to fatigue are not covered in this chapter.
Example J.1  Fillet Weld in Longitudinal Shear

Given:

An \( \frac{1}{4} \) in. \( \times \) 18-in. wide plate is fillet welded to a \( \frac{3}{8} \)-in. plate. Assume that the plates are ASTM A572 grade 50 and have been properly sized. Assume \( F_{y} = 70 \) ksi. Note that plates would normally be specified as ASTM A36, but \( F_{y} = 50 \) ksi plate has been used here to demonstrate requirements for long welds.

Size the welds for the loads shown.

Solution:

**Determine the maximum weld size**

Because the overlapping plate is \( \frac{1}{4} \) in., the maximum fillet weld size that can be used without special notation (built out to obtain full-throat thickness as required in AISC Specification Section J2.2b) is a \( \frac{3}{8} \)-in. fillet weld. A \( \frac{3}{8} \)-in. fillet weld can be deposited in the flat or horizontal position in a single pass (true up to \( \frac{1}{4} \)6-in).

**Determine the required strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{u} = 1.2(33 \text{ kips}) + 1.6(100 \text{ kips}) = 200 \text{ kips} )</td>
<td>( P_{a} = 33 \text{ kips} + 100 \text{ kips} = 133 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Determine the length of weld required**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>The design strength per inch of a ( \frac{3}{8} )-in. fillet weld is ( \phi R_{n} = 1.392 (3) = 4.17 \text{ kips/in.} )</td>
<td>The allowable strength per inch of a ( \frac{3}{8} )-in. fillet weld is ( R_{n}/\Omega = 0.928 (3) = 2.78 \text{ kips/in.} )</td>
</tr>
<tr>
<td>( \frac{P_{u}}{\phi R_{n}} = \frac{200 \text{ kips}}{4.17 \text{ kips/in.}} = 48 \text{ in.} )</td>
<td>( \frac{P_{a}}{R_{n}/\Omega} = \frac{133 \text{ kips}}{2.78 \text{ kips/in.}} = 48 \text{ in.} )</td>
</tr>
<tr>
<td>or 24 in. of weld on each side</td>
<td>or 24 in. of weld on each side.</td>
</tr>
</tbody>
</table>
Check the weld for length to weld size ratio

\[
\frac{l}{w} = \frac{24 \text{ in.}}{0.188 \text{ in.}} = 128 > 100,
\]

Therefore Specification Equation J2-1 must be applied, and the length of weld increased, since the resulting \( \beta \) will reduce the available strength below the required strength.

Try a weld length of 27 in.

The new length to weld size ratio is 27 in. \( /0.188 \text{ in.} = 144 \)

For this ratio

\[ \beta = 1.2 - 0.002\left(\frac{l}{w}\right) \leq 1.0; \quad 1.2 - 0.002(144) = 0.912 \]

Recheck the weld at its reduced strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = (0.912)(4.17 \text{ kips/in.})(54 \text{ in.}) )</td>
<td>( R_u = (0.912)(2.78 \text{ kips/in.})(54 \text{ in.}) )</td>
</tr>
<tr>
<td>( = 205 \text{ kips} &gt; P_u = 200 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 137 \text{ kips} &gt; P_u = 133 \text{ kips} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>Therefore, use 27 in. of weld on each side</td>
<td>Therefore, use 27 in. of weld on each side</td>
</tr>
</tbody>
</table>
Example J.2  Fillet Weld Loaded at an Angle

Given:

Design a fillet weld at the edge of a gusset plate to carry a force of 50 kips due to dead load and a force of 150 kips due to live load, at an angle of 60 degrees relative to the weld. Assume the beam and the gusset plate thickness and length have been properly sized.

Solution:

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(50 \text{ kips}) + 1.6(150 \text{ kips}) = 300 \text{ kips} )</td>
<td>( P_a = 50 \text{ kips} + 150 \text{ kips} = 200 \text{ kips} )</td>
</tr>
</tbody>
</table>

Assume a \( \frac{3}{4} \)-in. fillet weld is used on each side.

The shear strength of a \( \frac{3}{4} \)-in. fillet weld is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( 5(1.392) = 6.96 \text{ kip/in.} )</td>
<td>( 5(0.928) = 4.64 \text{ kip/in.} )</td>
</tr>
<tr>
<td>( 2(6.96 \text{ kip/in.}) = 13.9 \text{ kip/in.} )</td>
<td>( 2(4.64 \text{ kip/in.}) = 9.28 \text{ kip/in.} )</td>
</tr>
</tbody>
</table>

Because the angle of the force relative to the axis of the weld is 60 degrees, the strength of the weld can be increased as follows:

\[
 k_w = 0.60 F_{ext} \left( 1.0 + 0.50 \sin^{1.5} \theta \right) \\
= 0.60(70) \left( 1.0 + 0.50 \left( 0.866 \right)^{1.5} \right) \\
= 1.40
\]
Find the increased strength and the required length of weld

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculate the increased strength.</strong> 13.9 kip/in.(1.40) = 19.5 kip/in.</td>
<td><strong>Calculate the increased capacity.</strong> 9.280 kip/in.(1.40) = 13.0 kip/in.</td>
</tr>
<tr>
<td><strong>Determine the required length of weld.</strong> 300 kips/19.5 kip/in. = 15.4 in.</td>
<td><strong>Determine the required length of weld.</strong> 200 kips/13.0 kip/in. = 15.4 in.</td>
</tr>
<tr>
<td>Use 16 in. <strong>o.k.</strong></td>
<td>Use 16 in. <strong>o.k.</strong></td>
</tr>
</tbody>
</table>
**Example J.3 Combined Tension and Shear in Bearing Type Connections**

**Given:**

A ¾-in. diameter, ASTM A325-N bolt is subjected to a tension force of 3.5 kips due to dead load and 12 kips due to live load, and a shear force of 1.33 kips due to dead load and 4 kips due to live load.

Check the combined stresses according to the Equations J3-3a and J3-3b.

**Solution:**

**Calculate the required tensile and shear strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension = 1.2(3.5) +1.6(12.0) = 23.4 kips</td>
<td>Tension = 3.5 + 12.0 = 15.5 kips</td>
</tr>
<tr>
<td>Shear = 1.2(1.33) + 1.6(4.00) = 8.00 kips</td>
<td>Shear = 1.33 + 4.00 = 5.33 kips</td>
</tr>
</tbody>
</table>

Calculate $f_v$

\[
8.00/0.442 = 18.1 \text{ ksi} \leq \phi F_{nv}
\]

Check combined tension and shear.

\[
F_{nt'} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \leq F_{nt}
\]

\[
F_{nt} = 90 \text{ ksi}, F_{nv} = 48 \text{ ksi}
\]

\[
F_{nt'} = 1.3(90) - \frac{90}{0.75(48)}(18.1)
\]  
\[
= 71.7 < 90
\]

\[
R_n = F_{nt'} A_b = 71.7(0.442) = 31.7 \text{ kips}
\]

For combined tension and shear

\[
\phi = 0.75
\]

Design tensile strength

\[
\phi R_n = 0.75(31.7) = 23.8 \text{ kips} > 23.4 \text{ kips o.k.}
\]

---

**Check combined tension and shear.**

\[
F_{nt'} = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \leq F_{nt}
\]

\[
F_{nt} = 90 \text{ ksi}, F_{nv} = 48 \text{ ksi}
\]

\[
F_{nt'} = 1.3(90) - \frac{2.00(90)}{48}(12.1)
\]  
\[
= 71.6 < 90
\]

\[
R_n = F_{nt'} A_b = 71.6(0.442) = 31.6 \text{ kips}
\]

For combined tension and shear

\[
\Omega = 2.00
\]

Allowable tensile strength

\[
R_n/\Omega = 31.6/2.00 = 15.8 \text{ kips} > 15.5 \text{ kips o.k.}
\]
Example J.4  Slip-Critical Connection with Short Slotted Holes

High-strength bolts in slip-critical connections are permitted to be designed to prevent slip either as a serviceability limit state or as a strength limit state. The most common design case is design for slip as a serviceability limit state. The design of slip as a strength limit state should only be applied when bolt slip can result in a connection geometry that will increase the required strength beyond that of a strength limit state, such as bearing or bolt shear. Such considerations occur only when oversized holes or slots parallel to the load are used, and when the slipped geometry increases the demand on the connection. Examples include the case of ponding in flat-roofed long span trusses, or the case of shallow, short lateral bracing.

Given:
Select the number of ¾-in. ASTM A325 slip-critical bolts with a Class A faying surface that are required to support the loads shown when the connection plates have short slots transverse to the load. Select the number of bolts required for slip resistance only.

Assume that the connected pieces have short slots transverse to the load. Use a mean slip coefficient of 0.35, which corresponds to a Class A surface.

Solution:

**Calculate the required strength**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{tu}$</td>
<td>$1.2(17 \text{ kips}) + 1.6(51 \text{ kips}) = 102 \text{ kips}$</td>
<td>$P_a = 17 \text{ kips} + 51 \text{ kips} = 68 \text{ kips}$</td>
</tr>
</tbody>
</table>

For standard holes or slots transverse to the direction of the load, a connection can be designed on the basis of the serviceability limit state. For the serviceability limit state:

$$\phi = 1.00 \quad \Omega = 1.50$$

Specification

Find $R_n$ where:

$\mu = 0.35$ for Class A surface

$D_u = 1.13$

$h_{sc} = 0.85$ (short slotted holes)

$T_b = 28 \text{ kips}$

$N_s = 2$, number of slip planes

$$R_n = \mu D_u h_{sc} T_b N_s$$

$$R_n = 0.35(1.13)(0.85)(28)(2) = 18.8 \text{ kips/bolt}$$

Table J3.1

Eqn.J3-4
Determine the required number of bolts.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 7-3</th>
</tr>
</thead>
<tbody>
<tr>
<td>102 kips/1.00(18.8 kips/bolt) = 5.42 bolts</td>
<td>68 kips/18.8 kips/bolt = 5.42 bolts</td>
<td>Use 6 bolts</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Use 6 bolts o.k.

Given:

Repeat the problem with the same loads, but assuming that the connected pieces have long slotted holes in the direction of the load and that the deformed geometry of the connection would result in a critical load increase.

Solution:

\[ P_u = 102 \text{ kips and } P_a = 68 \text{ kips per the first solution} \]

For this connection, the designer has determined that oversized holes or slots parallel to the direction of the load will result in a deformed geometry of the connection that creates a critical load case. Therefore, the connection is designed to prevent slip at the required strength level.

\[ \phi = 0.85 \quad \Omega = 1.76 \quad \text{Specification} \quad \text{Section J3.8} \]

In addition, \( h_{sc} \) will change because we now have long slotted holes.

Find \( R_n \)

\[ \mu = 0.35 \text{ for Class A surface} \]
\[ D_a = 1.13 \]
\[ h_{sc} = 0.70 \text{ (long slotted holes)} \]
\[ T_b = 28 \text{ kips} \]
\[ N_s = 2, \text{ number of slip planes} \]

\[ R_n = \mu D_a h_{sc} T_b N_s \]

\[ R_n = 0.35(1.13)(0.70)(28)(2) = 15.5 \text{ kips/bolt} \quad \text{Specification} \quad \text{Eqn. J3-4} \]
Determine the required number of bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>102 kips / 0.85(15.5 kips/bolt) = 7.73 bolts</td>
<td>68 kips (1.76) / 15.5 kips/bolt = 7.63 bolts</td>
<td>Table 7-4</td>
</tr>
<tr>
<td>Use 8 bolts o.k.</td>
<td>Use 8 bolts o.k.</td>
<td></td>
</tr>
</tbody>
</table>
Example J.5  Combined Tension and Shear in a Slip-Critical Connection.

Because the pretension of a bolt in a slip-critical connection is used to create the clamping force that produces the shear strength of the connection, the available shear strength must be reduced for any load that produces tension in the connection.

Given:

The slip-critical bolt group shown below is subjected to tension and shear. This connection is designed for slip as a serviceability limit state. Use ¾-in. diameter ASTM A325 slip-critical class A bolts in standard holes. This example shows the design for bolt slip resistance only, and assumes that the beams and plates are adequate to transmit loads in a rigid fashion.

Solution:

The fastener pretension for a ¾-in. diameter ASTM A325 bolt is 28 kips

\[ D_u = 1.13 \text{ per Specification Section J3.8.} \]

\[ N_b = \text{number of bolts carrying the applied tension.} \]

Determine the tension on bolts, check tension on the bolts and find \( k_s \).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_u = 1.2(10 \text{kips}) + 1.6(30 \text{kips}) = 60 \text{kips} ]</td>
<td>[ P_u = 10 \text{kips} + 30 \text{kips} = 40 \text{kips} ]</td>
</tr>
<tr>
<td>By geometry,</td>
<td>By geometry,</td>
</tr>
<tr>
<td>[ T_u = \frac{4}{5} \left( \frac{60 \text{kips}}{8 \text{ bolts}} \right) = 6 \text{kips/bolt} ]</td>
<td>[ T_u = \frac{4}{5} \left( \frac{40 \text{kips}}{8 \text{ bolts}} \right) = 4 \text{kips/bolt} ]</td>
</tr>
<tr>
<td>[ V_u = \frac{3}{5} \left( \frac{60 \text{kips}}{8 \text{ bolts}} \right) = 4.5 \text{kips/bolt} ]</td>
<td>[ V_u = \frac{3}{5} \left( \frac{40 \text{kips}}{8 \text{ bolts}} \right) = 3 \text{kips/bolt} ]</td>
</tr>
<tr>
<td>Check bolt tension</td>
<td>Check bolt tension</td>
</tr>
<tr>
<td>[ \phi R_n = 29.8 \text{kips/bolt} &gt; 6 \text{kips/bolt} \text{ o.k.} ]</td>
<td>[ R_n/\Omega = 19.9 \text{kips/bolt} &gt; 4 \text{kips/bolt} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>
Combined tension and shear factor

\[
k_s = 1 - \frac{T_u}{D_u T_b N_b} = 1 - \frac{6 \text{kips}}{1.13(28 \text{kips})(1)} = 0.810
\]

Combined tension and shear factor

\[
k_s = 1 - \frac{1.5T_u}{D_u T_b N_b} = 1 - \frac{1.5 \times 4 \text{kips}}{1.13(28 \text{kips})(1)} = 0.810
\]

Eqn. J3-5a and J3-5b

Multiply the available shear strength of the bolts by the reduction factor \(k_s\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_u = 11.1 \text{kips/bolt})</td>
<td>(R_u / \Omega = 7.38 \text{kips/bolt})</td>
</tr>
</tbody>
</table>

Modify the slip resistance by \(k_s\) and check bolt shear

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(k_s \phi R_u = (0.810)(11.1 \text{kips/bolt}) = 8.99 \text{kips/bolt} &gt; 4.50 \text{kips/bolt} \text{ o.k.})</td>
<td>(k_s R_u / \Omega = (0.810)(7.38 \text{kips/bolt}) = 5.98 \text{kips/bolt} &gt; 3.00 \text{kips/bolt} \text{ o.k.})</td>
</tr>
</tbody>
</table>
Example J.6  Bearing Strength of a Pin in a Drilled Hole

Given:

A 1-in. diameter pin is placed in a drilled hole in a 1½-in. thick steel plate.

Determine the available bearing strength of the pinned connection.

Material Properties:
Plate  ASTM A36  \( F_y = 36 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)

Solution:

Calculate the projected bearing area

\[ A_{pb} = d \cdot t_p = (1 \text{ in.})(1\frac{1}{2} \text{ in.}) = 1\frac{1}{2} \text{ in.}^2 \]

Calculate nominal bearing strength

\[ R_n = 1.8F_yA_{pb} = 1.8(36 \text{ ksi})(1\frac{1}{2} \text{ in.}) = 97.2 \text{ kips} \]

Calculate the available bearing strength

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
\phi = 0.75 & \Omega = 2.00 \\
\phi R_n = 0.75(97.2 \text{ kips}) = 72.9 \text{ kips} & \frac{R_n}{\Omega} = \frac{97.2 \text{ kips}}{2.00} = 48.6 \text{ kips} \\
\hline
\end{array}
\]
Example J.7  Base Plate Bearing on Concrete

Given:

A W12×96 column bears on a 24 in. × 24 in. concrete pedestal. The space between the base plate and the concrete pedestal is grouted. Design the base plate to support the following loads in axial compression:

\[ P_D = 115 \text{ kips} \]
\[ P_L = 345 \text{ kips} \]

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column W12×96</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Base Plate</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
<tr>
<td>Concrete Pedestal</td>
<td></td>
<td>3 ksi</td>
<td></td>
</tr>
<tr>
<td>Grout</td>
<td></td>
<td>4 ksi</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column W12×96</td>
<td>( d )</td>
<td>12.7 in.</td>
</tr>
<tr>
<td></td>
<td>( b_f )</td>
<td>12.2 in.</td>
</tr>
<tr>
<td></td>
<td>( t_f )</td>
<td>0.900 in.</td>
</tr>
<tr>
<td></td>
<td>( t_w )</td>
<td>0.550 in.</td>
</tr>
</tbody>
</table>

Solution:

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u ) = 1.2(115 \text{ kips}) + 1.6(345 \text{ kips}) = 690 \text{ kips}</td>
<td>( P_a = 115 \text{ kips} + 345 \text{ kips} = 460 \text{ kips}</td>
<td></td>
</tr>
</tbody>
</table>

\[ A_{ped} = (24 \text{ in.})(24 \text{ in.}) = 576 \text{ in.}^2 \]
\[ A_{col} = (12.7 \text{ in.})(12.2 \text{ in.}) = 155 \text{ in.}^2 \]

Since the pedestal area is larger than the column footprint area, but less than less than 4 times column footprint area, the concrete bearing area will be the geometrically similar area of the pedestal to the base plate.
Calculate the base plate area

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( A_{i(\text{req})} = \frac{P_p}{\phi_c 0.85 f'_{c}} )</td>
<td>( A_{i(\text{req})} = \frac{P_p \Omega_c}{0.85 f'_{c}} )</td>
</tr>
<tr>
<td>( = \frac{690 \text{ kips}}{0.6(0.85)(3 \text{ ksi})} = 451 \text{ in.}^2 )</td>
<td>( = \frac{(460 \text{ kips})(2.5)}{(0.85)(3 \text{ ksi})} = 451 \text{ in.}^2 )</td>
</tr>
</tbody>
</table>

Note: The strength of the grout has conservatively been neglected, as its strength is greater than that of the concrete pedestal.

Try a 22 in.\( \times \)22 in. base plate

Check base plate dimensions

Verify \( N \geq d + 2(3 \text{ in.}) \) and \( B \geq b_y + 2(3 \text{ in.}) \)

\( d + 2(3 \text{ in.}) = 12.7 \text{ in.} + 2(3 \text{ in.}) = 18.7 \text{ in.} < 22 \text{ in.} \quad \text{o.k.} \)

\( b_y + 2(3 \text{ in.}) = 12.2 \text{ in.} + 2(3 \text{ in.}) = 18.2 \text{ in.} < 22 \text{ in.} \quad \text{o.k.} \)

Base plate area, \( A_i = NB = (22 \text{ in.})(22 \text{ in.}) = 484 \text{ in.}^2 > 451 \text{ in.}^2 \quad \text{o.k.} \)

Note: A square base plate with a square anchor rod pattern will be used to minimize the chance for field and shop problems.

Calculate the geometrically similar concrete bearing area

Since the pedestal is square and the base plate is a concentrically located square, the full pedestal area is also the geometrically similar area. Therefore,

\( A_c = (24 \text{ in.})(24 \text{ in.}) = 576 \text{ in.}^2 \)

Verify the concrete bearing strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( \phi_c P_p = \phi_c 0.85 f'_{c} A_i \sqrt{\frac{A_c}{A_i}} )</td>
<td>( P_p / \Omega_c = \frac{0.85 f'_{c} A_i \sqrt{\frac{A_c}{A_i}}}{\Omega_c} )</td>
</tr>
<tr>
<td>( = 0.6(0.85)(3 \text{ ksi})(484 \text{ in.}^2) \sqrt{\frac{576 \text{ in.}^2}{484 \text{ in.}^2}} )</td>
<td>( = \frac{(0.85)(3 \text{ ksi})(484 \text{ in.}^2)}{2.5} \sqrt{\frac{576 \text{ in.}^2}{484 \text{ in.}^2}} )</td>
</tr>
<tr>
<td>( = 808 \text{ kips} &gt; 690 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 539 \text{ kips} &gt; 460 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>
Note: It is permitted to take $\phi_c = 0.65$ per ACI 318-02

Also note that as the area of the base plate approaches the area of concrete, the modifying ratio, $\sqrt[3]{\frac{A_b}{A_t}}$, approaches unity and Specification Eqn. J8-2 converges to Specification Eqn. J8-1.

Calculate required base plate thickness

$$m = \frac{N - 0.95d}{2} = \frac{22 \text{ in.} - 0.95(12.7 \text{ in.})}{2} = 4.97 \text{ in.}$$

$$n = \frac{B - 0.8b_f}{2} = \frac{22 \text{ in.} - 0.8(12.2 \text{ in.})}{2} = 6.12 \text{ in.}$$

$$n' = \frac{\sqrt{db_f}}{4} = \frac{\sqrt{(12.7 \text{ in.})(12.2 \text{ in.})}}{4} = 3.11 \text{ in.}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate required base plate thickness</td>
<td>Calculate required base plate thickness</td>
</tr>
<tr>
<td>$X = \frac{4db_f P_w}{(d + b_f)^2 \phi_c P_p}$</td>
<td>$X = \frac{4db_f P_w \Omega_c}{(d + b_f)^2 P_p}$</td>
</tr>
<tr>
<td>$= \frac{4(12.7 \text{ in.})(12.2 \text{ in.})(690 \text{ kips})}{(12.7 \text{ in.} + 12.2 \text{ in.})^2(808 \text{ kips})}$</td>
<td>$= \frac{4(12.7 \text{ in.})(12.2 \text{ in.})(460 \text{ kips})}{(12.7 \text{ in.} + 12.2 \text{ in.})^2 (539 \text{ kips})}$</td>
</tr>
<tr>
<td>$= 0.854$</td>
<td>$= 0.853$</td>
</tr>
<tr>
<td>$\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1 - X}} \leq 1$</td>
<td>$\lambda = \frac{2\sqrt{X}}{1 + \sqrt{1 - X}} \leq 1$</td>
</tr>
<tr>
<td>$= \frac{2\sqrt{0.854}}{1 + \sqrt{1 - 0.854}}$</td>
<td>$= \frac{2\sqrt{0.853}}{1 + \sqrt{1 - 0.853}}$</td>
</tr>
<tr>
<td>$= 1.34 &gt; 1$, use $\lambda = 1.$</td>
<td>$= 1.34 &gt; 1$, use $\lambda = 1.$</td>
</tr>
</tbody>
</table>

Note: $\lambda$ can always be conservatively taken as being equal to 1

$$\lambda n' = (1)(3.11 \text{ in.}) = 3.11 \text{ in.}$$

$$l = \max(m, n, \lambda n') = \max(4.97 \text{ in.}, 6.12 \text{ in.}, 3.11 \text{ in.}) = 6.12 \text{ in.}$$
Use a 2 in. thick base plate.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{pu} = \frac{P_u}{BN} = \frac{690 \text{ kips}}{(22 \text{ in.})(22 \text{ in.})} = 1.43 \text{ ksi}$</td>
<td>$f_{pu} = \frac{P_u}{BN} = \frac{460 \text{ kips}}{(22 \text{ in.})(22 \text{ in.})} = 0.95 \text{ ksi}$</td>
</tr>
<tr>
<td>$t_{p,(req)} = l \sqrt{\frac{2 f_{pu}}{0.9 F_y}}$</td>
<td>$t_{p,(req)} = l \sqrt{\frac{3.33 f_{pu}}{F_y}}$</td>
</tr>
<tr>
<td>$= (6.12 \text{ in.}) \sqrt{\frac{2(1.43 \text{ ksi})}{0.9(36 \text{ ksi})}}$</td>
<td>$= (6.12 \text{ in.}) \sqrt{\frac{3.33(0.950 \text{ ksi})}{36 \text{ ksi}}}$</td>
</tr>
<tr>
<td>$= 1.82 \text{ in.}$</td>
<td>$= 1.82 \text{ in.}$</td>
</tr>
</tbody>
</table>
CHAPTER K
Design of HSS and Box Member Connections

Examples K.1 through K.6 illustrate common beam to column shear connections that have been adapted for use with HSS columns. Example K.7 illustrates a through plate shear connection, which is unique to HSS columns. Calculations for transverse and longitudinal forces applied to HSS are illustrated in Examples K.8 and K.9. An example of an HSS truss connection is given in Example K.10. Examples on HSS cap plate and base plate connections are given in Examples K.11 through K.13.
Example K.1  Welded/bolted Wide Tee Connection to an HSS Column

Given:
Design a connection between a W16×50 beam and a HSS8×8×¼ column using a WT5×24.5. Use ⅜-in. diameter ASTM A325-N bolts in standard holes with a bolt spacing, s, of 3 in., vertical edge distance Lev of 1¼ in. and 3 in. from the weld line to the bolt line. Design as a flexible connection.

\[ P_D = 6.2 \text{ kips} \]
\[ P_L = 18.5 \text{ kips} \]

Note: A tee with a flange width wider than 8 in. was selected to provide sufficient surface for flare bevel groove welds on both sides of the column, since the tee will be slightly offset from the column centerline.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam</th>
<th>ASTM A992</th>
<th>( F_y = 50 \text{ ksi} )</th>
<th>( F_u = 65 \text{ ksi} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Tee</td>
<td>ASTM A992</td>
<td>( F_y = 50 \text{ ksi} )</td>
<td>( F_u = 65 \text{ ksi} )</td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>( F_y = 46 \text{ ksi} )</td>
<td>( F_u = 58 \text{ ksi} )</td>
</tr>
</tbody>
</table>

Manual

Table 2-3

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam</th>
<th>W16×50</th>
<th>( t_w = 0.380 \text{ in.} )</th>
<th>( t_f = 0.630 \text{ in.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>T = 13 ¾ in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Tee</td>
<td>WT5×24.5</td>
<td>( t_w = t_f = 0.340 \text{ in.} )</td>
<td>( t_f = 0.560 \text{ in.} )</td>
</tr>
<tr>
<td></td>
<td>( b_f = 10.0 \text{ in.} )</td>
<td>( k_{f} = \frac{1}{6} \text{ in.} )</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Column</td>
<td>HSS8×8×¼</td>
<td>( t = 0.233 \text{ in.} )</td>
<td>( B = 8.00 \text{ in.} )</td>
</tr>
</tbody>
</table>

Manual

Tables

1-1, 1-8, and 1-12

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ P_u = 1.2(6.20 \text{ kips}) + 1.6(18.5 \text{ kips}) ] = 37.0 kips</td>
<td>[ P_u = 6.20 \text{ kips} + 18.5 \text{ kips} ] = 24.7 kips</td>
</tr>
</tbody>
</table>
Calculate the available strength assuming the connection is flexible

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the number of bolts</td>
<td>Determine the number of bolts</td>
</tr>
<tr>
<td>Determine the single bolt shear strength</td>
<td>Determine the single bolt shear strength</td>
</tr>
<tr>
<td>( \phi r_n = 15.9 \text{kips} )</td>
<td>( r_n / \Omega = 10.6 \text{kips} )</td>
</tr>
<tr>
<td>Determine single bolt bearing strength based on edge distance</td>
<td>Determine single bolt bearing strength based on edge distance</td>
</tr>
<tr>
<td>( L_{ev} = 1\frac{3}{4} \text{in.} \geq 1.25 \text{in.} \quad \text{o.k.} )</td>
<td>( L_{ev} = 1\frac{3}{4} \text{in.} \geq 1.25 \text{in.} \quad \text{o.k.} )</td>
</tr>
<tr>
<td>( \phi r_n = 49.4 \text{kips/in.(0.340 in.)} = 16.8 \text{kips} )</td>
<td>( r_n / \Omega = 32.9 \text{kips/in.(0.340 in.)} = 11.2 \text{kips} )</td>
</tr>
<tr>
<td>Determine single bolt bearing capacity based on spacing</td>
<td>Determine single bolt bearing capacity based on spacing</td>
</tr>
<tr>
<td>( s = 3.00 \text{in.} &gt; 3(\frac{3}{8}) = 2.25 \text{in.} )</td>
<td>( s = 3.00 \text{in.} &gt; 3(\frac{3}{8}) = 2.25 \text{in.} )</td>
</tr>
<tr>
<td>( \phi r_n = 87.8 \text{kips/in.(0.340 in.)} = 29.8 \text{kips} )</td>
<td>( r_n / \Omega = 58.5 \text{kips/in.(0.340 in.)} = 19.9 \text{kips} )</td>
</tr>
<tr>
<td>Therefore bolt shear controls,</td>
<td>Therefore bolt shear controls,</td>
</tr>
<tr>
<td>( C_{\min} = \frac{P}{\phi r_n} = \frac{37.0 \text{kips}}{15.9 \text{kips}} = 2.33 )</td>
<td>( C_{\min} = \frac{P}{r_n / \Omega} = \frac{24.7 \text{kips}}{10.6 \text{kips}} = 2.33 )</td>
</tr>
<tr>
<td>Using ( e = 3 \text{in.} ) and ( s = 3 \text{in.} ), determine ( C ).</td>
<td>Using ( e = 3 \text{in.} ) and ( s = 3 \text{in.} ), determine ( C ).</td>
</tr>
<tr>
<td>Try 4 bolts, ( C = 2.81 &gt; 2.33 \quad \text{o.k.} )</td>
<td>Try 4 bolts, ( C = 2.81 &gt; 2.33 \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Check WT stem thickness limit

\[ t_{\max} = \frac{d_t}{2} + \frac{\gamma_0}{6} \text{ in.} = \frac{\left(\frac{3}{4} \text{ in.}\right)}{2} + \frac{\gamma_0}{6} \text{ in.} = 0.438 \text{ in.} > 0.340 \text{ in.} \quad \text{o.k.} \]

Note: beam web thickness is greater than WT stem thickness. If the beam web were thinner than the WT stem, this check could be satisfied by checking the thickness of the beam web

Determine WT length required

A W16\times50 has a \( T \)-dimension of 13\% in.

\[ L_{\min} = T/2 = (13\% \text{ in.})/2 = 6.81 \text{ in.} \]

Determine WT length required for bolt spacing and edge distances

\[ L = 3(3.00 \text{in.}) + 2 (1.25 \text{in.}) = 11.5 \text{ in.} < 13\% \text{ in.} \quad \text{o.k.} \]

Try \( L = 11.5 \text{ in.} \).
Calculate the stem shear yielding strength

\[ R_a = 0.6 F_y A_g = 0.6(50 \text{ ksi})(11.5 \text{ in.})(0.340 \text{ in.}) = 117 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_a = 1.00(117 \text{ kips}) = 117 \text{ kips} )</td>
<td>( R_a / \Omega = \frac{117 \text{ kips}}{1.5} = 78.0 \text{ kips} )</td>
</tr>
<tr>
<td>117 kips &gt; 37.0 kips \textbf{o.k.}</td>
<td>78.0 kips &gt; 24.7 kips \textbf{o.k.}</td>
</tr>
</tbody>
</table>

Calculate the stem shear rupture strength

\[ R_a = [L-n(d_s + 1/16)][t] (0.6 F_y) = [11.5 \text{ in.} - 4(0.875 \text{ in.})](0.340 \text{ in.})(0.6)(65 \text{ ksi}) = 106 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_a = 0.75(106 \text{ kips}) = 79.6 \text{ kips} )</td>
<td>( R_a / \Omega = \frac{106 \text{ kips}}{2.00} = 53.0 \text{ kips} )</td>
</tr>
<tr>
<td>79.6 kips &gt; 37.0 kips \textbf{o.k.}</td>
<td>53.0 kips &gt; 24.7 kips \textbf{o.k.}</td>
</tr>
</tbody>
</table>

Calculate the stem available block shear rupture strength

For this case \( U_{bs} = 1.0 \)

\[ \phi F_u A_{ult} = 76.2 \text{ kips/in.} \]
\[ \phi 0.6 F_y A_{ugv} = 231 \text{ kips/in.} \]
\[ \phi 0.6 F_u A_{wul} = 210 \text{ kips/in.} \]
\[ \phi R_a = \phi 0.6 F_y A_{ugv} + \phi U_{bs} F_u A_{ult} \leq \phi 0.6 F_y A_{ugv} + \phi U_{bs} F_u A_{ult} \\ \]
\[ \phi R_a = 0.340 \text{ in.}(210 \text{ kips/in.} + 76.2 \text{ kips/in.}) \leq 0.340 \text{ in.}(231 \text{ kips/in.} + 76.2 \text{ kips/in.}) = 97.3 \text{ kips} \leq 104 \text{ kips} \]

\[ 97.3 \text{ kips} > 37.0 \text{ kips} \textbf{o.k.} \]

Check stem bending

Calculate the required flexural strength

\[ M_a = P_a e = 37.0 \text{ kips}(3.00 \text{ in.}) = 111 \text{ kip-in.} \]
\[ M_a = P_a e = 24.7 \text{ kips}(3.00 \text{ in.}) = 74.1 \text{ kip-in.} \]
Calculate the stem nominal flexural yielding strength

\[
Z = \frac{td^2}{4} = \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{4} = 11.2 \text{ in.}^3
\]

\[
S = \frac{td^2}{6} = \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{6} = 7.49 \text{ in.}^3
\]

\[
M_a = M_p = F_sZ \leq 1.6M_y
\]

\[
= 50 \text{ ksi}(11.2 \text{ in.}^3) \leq 1.6(50 \text{ ksi})(7.49 \text{ in.}^3)
\]

\[
= 560 \text{ kip-in.} < 599 \text{ kip-in.} \text{ o.k.}
\]

Note: the 1.6 limit will never control, because the shape factor for a plate is 1.5

Calculate the stem available flexural yielding strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi M_n = 0.90(560 \text{ kip-in.}))</td>
<td>(M_n / \Omega = 560 \text{ kip-in.} \div 1.67)</td>
</tr>
<tr>
<td>= 504 kip-in. &gt; 111 kip-in. \text{ o.k.}</td>
<td>= 335 kip-in. &gt; 74.1 kip-in. \text{ o.k.}</td>
</tr>
</tbody>
</table>

Calculate the stem flexural rupture strength

\[
Z_{net} = \frac{td^2}{4} - 2t(d_h + \frac{\sqrt{3}}{6} \text{ in.})(1.5 \text{ in.} + 4.5 \text{ in.})
\]

\[
= \frac{0.340 \text{ in.}(11.5 \text{ in.})^2}{4} - 2(0.340 \text{ in.})(\frac{\sqrt{3}}{6} \text{ in.} + \frac{\sqrt{3}}{6} \text{ in.})(1.5 \text{ in.} + 4.5 \text{ in.}) = 7.67 \text{ in.}^3
\]

\[
M_a = F_sZ_{net} = 65 \text{ ksi}(7.67 \text{ in.}^3) = 499 \text{ kip-in.}
\]

Calculate the stem available flexural rupture strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi M_n = 0.75(499 \text{ kip-in.}))</td>
<td>(M_n / \Omega = \frac{499 \text{ kip-in.}}{2.00})</td>
</tr>
<tr>
<td>= 374 kip-in. &gt; 111 kip-in. \text{ o.k.}</td>
<td>= 250 kip-in. &gt; 74.1 kip-in. \text{ o.k.}</td>
</tr>
</tbody>
</table>

Check beam web bearing

\(t_w > t_s \rightarrow 0.380 \text{ in.} > 0.340 \text{ in.}\)

Beam web is satisfactory for bearing by comparison with WT.

Calculate weld size

Since the flange width of the WT is larger than the width of the HSS, a flare bevel groove weld is required. Taking the outside radius as \(2(\frac{\sqrt{3}}{4} \text{ in.}) = \frac{\sqrt{3}}{2} \text{ in.}\) and using AISC Specification Table J2.2, the effective throat thickness of the flare bevel weld is \(E = \frac{\sqrt{3}}{2}(\frac{\sqrt{3}}{2} \text{ in.}) = 0.156 \text{ in.}\)
The equivalent fillet weld that provides the same throat dimension is

$$\left( \frac{D}{16}, \frac{1}{\sqrt{2}} \right) = 0.156 \rightarrow D = 16\sqrt{2}(0.156) = 3.53 \text{ sixteenths of an inch}$$

The equivalent fillet weld size is used in the following calculations

**Check weld ductility**

$$b = \frac{b_f - 2k_v}{2} = \frac{8.00 \text{ in.} - 2(0.156 \text{ in.})}{2} = 3.19 \text{ in.}$$

$$w_{\text{min}} = \left( 0.0158 \right) \frac{F_n t_f^2}{b} \left( \frac{b^2}{L^2} + 2 \right) \leq 16 \left( 0.625 t_f \right)$$

$$= \left( 0.0158 \right) \left( 50 \text{ ksi} \right) \left( 0.560 \text{ in.} \right)^2 \left[ \left( 3.19 \text{ in.} \right)^2 + 2 \right] \leq \left( 0.625 \right) \left( 0.340 \text{ in.} \right)$$

$$= 0.161 \text{ in.} < 0.213 \text{ in.}$$

0161 in. = 2.58 sixteenths of an in.

$$D_{\text{min}} = 2.58 < 3.53 \text{ sixteenths of an in.} \quad \text{o.k.}$$

**Calculate the nominal weld shear strength**

The load is assumed to act concentrically with the weld group.

$$a = 0, \text{ therefore, } C = 3.71$$

$$R_n = CC_D = 3.71(1.00)(3.53 \text{ sixteenths of an in.})(11.5 \text{ in.}) = 151 \text{ kips}$$

**Check shear rupture in HSS at weld**

$$t_{\text{min}} = \frac{3.09D}{F_n} = \frac{3.09(3.53 \text{ sixteenths})}{58 \text{ ksi}} = 0.188 \text{ in.}$$

Therefore the weld controls

**Calculate the available weld strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(151 \text{ kips}) = 113 \text{ kips} &gt; 37.0 \text{ kips} \quad \text{o.k.}$</td>
<td>$R_n / \Omega = 151 \text{ kips} / 2.00 = 75.3 \text{ kips} &gt; 24.7 \text{ kips} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>
Example K.2  Welded/bolted Narrow Tee Connection to an HSS Column

Given:
Design a connection for a \(W16\times50\) beam to an HSS8\(\times8\times\frac{3}{4}\) column using a tee with fillet welds against the flat width of the HSS. Use \(\frac{3}{4}\)-in. diameter A325-N bolts in standard holes with a bolt spacing, \(s\), of 3.00 in., vertical edge distance \(L_{cv}\) of 1\(\frac{1}{4}\)-in. and 3.00 in. from the weld line to the center of the bolt line. Design this as a connection to a flexible support. Assume that, for architectural purposes, the flanges of the WT from the previous example have been stripped down to a width of 6.5 in.

\[P_D = 6.2\text{ kips}\]
\[P_L = 18.5\text{ kips}\]

Note: This is the same problem as Example K.1 with the exception that a narrow tee will be selected which will permit fillet welds on the flat of the column. The beam will still be centered; therefore the tee will be slightly offset.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>(F_y) (ksi)</th>
<th>(F_u) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Tee</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
</tr>
</tbody>
</table>

Geometric Properties:

| Component | Section | \(t_w\) (in.) \(d\) (in.) \(t_f\) (in.) |
|-----------|---------|----------------|----------------|
| Beam      | W16\times50 | 0.380         | 16.3          | 0.630          |
| Column    | HSS8\times8\times\frac{3}{4} | 0.233         | 8.00          |
| Tee       | WT5\times24.5 | 0.340        | 4.99          | 0.560          |

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(P_u = 1.2(6.20\text{ kips}) + 1.6(18.5\text{ kips}) = 37.0\text{ kips})</td>
<td>(P_u = 6.20\text{ kips} + 18.5\text{ kips} = 24.7\text{ kips})</td>
</tr>
</tbody>
</table>
Verify the strength of the WT

Maximum flange width assuming 1/4-in. welds and HSS corner radius equal to 1.5 times the nominal thickness \((1.5)(1/4\text{ in.}) = 3/8\text{ in.})

\[
b_f \leq 8.00\text{ in.} - 2(1/4\text{ in.}) - 2(1/6\text{ in.}) = 6.75\text{ in.}
\]

The strength of the stem thickness was verified in Example K.1.

Determine the number of bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the bolt shear strength</td>
<td>Determine the bolt shear strength</td>
</tr>
<tr>
<td>(\phi r_u = 15.9\text{ kips} )</td>
<td>(r_n/\Omega = 10.6\text{ kips} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine bolt bearing strength based on edge distance</td>
<td>Determine bolt bearing strength based on edge distance</td>
</tr>
<tr>
<td>(L_{ev} = 1\frac{1}{4}\text{-in.} \geq 1.25\text{ in.} )</td>
<td>(L_{ev} = 1\frac{1}{4}\text{-in.} \geq 1.25\text{ in.} )</td>
</tr>
<tr>
<td>(\phi r_u = 49.4\text{ kips/in.}(0.340\text{ in.}) = 16.8\text{ kips} )</td>
<td>(r_n/\Omega = 32.9\text{ kips/in.}(0.340\text{ in.}) = 11.2\text{ kips} )</td>
</tr>
<tr>
<td>Bolt shear strength controls</td>
<td>Bolt shear strength controls</td>
</tr>
</tbody>
</table>

Determine the coefficient for the eccentrically loaded bolt group

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi r_u = 15.9\text{ kips} )</td>
<td>(r_n/\Omega = 10.6\text{ kips} )</td>
</tr>
<tr>
<td>(C_{min} = \frac{P_u}{\phi r_u} = \frac{37.0\text{ kips}}{15.9\text{ kips}} = 2.33 )</td>
<td>(C_{min} = \frac{P_u}{r_n/\Omega} = \frac{24.7\text{ kips}}{10.6\text{ kips}} = 2.33 )</td>
</tr>
<tr>
<td>Using (e = 3.00\text{ in.} ) and (s = 3.00\text{ in.} ), enter Manual Table 7-7</td>
<td>Using (e = 3.00\text{ in.} ) and (s = 3.00\text{ in.} ), enter Manual Table 7-7</td>
</tr>
<tr>
<td>Try 4 bolts, (C = 2.81 &gt; 2.33 \text{ o.k.} )</td>
<td>Try 4 bolts, (C = 2.81 &gt; 2.33 \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Determine the minimum fillet weld size based on the thinner part

Minimum size = 1/8 in. (D = 2) for welding to 1/4 in. material

Check weld ductility

\[
b = \left(\frac{b_f - 2k_i}{2}\right) = \left[\frac{5\frac{3}{8}\text{ in.} - 2(1/6\text{ in.})}{2}\right] = 2\frac{3}{8}\text{ in.}
\]

\[
w_{min} = \left(0.0158\right)\frac{F_{wu} t_f^2 (b_f^2 + 2)}{b} \leq \left(0.625 t_j\right)
\]
\[
= (0.0158) \frac{(50 \text{ ksi})(0.360 \text{ in.})}{2 \frac{3}{8} \text{ in.}} \left( \frac{(2 \frac{3}{4} \text{ in.})^2}{(11 \frac{1}{2} \text{ in.})^2} + 2 \right) \leq (0.625)(0.240 \text{ in.})
\]

\[
= 0.093 \text{ in.} \leq 0.150 \text{ in.}
\]

\[D_{\text{min}} = 0.093(0.625) = 0.058 \text{ sixteenths of an in.} < 2 \text{ o.k.}\]

*Calculate minimum wall thickness in HSS to match weld strength*

\[t_{\text{min}} = \frac{3.09D}{F_u} = \frac{3.09(2)}{58 \text{ ksi}} = 0.107 \text{ in.} < 0.233 \text{ in.}\]

Therefore weld controls

*Calculate the weld nominal shear strength*

The load is assumed to act concentrically with the weld group.

\[a = 0; \text{ therefore } C = 3.71\]

\[R_n = CC_D = 3.71(1.00)(2 \text{ sixteenths of an in.})(11.5 \text{ in.}) = 85.3 \text{ kips}\]

*Calculate the available weld shear strength*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75(85.3 \text{ kips}) = 64.0 \text{ kips})</td>
<td>(R_n / \Omega = \frac{85.3 \text{ kips}}{2.00} = 42.7 \text{ kips})</td>
</tr>
</tbody>
</table>

\[64.0 \text{ kips} > 37.0 \text{ kips} \text{ o.k.} \quad 42.7 \text{ kips} > 24.7 \text{ kips} \text{ o.k.}\]

Note: Use a \(\frac{3}{16}\)-in. fillet weld as a practical minimum.
Example K.3  
Double Angle Connection to an HSS Column

Given:
Use Tables 10-1 and 10-2 to design a double-angle connection for a W36×231 beam to an HSS14×14×½ column. Use 3/8-in. diameter ASTM A325-N bolts in standard holes.

\[ P_D = 37.5 \text{ kips} \]
\[ P_L = 113 \text{ kips} \]

Solution:

**Material Properties:**
- Beam: ASTM A992  
  \[ F_y = 50 \text{ ksi} \]
  \[ F_u = 65 \text{ ksi} \]
- Column: ASTM A500 Gr.  
  \[ F_y = 46 \text{ ksi} \]
  \[ F_u = 58 \text{ ksi} \]
- Angles: ASTM A36  
  \[ F_y = 36 \text{ ksi} \]
  \[ F_u = 58 \text{ ksi} \]

**Geometric Properties:**
- Beam: W36×231  
  \[ t_w = 0.760 \text{ in.} \]
- Column: HSS14×14×½  
  \[ t = 0.465 \text{ in.} \]
  \[ B = 14.0 \text{ in.} \]

**Compute the required strength**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R_u = 1.2(37.5 \text{ kips}) + 1.6(113 \text{ kips}) ]</td>
<td>[ R_u = 37.5 \text{ kips} + 113 \text{ kips} ]</td>
<td></td>
</tr>
<tr>
<td>= 225 kips</td>
<td>= 151 kips</td>
<td></td>
</tr>
</tbody>
</table>
Design bolts and welds

Try 8 rows of bolts and 3/16-in. welds B

Obtain the bolt group and angle available strengths from Table 10-1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_d = 254$ kips &gt; 225 kips o.k.</td>
<td>$R_n / \Omega = 170$ kips &gt; 151 kips o.k.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Obtain the available weld strength from Table 10-2

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_d = 279$ kips &gt; 225 kips o.k.</td>
<td>$R_n / \Omega = 186$ kips &gt; 151 kips o.k.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Check the minimum support thickness

Minimum support thickness = 0.238 in. < 0.465 in. o.k.

Calculate the minimum angle thickness

$$t_{\text{min}} = w + \frac{1}{16} = \frac{3}{16} + \frac{1}{16} = \frac{3}{8} \text{ in.}$$

Use 3/8-in. angle thickness to accommodate the welded legs of the double angle connection.

Use 2L4×3 1/2 ×3/8×1'-11 1/2".

L = 23.5 in. > T/2 o.k.

B of the HSS is greater than 12 in., the minimum acceptable width for 4-in. OSL angles.

Calculate the available beam web strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_d = (702 \text{ kips/in.})(0.760 \text{ in.}) = 534$ kips 534 kips &gt; 225 kips o.k.</td>
<td>$R_n / \Omega = (468 \text{ kips/in.})(0.760 \text{ in.}) = 356$ kips 356 kips &gt; 151 kips o.k.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example K.4 Unstiffened Seated Connection to an HSS Column

Given:
Use Table 10-6 to design an unstiffened seated connection for a W21×62 beam to an HSS12×12×½ column.

\[ P_D = 9 \text{ kips} \]
\[ P_L = 27 \text{ kips} \]

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Grade</th>
<th>( F_y ) (ksi)</th>
<th>( F_u ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
</tr>
<tr>
<td>Angles</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Manual Table 2-3

Geometric Properties:

<table>
<thead>
<tr>
<th>Beam</th>
<th>W21×62</th>
<th>( t_w ) (in.)</th>
<th>( d ) (in.)</th>
<th>( k ) (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>HSS12×12×½</td>
<td>0.400</td>
<td>21.0</td>
<td>1.12</td>
</tr>
</tbody>
</table>

Manual Tables 1-1 and 1-12

Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(9.00 \text{ kips}) + 1.6(27.0 \text{ kips}) )</td>
<td>( R_u = 9.00 \text{ kips} + 27.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 54.0 kips</td>
<td>= 36.0 kips</td>
</tr>
</tbody>
</table>
Design the seat angle and weld

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 9-4</th>
<th>Manual</th>
<th>Section 10</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check local web yielding</td>
<td>Check local web yielding</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(N_{\text{min}} = \frac{R_w - \phi R_z}{\phi R_z} \geq k)</td>
<td>(N_{\text{min}} = \frac{R_z - R_s / \Omega}{R_z / \Omega} \geq k)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>[= \frac{54.0 \text{ kips} - 55.8 \text{ kips}}{20.0 \text{ kips/in.}} \geq 1.12 \text{ in.}]</td>
<td>[= \frac{36.0 \text{ kips} - 37.2 \text{ kips}}{13.3 \text{ kips/in.}} \geq 1.12 \text{ in.}]</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Use (N_{\text{min}} = 1.12 \text{ in.})</td>
<td>Use (N_{\text{min}} = 1.12 \text{ in.})</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Check web crippling when \(N / d \leq 0.2\),

\(N_{\text{min}} = \frac{R_w - \phi R_z}{\phi R_z} = 54.0 \text{ kips} - 71.7 \text{ kips} \frac{5.37 \text{ kips/in.}}{\phi R_z} \),
which results in a negative quantity.

Check web crippling when \(N / d > 0.2\),

\(N_{\text{min}} = \frac{R_w - \phi R_z}{\phi R_z} = 54.0 \text{ kips} - 64.2 \text{ kips} \frac{7.16 \text{ kips/in.}}{\phi R_z} \),
which results in a negative quantity.

Check web crippling when \(N / d > 0.2\),

\(N_{\text{min}} = \frac{R_w - \phi R_z}{\phi R_z} = 36.0 \text{ kips} - 47.8 \text{ kips} \frac{3.58 \text{ kips/in.}}{\phi R_z} \),
which results in a negative quantity.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_w = 81.0 \text{ kips})</td>
<td>(\phi R_z = 81.0 \text{ kips})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(81.0 \text{ kips} &gt; 54.0 \text{ kips})</td>
<td>(R_s / \Omega = 53.9 \text{ kips})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o.k.</td>
<td>53.9 kips &gt; 36.0 kips</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Determine outstanding angle leg available strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_w = 66.7 \text{ kips} &gt; 54.0 \text{ kips})</td>
<td>(R_s / \Omega = 44.5 \text{ kips} &gt; 36.0 \text{ kips})</td>
<td></td>
<td></td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Since the \(t\) of the HSS is greater than the \(t_{\text{min}}\) for the \(\frac{1}{16}\)-in. weld, no reduction in the weld strength is required to account for the shear in the HSS.

Connection to beam and top angle

Use a L8×4×\(\frac{3}{16}\) in. top angle. Use \(\frac{3}{16}\)-in. fillet weld across the toe of the angle for attachment to the HSS. Attach both the seat and top angles to the beam flanges with two \(\frac{1}{4}\)-in. diameter ASTM A325-N bolts.
Example K.5  Stiffened Seated Connection to an HSS Column

Given:
Use Manual Tables 10-8 and 10-14 to design a stiffened seated connection for a W21×68 beam to a HSS12×12×½ column.

\[ P_D = 20 \text{ kips} \]
\[ P_L = 60 \text{ kips} \]

Use ¾-in. diameter ASTM A325-N bolts in standard holes to connect the beam to the seat plate.
Use 70 ksi electrode welds to connect the stiffener, seat plate and top angle to the HSS.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>( F_y ) (ksi)</th>
<th>( F_u ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
</tr>
<tr>
<td>Angles and Plates</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Manual Table 2-3
Geometric Properties:

<p>| | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W21×68</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column</td>
<td>HSS12×12×½</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Manual Tables 1-1 and 1-12

Check limits of applicability for Specification Section K1

Strength: \( F_y = 46 \text{ ksi} < 52 \text{ ksi} \)   \( \text{o.k.} \)  Section K1.2

Ductility: \( \frac{F_y}{F_u} = \frac{46 \text{ ksi}}{58 \text{ ksi}} = 0.79 < 0.80 \)   \( \text{o.k.} \)

Calculate the required strength

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>( P_u = 1.2(20.0 \text{ kips}) + 1.6(60.0 \text{ kips}) )</td>
<td>( P_a = 20.0 \text{ kips} + 60.0 \text{ kips} )</td>
</tr>
<tr>
<td>= 120 \text{ kips}</td>
<td>= 80.0 \text{ kips}</td>
</tr>
</tbody>
</table>

Determine stiffener width \( W \) required for web crippling and local web yielding

For web crippling, assume \( N/d > 0.2 \) and use constants \( R_5 \) and \( R_6 \) from Manual Table 9-4. Assume a \( \frac{3}{4} \)-in. setback.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>( W_{\min} = \frac{R_u - \phi R_5}{\phi R_6} + \text{setback} \geq k + \text{setback} )</td>
<td>( W_{\min} = \frac{R_u - R_5 / \Omega}{R_6 / \Omega} + \text{setback} \geq k + \text{setback} )</td>
</tr>
<tr>
<td>= 120 \text{ kips} - 75.9 \text{ kips} \text{ ( / ) setback \text{( \geq ) ( k ) \text{ ( + ) ( \text{setback} )} \text{( \geq ) ( \frac{7.94 \text{ kips/in.}}{\text{in.}})} ))</td>
<td>= 80.0 \text{ kips} - 50.6 \text{ kips} \text{ ( / ) setback \text{( \geq ) ( k ) \text{ ( + ) ( \text{setback} )} \text{( \geq ) ( \frac{5.29 \text{ kips/in.}}{\text{in.}})} ))</td>
</tr>
<tr>
<td>= 6.31 \text{ in.}</td>
<td>= 6.34 \text{ in.}</td>
</tr>
</tbody>
</table>

For local web yielding, use constants \( R_1 \) and \( R_2 \) from Manual Table 9-4. Assume a \( \frac{3}{4} \)-in. setback.

<p>| | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>( W_{\min} = \frac{R_u - \phi R_1}{\phi R_2} + \text{setback} )</td>
<td>( W_{\min} = \frac{R_u - R_1 / \Omega}{R_2 / \Omega} + \text{setback} )</td>
</tr>
<tr>
<td>= 126 \text{ kips} - 63.7 \text{ kips} \text{ ( / ) setback \text{( \geq ) ( k ) \text{ ( + ) ( \text{setback} )} \text{( \geq ) ( \frac{21.5 \text{ kips/in.}}{\text{in.}})} ))</td>
<td>= 84.0 \text{ kips} - 42.5 \text{ kips} \text{ ( / ) setback \text{( \geq ) ( k ) \text{ ( + ) ( \text{setback} )} \text{( \geq ) ( \frac{14.3 \text{ kips/in.}}{\text{in.}})} ))</td>
</tr>
<tr>
<td>= 3.65 \text{ in.}</td>
<td>= 3.65 \text{ in.}</td>
</tr>
</tbody>
</table>

The minimum stiffener width, \( W \), for web crippling controls. Use \( W = 7.00 \text{ in.} \)

Check bearing width assumption

\[ N = 7.00 \text{ in.} - \frac{\frac{3}{4} \text{ in.}}{21.1 \text{ in.}} = 6 \frac{3}{4} \text{ in.} \]

\[ N/d = \frac{6 \frac{3}{4} \text{ in.}}{21.1 \text{ in.}} = 0.30 > 0.2, \text{ as assumed.} \]

Determine the weld strength requirements for the seat plate

Try \( L = 24 \text{ in.} \) with \( \frac{3}{16} \) in. fillet welds. Enter Manual Table 10-8 using \( W = 7 \text{ in.} \) as determined above.
Since $t$ of the HSS is greater than $t_{\text{min}}$ for the $\frac{3}{16}$-in. weld, no reduction in the weld strength to account for shear in the HSS is required.

The minimum length of the seat-plate-to-HSS weld on each side of the stiffener is $0.2L = 4.8$ in. This establishes the minimum weld between the seat plate and stiffener; use 5 in. of $\frac{3}{16}$-in. weld on each side of the stiffener.

**Determine the stiffener plate thickness**

To develop the stiffener-to-seat plate welds, the minimum stiffener thickness is

$$t_{\text{min}} = \dfrac{2w}{2} = \dfrac{2}{\frac{9}{16} \text{ in.}} = \frac{32}{9} \text{ in.}$$

For a stiffener with $F_y = 36$ ksi and a beam with $F_y = 50$ ksi, the minimum stiffener thickness,

$$t_{\text{min}} = \dfrac{F_{y_{\text{beam}}}}{F_{y_{\text{stiffener}}}} \times t_w = \dfrac{50}{36} \dfrac{\text{ksi}}{\text{ksi}} \times 0.430 \text{ in.} = 0.597 \text{ in.}$$

For a stiffener with $F_y = 36$ ksi and a column with $F_u = 58$ ksi, the maximum stiffener thickness is

$$t_{\text{max}} = \dfrac{F_{y}}{F_{u}} \times \dfrac{58}{36} \dfrac{\text{ksi}}{\text{ksi}} \times 0.465 \text{ in.} = 0.749 \text{ in.}$$

Use stiffener thickness of $\frac{3}{8}$ in.

**Determine the stiffener length using Manual Table 10-14**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R/W$</td>
<td>$R/W$</td>
</tr>
<tr>
<td>$t^2$</td>
<td>$t^2$</td>
</tr>
<tr>
<td>req</td>
<td>req</td>
</tr>
<tr>
<td>$120 \text{kips} \left( \frac{7.00 \text{in.}}{0.465 \text{in.}} \right) = 4100 \text{kips/in.}$</td>
<td>$80.0 \text{kips} \left( \frac{7.00 \text{in.}}{0.465 \text{in.}} \right) = 2720 \text{kips/in.}$</td>
</tr>
</tbody>
</table>

To satisfy the minimum above, select a stiffener $L = 24$ in. from Table 10-14

$$R/W = 4310 \text{kips/in.} > 4100 \text{kips/in.} \quad \text{o.k.}$$

Use PL5\%x7x2'-0 for the stiffener.

**Check the HSS width**

The minimum width is $0.4L + t_p + 3t$

$$B = 12.0 \text{ in.} + 0.4(24.0 \text{ in.}) + \frac{3}{8} \text{ in.} + 3(0.465 \text{ in.}) = 11.6 \text{ in.}$$
Determine the seat plate dimensions

To accommodate two $\frac{3}{4}$-in. diameter ASTM A325-N bolts on a $5\frac{1}{2}$ in. gage connecting the beam flange to the seat plate, a width of 8 in. is required. To accommodate the seat-plate-to-HSS weld, the required width is:

$$0.4(24 \text{ in.}) + 0.625 \text{ in.} = 10.2 \text{ in.}$$

Note: To allow room to start and stop welds, an 11.5 in. width is used.

Use PL$\frac{3}{4}$×7×0'-11½ for the seat plate.

Select the top angle, bolts and welds

The minimum weld size for the HSS thickness according to Specification Table J2.4 is $\frac{3}{8}$ in. The angle thickness should be $\frac{3}{16}$ in. larger.

Use L$4\times4\times\frac{1}{4}$ with $\frac{3}{16}$-in. fillet welds along the toes of the angle to the beam flange and HSS. Alternatively two $\frac{3}{4}$-in. diameter ASTM A325-N bolts may be used to connect the beam leg of the angle to the beam flange.
Example K.6 Single-Plate Connection to Rectangular HSS Column

Given:
Use Manual Table 10-9 to design a single-plate connection for between a W18×35 beam and a HSS6×6×⅜ column.

\[ P_D = 6.5 \text{ kips} \]
\[ P_L = 19.5 \text{ kips} \]

Use ¾-in. diameter ASTM A325-N bolts in standard holes and 70 ksi weld electrode.

![Diagram of single-plate connection to rectangular HSS column]

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>( F_y ) (ksi)</th>
<th>( F_u ) (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
</tr>
<tr>
<td>Plate</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Manual Tables 2-3 and 2-4

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
<th>( d ) (in.)</th>
<th>( t_w ) (in.)</th>
<th>( B ) (in.)</th>
<th>( H ) (in.)</th>
<th>( b/t )</th>
<th>Manual Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×35</td>
<td>17.7</td>
<td>0.30</td>
<td>6.00</td>
<td>6.00</td>
<td>14.2</td>
<td>1-1</td>
</tr>
<tr>
<td>Column</td>
<td>HSS6×6×⅜</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>1-12</td>
</tr>
</tbody>
</table>

Determine applicability of Specification Section K1

Strength:
\[ F_y = 46 \text{ ksi} < 52 \text{ ksi} \quad \text{o.k.} \]

Ductility:
\[ \frac{F_u}{F_y} = \frac{46 \text{ ksi}}{58 \text{ ksi}} = 0.79 < 0.80 \quad \text{o.k.} \]

Determine if a single plate connection is suitable (the HSS wall is not slender)

Slenderness:
\[ \lambda = 1.40 \sqrt{\frac{E}{F_y}} \approx 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2 > 14.2 \quad \text{o.k.} \]

Table B4.1 Case 12
Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2 \left( 6.50 \text{ kips} \right) + 1.6 \left( 19.5 \text{ kips} \right) )</td>
<td>( R_u = 6.50 \text{ kips} + 19.5 \text{ kips} )</td>
</tr>
<tr>
<td>= 39.0 kips</td>
<td>= 26.0 kips</td>
</tr>
</tbody>
</table>

Calculate maximum single-plate thickness

\[
t_{p,\text{max}} = \frac{F_t}{F_{y,p}} = \frac{58 \text{ ksi} \left( 0.349 \text{ in.} \right)}{36 \text{ ksi}} = 0.562 \text{ in.}
\]

Note: Limiting the single-plate thickness precludes a shear yielding failure of the HSS wall.

Design the single-plate connection

Try 3 bolts and a 1/8-in. plate thickness with 3/16-in. fillet welds.

\( t_p = \frac{1}{8} \text{ in.} < 0.562 \text{ in.} \quad \text{o.k.} \)

Obtain the available single plate connection strength from Manual Table 10-9

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 47.7 \text{ kips} &gt; 39.0 \text{ kips} )</td>
<td>( R_u / \Omega = 31.8 \text{ kips} &gt; 26.0 \text{ kips} )</td>
</tr>
<tr>
<td>\text{o.k.}</td>
<td>\text{o.k.}</td>
</tr>
</tbody>
</table>

Use a PL\( \frac{1}{16} \times 4\frac{1}{2} \times 0' \)-9"

Check HSS shear rupture strength at welds

\[
t_{\text{min}} = \frac{6.19 D}{F_n} = \frac{6.19 \left( 4 \right)}{58 \text{ ksi}} = 0.427 \text{ in.} > t = 0.349 \text{ in.} \quad \text{n.g.}
\]

Since \( t < t_{\text{min}} \), the shear rupture strength of the HSS is less than the weld strength. Consider reducing the weld size to 1/8 in.

\[
t_{\text{min}} = \frac{6.19 D}{F_n} = \frac{6.19 \left( 3 \right)}{58 \text{ ksi}} = 0.320 \text{ in.} < t = 0.349 \text{ in.} \quad \text{o.k.}
\]
Determine the weld shear strength

This is the special case where the load is not in the plane of the weld group. Use \( k = 0 \).

\[
a = \frac{e}{l} = \frac{3.00 \text{ in.}}{9.00 \text{ in.}} = 0.333 \quad \text{Therefore } C = 2.95 \text{ (interpolated)}
\]

\[
R_n = C \tau Dl = 2.95(1.00)(3 \text{ sixteenths of an in.})(9.00 \text{ in.}) = 79.6 \text{ kips}
\]

**Calculate the available weld shear strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(79.6 \text{ kips}) = 59.7 \text{ kips} )</td>
<td>( R_n / \Omega = \frac{79.6 \text{ kips}}{2.00} = 39.8 \text{ kips} )</td>
</tr>
<tr>
<td>59.7 kips &gt; 39.0 kips <strong>o.k.</strong></td>
<td>39.8 kips &gt; 26.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Calculate the available beam web bearing strength from Manual Table 10-1**

For three \( \frac{3}{8} \)-in. diameter bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = (263 \text{ kips/in.})(0.300 \text{ in.}) )</td>
<td>( R_n / \Omega = (175 \text{ kips/in.})(0.300 \text{ in.}) )</td>
</tr>
<tr>
<td>= 78.9 kips &gt; 39.0 kips <strong>o.k.</strong></td>
<td>52.5 kips &gt; 26.0 kips <strong>o.k.</strong></td>
</tr>
</tbody>
</table>
Example K.7  Through-Plate Connection

Given:
Use Table 10-9 to design a through-plate connection between a W18×35 beam and a HSS6×4×⅛ with the connection to one of the 6 in. faces. A thin-walled column is used to illustrate the design of a through-plate connection.

\[ P_D = 6.5 \text{ kips} \]
\[ P_L = 19.5 \text{ kips} \]

Use ⅛-in. diameter ASTM A325-N bolts in standard holes and 70 ksi weld electrode.

Solution:

Material Properties:
- Beam: ASTM A992, \( F_y = 50 \text{ ksi} \), \( F_u = 65 \text{ ksi} \)
- Column: ASTM A500 Gr. B, \( F_y = 46 \text{ ksi} \), \( F_u = 58 \text{ ksi} \)
- Plate: ASTM A36, \( F_y = 36 \text{ ksi} \), \( F_u = 58 \text{ ksi} \)

Geometric Properties:
- Beam: W18×35, \( d = 17.7 \text{ in.} \), \( t_w = 0.300 \text{ in.} \), \( T = 15\frac{1}{2} \text{ in.} \)
- Column: HSS6×4×⅛, \( B = 4.00 \text{ in.} \), \( H = 6.00 \text{ in.} \), \( t = 0.116 \text{ in.} \), \( h/t = 48.7 \)

Determine applicability of Specification Section K1

Strength: \( F_y = 46 \text{ ksi} < 52 \text{ ksi} \)  o.k.  \( F_u = 58 \text{ ksi} \)

Ductility: \( \frac{F_y}{F_u} = \frac{46 \text{ ksi}}{58 \text{ ksi}} = 0.79 < 0.80 \)  o.k.

Determine if a single-plate connection is suitable (HSS wall is not slender)

Slenderness: \( \lambda = 1.40 \sqrt{\frac{F_y}{F_u}} = 1.40 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}} = 35.2 < 48.7 \)  n.g.

Since the HSS6×4×⅛ is slender, a through-plate connection should be used instead of a single-plate connection. Through plate connections are typically very expensive. When a single-plate connection is not adequate, another type of connection, such as a double-angle

---

Manual
Tables 2-3 and 2-4
Manual
Tables 1-1 and 1-11
Section K1.2
Table B4.1
Case 12
connection may be preferable to a through-plate connection.

**Calculate the required strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(3.30 \text{ kips}) + 1.6(9.90 \text{ kips}) )</td>
<td>( R_u = 3.30 \text{ kips} + 9.90 \text{ kips} )</td>
</tr>
<tr>
<td>= 19.8 \text{ kips}</td>
<td>= 13.2 \text{ kips}</td>
</tr>
</tbody>
</table>

**Design the portion of the through-plate connection that resembles a single-plate**

Try 3 rows of bolts (\( L = 8\frac{1}{2} \)) and a \( \frac{1}{4} \) in. plate thickness with \( \frac{1}{6} \)-in. fillet welds.

**Obtain the available single plate connection strength from Manual Table 10-9**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 38.3 \text{ kips} &gt; 19.8 \text{ kips} )</td>
<td><strong>o.k.</strong></td>
</tr>
<tr>
<td>( R_u / \Omega = 25.6 \text{ kips} &gt; 13.2 \text{ kips} )</td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Check weld size for through-plate connection**

\( e = 3.00 \text{ in.} \)

\( t_{\text{min}} = \frac{6.19D}{F_u} = \frac{6.19(3)}{58 \text{ ksi}} = 0.320 \text{ in.} > 0.116 \text{ in.} \) **(proration required)**

**Calculate the required weld strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{fu} = \frac{R_u (B + e)}{B} )</td>
<td>( V_{fu} = \frac{R_u (B + e)}{B} )</td>
</tr>
<tr>
<td>= \frac{(19.8 \text{ kips})(4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}} )</td>
<td>= \frac{(13.2 \text{ kips})(4.00 \text{ in.} + 3.00 \text{ in.})}{4.00 \text{ in.}} )</td>
</tr>
<tr>
<td>= 34.7 \text{ kips}</td>
<td>= 23.1 \text{ kips}</td>
</tr>
</tbody>
</table>

\( V_{fu} = \frac{V_{fu}}{L} = \frac{34.7 \text{ kips}}{8.50 \text{ in.}} = 4.08 \text{ kips/in.} \)

\( V_{fu} = \frac{V_{fu}}{L} = \frac{23.1 \text{ kips}}{8.50 \text{ in.}} = 2.72 \text{ kips/in.} \)

**Calculate the available weld strength**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_u = 1.392 D n_r \left( \frac{t}{t_{\text{min}}} \right) )</td>
<td>( \phi r_u = 0.928 D n_r \left( \frac{t}{t_{\text{min}}} \right) )</td>
</tr>
<tr>
<td>= 1.392(3)(2 \text{ welds})\left( \frac{0.116 \text{ in.}}{0.320 \text{ in.}} \right) )</td>
<td>= 0.928(3)(2 \text{ welds})\left( \frac{0.116 \text{ in.}}{0.320 \text{ in.}} \right) )</td>
</tr>
<tr>
<td>= 3.03 \text{ kips/in.} &lt; 4.08 \text{ kips/in.}</td>
<td>= 2.02 \text{ kips/in.} &lt; 2.72 \text{ kips/in.} ) <strong>n.g.</strong></td>
</tr>
</tbody>
</table>

A deeper plate is required.

\( L_{req} = \frac{V_{fu}}{\phi r_u} = \frac{34.7 \text{ kips}}{3.03 \text{ kips/in.}} = 11.5 \text{ in.} \)

\( L_{req} = \frac{V_{fu}}{r_u / \Omega} = \frac{23.1 \text{ kips}}{2.02 \text{ kips/in.}} = 11.5 \text{ in.} \)
Use an 11½ in. long plate and increase the vertical edge distance to 2¾ in.

*Recheck the plate length*

\[ L = 11\frac{1}{2} \text{ in.} < T = 15\frac{1}{2} \text{ in.} \quad \text{o.k.} \]

*Calculate the available beam web bearing strength*

For three \( \frac{3}{4} \) in. diameter bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
<th>Table 10-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_w = (263 \text{ kips/in.})(0.300 \text{ in.}) )</td>
<td>( R_w \sqrt{\Omega} = (175 \text{ kips/in.})(0.300 \text{ in.}) )</td>
<td></td>
<td></td>
</tr>
<tr>
<td>( = 78.9 \text{ kips} &gt; 19.8 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 52.5 \text{ kips} &gt; 13.2 \text{ kips} \quad \text{o.k.} )</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Example K.8 Transverse Plate Loaded Perpendicular to the HSS Axis on a Rectangular HSS.

Given:
Verify the local strength of the HSS column subject to the transverse loadings given below, applied through a 5 in. wide plate. The HSS 8×8×½ is in compression with nominal axial loads of $P_D\text{column} = 54\text{ kips}$ and $P_L\text{column} = 162\text{ kips}$. The HSS has negligible flexural required strength and the connecting member is 4 in. wide.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th>ASTM A500 Gr. B</th>
<th>$F_y = 46$ ksi</th>
<th>$F_u = 58$ ksi</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>ASTM A36</td>
<td></td>
<td>$F_{yp} = 36$ ksi</td>
<td>$F_u = 58$ ksi</td>
<td>Table 2-3</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th>HSS8×8×½</th>
<th>$B = 8.00$ in.</th>
<th>$t = 0.465$ in.</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Plate</td>
<td>$B_p = 5.50$ in.</td>
<td>$t_p = 0.500$ in.</td>
<td></td>
<td>Table 1-12</td>
<td></td>
</tr>
</tbody>
</table>

Check the limits of applicability of Specification Section K1

1) Strength: $F_y$ less than or equal to 52 ksi for HSS $F_y = 46$ ksi o.k. Section K1.2

2) Ductility: $F_y/F_u \leq 0.8$ for HSS $F_y/F_u = 0.79$ o.k.

3) $0.25 < B_p/B < 1.0$ $B_p/B = 0.69$ o.k. Section

4) $B/t \leq 35$ $B/t = 17$ o.k. K1.3b

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Transverse force from the plate</strong></td>
<td>$P_u = 1.2(10.0\text{ kips}) + 1.6(30.0\text{ kips}) = 60.0\text{ kips}$</td>
<td>$P_a = 10.0\text{ kips} + 30.0\text{ kips} = 40.0\text{ kips}$</td>
</tr>
<tr>
<td><strong>Column axial force</strong></td>
<td>$P_r = P_u\text{column} = 1.2(54.0\text{ kips}) + 1.6(162\text{ kips}) = 324\text{ kips}$</td>
<td>$P_r = P_a\text{column} = 54.0\text{ kips} + 162\text{ kips} = 216\text{ kips}$</td>
</tr>
</tbody>
</table>
Calculate available local yielding strength for uneven load distribution in the loaded plate

\[
R_u = \frac{10F_t}{B} 
\]

\[
R_u = \left( \frac{8.00 \text{ in.}}{0.465 \text{ in.}} \right) 5.50 \text{ in.} \leq 36 \text{ ksi}(0.500 \text{ in.})(5.50 \text{ in.})
\]

\[
= 68.4 \text{ kips} \leq 99.0 \text{ kips} \quad \text{o.k.}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.95 )</td>
<td>( \Omega = 1.58 )</td>
<td>K1.3b</td>
</tr>
<tr>
<td>( \phi R_u = 0.95(68.4 \text{ kips}) = 65.0 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{68.4 \text{ kips}}{1.58} = 43.3 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>65.0 kips &gt; 60.0 kips \quad \text{o.k.}</td>
<td>43.3 kips &gt; 40.0 kips \quad \text{o.k.}</td>
<td></td>
</tr>
</tbody>
</table>

Check shear yielding (punching)

This limit state does not control when \( B_p > B-2t \), nor when \( B_p < 0.85B \).

\( B-2t = 8.00 \text{ in.} - 2(0.500 \text{ in.}) = 7.00 \text{ in.} \)

\( 0.85B = 0.85(8.00 \text{ in.}) = 6.80 \text{ in.} \)

Therefore, since \( B_p < 6.80 \text{ in.} \) this limit state does not control.

Check sidewall strength

This limit state does not control unless the chord member and branch member (connecting element) have the same width.

Therefore, since \( B_p < 6.80 \text{ in.} \) this limit state does not control.
Example K.9  Longitudinal Plate Loaded Perpendicular to the HSS Axis on a Round HSS.

Given:
Verify the local strength of the HSS tension member subject to the transverse loadings given below, applied through a 4 in. wide plate. The HSS6.000×0.375 is in tension with $P_D = 4$ kips and $P_L = 12$ kips.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th>Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM</td>
<td>ASTM A500 Gr. B</td>
<td>ASTM A36</td>
</tr>
<tr>
<td>$F_y$</td>
<td>42 ksi</td>
<td>36 ksi</td>
</tr>
<tr>
<td>$F_u$</td>
<td>58 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Column</th>
<th>Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>HSS6.000×0.375</td>
<td>D = 6.00 in.</td>
<td>N = 4.00 in.</td>
</tr>
<tr>
<td></td>
<td>$t = 0.349$ in.</td>
<td>Manual</td>
</tr>
</tbody>
</table>

Check the limits of applicability of Specification Section K1

1) Strength: $F_y$ less than or equal to 52 ksi for HSS

2) Ductility: $F_y/F_u \leq 0.8$ for HSS

3) $D/t \leq 50$ for T-connections

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>1.2(4.00 kips) + 1.6(12.0 kips) = 24.0 kips</td>
<td>$P_u = 4.00$ kips + 12.0 kips = 16.0 kips</td>
</tr>
</tbody>
</table>

Check the limit state of chord plastification

$$R_u = 5.5F_y t^2 \left(1 + \frac{0.25N}{D}\right)Q_f$$

Since the column is in tension, $Q_f = 1.0$

$$R_u = 5.5(42$ ksi)(0.349 in.$)^2 \left(1 + \frac{0.25(4.00 \text{ in.})}{6.00 \text{ in.}}\right)1.0 = 32.8$ kips
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Section</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 0.90(32.8 \text{ kips}) = 29.5 \text{ kips} )</td>
<td>( \frac{R_n}{\Omega} = \frac{32.8 \text{ kips}}{1.67} = 19.6 \text{ kips} )</td>
<td>K1.4a</td>
</tr>
<tr>
<td>29.5 \text{ kips} &gt; 24.0 \text{ kips} \textbf{ o.k.}</td>
<td>19.6 \text{ kips} &gt; 16.0 \text{ kips} \textbf{ o.k.}</td>
<td></td>
</tr>
</tbody>
</table>
Example K.10  HSS Brace Connection to a W-shape Column.

**Given:**
Design the connection shown below. The required axial strength in the brace is 80 kips (LRFD) and 52 kips (ASD). The axial force may be either tension or compression. The length of the brace is 6 ft.

**Solution:**

**Material Properties:**
- Brace: ASTM A500 Gr. B  \( F_y = 46 \text{ ksi} \)  \( F_u = 58 \text{ ksi} \)  
- Plate: ASTM A36  \( F_y = 58 \text{ ksi} \)

**Geometric Properties:**
- Brace: HSS3½x3½x¼  \( A_g = 2.91 \text{ in.}^2 \)  \( r = 1.32 \text{ in.} \)  \( t = 0.233 \text{ in.} \)  
- Plate  \( t = 0.375 \text{ in.} \)
Obtain the available axial compression strength of the brace from Manual Table 4-4

\[ K = 1.0 \]
\[ L_b = 6.00 \text{ ft} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 98.4 \text{ kips} )</td>
<td>( P_n/\Omega_c = 65.4 \text{ kips} )</td>
</tr>
<tr>
<td>98.4 kips &gt; 80.0 kips o.k.</td>
<td>65.4 kips &gt; 52.0 kips o.k.</td>
</tr>
</tbody>
</table>

Obtain the available tension yielding strength of the brace from Manual Table 5-5

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi P_n = 120 \text{ kips} )</td>
<td>( P_n/\Omega_c = 80.2 \text{ kips} )</td>
</tr>
<tr>
<td>120 kips &gt; 80.0 kips o.k.</td>
<td>80.2 kips &gt; 52.0 kips o.k.</td>
</tr>
</tbody>
</table>

Calculate the tensile rupture strength of the brace

\[ A_c = A_g - 2(\ell_0)(\text{slot}) = 2.91 - 2(0.233)(\frac{3}{8} + \frac{1}{8}) = 2.71 \text{ in.}^4 \]
\[ \bar{x} = \frac{B^2 + 2BH}{4(B + H)} = \frac{(3\frac{1}{2} \text{ in.})^2 + 2(3\frac{1}{2} \text{ in.})(3\frac{1}{2} \text{ in.})}{4(3\frac{1}{2} \text{ in.} + 3\frac{1}{2} \text{ in.})} = 1.31 \text{ in.} \]

8\( \frac{3}{4} \) in. of overlap occurs. Try four \( \frac{3}{16} \)-in. fillet welds, each 7-in. long

\[ U = 1 - \frac{\bar{x}}{l} = 1 - \frac{1.31 \text{ in.}}{7.00 \text{ in.}} = 0.813 \]
\[ A_c = UA_n = 0.813(2.71 \text{ in.}^2) = 2.20 \text{ in.}^2 \]
\[ P_n = F_uA_c = 58 \text{ ksi}(2.20 \text{ in.}^2) = 128 \text{ kips} \]

Calculate available strength of \( \frac{7}{8} \)-in. weld of HSS to plate

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 4(7 \text{ in.})(3 \text{ sixteenths})(1.392 \text{kips/in.}) )</td>
<td>( \frac{R_n}{\Omega} = 4(7 \text{ in.})(3 \text{ sixteenths})(0.928 \text{kips/in.}) )</td>
</tr>
<tr>
<td>= 117 kips &gt; 80.0 kips o.k.</td>
<td>= 78.0 kips &gt; 52.0 kips o.k.</td>
</tr>
</tbody>
</table>
Calculate the available strength of the weld of gusset plate to column

The slope of the member relative to the longitudinal weld along the column is 50 degrees. Therefore the standard values for longitudinal fillet welds can be increased by:

\[ (1.0 + 0.50 \sin^{1/2} \theta) = 1.34 \]

or 34-percent.

Minimum weld is \( \frac{3}{16} \)-in. fillet each side. Therefore:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 2(16.0 \text{ in.})(3)(1.392)(1.34) )</td>
<td>( R_n/\Omega = 2(16.0 \text{ in.})(3)(0.928)(1.34) )</td>
</tr>
<tr>
<td>( = 179 \text{ kips} &gt; 80.0 \text{ kips} )</td>
<td>( = 119 \text{ kips} &gt; 52.0 \text{ kips} )</td>
</tr>
</tbody>
</table>
Example K.11  Rectangular HSS Column with a Cap Plate, Supporting a Continuous Beam.

Given:
Verify the strength of the HSS column subject to the given gravity beam reactions through the cap plate. Out of plane stability of the column top is provided by the stiffeners shown. The column axial forces are RDL = 24 kips and RLL = 30 kips.

![Diagram of rectangular HSS column with a cap plate supporting a continuous beam.]

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Type</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>ASTM A992</td>
<td>$F_y = 50$ ksi, $F_u = 65$ ksi</td>
</tr>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>$F_y = 46$ ksi, $F_u = 58$ ksi</td>
</tr>
<tr>
<td>Cap Plate</td>
<td>ASTM A36</td>
<td>$F_{yp} = 36$ ksi, $F_u = 58$ ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Type</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18x35</td>
<td>$d = 17.7$ in., $b_y = 6.00$ in., $t_y = 0.300$ in.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$t_f = 0.425$ in., $k_f = 0.750$ in.</td>
</tr>
<tr>
<td>Column</td>
<td>HSS8x8x1/4</td>
<td>$t = 0.233$ in.</td>
</tr>
<tr>
<td>Cap Plate</td>
<td></td>
<td>$t = 0.500$ in.</td>
</tr>
</tbody>
</table>

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(24.0 \text{ kips}) + 1.6(30.0 \text{ kips})$</td>
<td>$R_u = 24.0 \text{ kips} + 30.0 \text{ kips}$</td>
</tr>
<tr>
<td></td>
<td>$= 76.8 \text{ kips}$</td>
<td>$= 54.0 \text{ kips}$</td>
</tr>
</tbody>
</table>

Assume the vertical beam reaction is transmitted to the HSS through bearing of the cap plate at the two column faces perpendicular to the beam.

Calculate bearing length, $N$, at bottom of W18x35

$N = 2k_f + 5t_f = 2(0.750 \text{ in.}) + 5(0.425 \text{ in.}) = 3.63 \text{ in.}$

Check limit for number of HSS faces contributing

$5t_p + N = 5(0.500 \text{ in.}) + 3.63 \text{ in.} = 6.13 \text{ in.} < 8.00 \text{ in.}$ therefore, only 2 walls contribute.
Calculate the nominal local wall yielding strength of the HSS

For each of the two walls:

\[ R_u = F_y t \left( 5 t_p + N \right) \leq BF_y t \]

\[ = 46 \text{ ksi} \left( 0.233 \text{ in.} \right) \left( 5(0.500 \text{ in.}) + 3.63 \text{ in.} \right) \leq 8.00 \text{ in.} \left( 46 \text{ ksi} \right) \left( 0.233 \text{ in.} \right) \]

65.7 kips \leq 85.7 kips

Use \( R_u = 65.7 \) kips per wall

Calculate the available local wall yielding strength of the HSS

\[ \phi R_u = 65.7 \text{ kips per wall} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_u = 1.00(65.7 \text{ kips})(2 \text{ walls}) )</td>
<td>( R_u/\Omega = (65.7 \text{ kips}/1.50)(2 \text{ walls}) )</td>
</tr>
<tr>
<td>= 131 kips &gt; 76.8 kips o.k.</td>
<td>= 87.6 kips &gt; 54.0 kips o.k.</td>
</tr>
</tbody>
</table>

Calculate the nominal wall local crippling strength of the HSS

Section K1.6

For each of the two walls:

\[ R_u = 0.8 t^2 \left[ 1 + \frac{6 \left( t/t_p \right)}{0.8(0.233 \text{ in.})} \right] \left[ \frac{29,000 \text{ ksi}(46 \text{ ksi})(0.500 \text{ in.})}{0.233 \text{ in.}} \right]^{0.5} \]

\[ = 0.8 \left( 0.233 \text{ in.} \right)^2 \left[ 1 + \frac{6(3.63 \text{ in.})}{8.00 \text{ in.}} \right] \left[ \frac{29,000 \text{ ksi}(46 \text{ ksi})(0.500 \text{ in.})}{0.233 \text{ in.}} \right]^{0.5} \]

\[ = 137 \text{ kips per wall} \]

Calculate the available local wall local crippling strength of the HSS

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_u = 0.75(137 \text{ kips})(2 \text{ walls}) )</td>
<td>( R_u/\Omega = (137 \text{ kips}/2.00)(2 \text{ walls}) )</td>
</tr>
<tr>
<td>= 207 kips &gt; 76.8 kips o.k.</td>
<td>= 137 kips &gt; 54.0 kips o.k.</td>
</tr>
</tbody>
</table>

The presence of the stiffeners eliminates the need to check the beam web for local web yielding and local web crippling.
Example K.12 Rectangular HSS Column Base Plate.

Given:
A HSS 6×6×½ column is supporting nominal loads of 40 kips from dead load and 120 kips from live load. The column is supported by a 7’-6” × 7’-6” concrete spread footing with a $f'_{c} = 3000$ psi. Size a base plate for this column.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Column</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
</tr>
<tr>
<td>Base Plate</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Geometric Properties:
HSS 6×6×½
B = H = 6.00 in.

Calculate the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$ = 1.2(40.0 kips) + 1.6(120 kips)</td>
<td>$P_u = 40.0$ kips + $120$ kips</td>
<td></td>
</tr>
<tr>
<td>= 240 kips</td>
<td>= 160 kips</td>
<td></td>
</tr>
</tbody>
</table>

Note: The procedure illustrated here is similar to that presented in AISC Design Guide 1 - Column Base Plates.

Try a base plate which extends 3½ in. from each face of the HSS column, or 13 in. by 13 in.

Calculate the available strength for the limit state of concrete crushing

$$P_p = 0.85 f'_{c} A_i \sqrt{A_z / A_i} \text{ where } A_z / A_i \leq 4$$

$$A_i = (13.0 \text{ in.})(13.0 \text{ in.}) = 169 \text{ in.}^2$$

$$A_z = (90.0 \text{ in.})(90.0 \text{ in.}) = 8100 \text{ in.}^2$$
\[ P_p = 0.85(3 \text{ ksi})(169 \text{ in.}^3) \sqrt{\frac{8100 \text{ in.}^2}{169 \text{ in.}^2}} \leq 1.7(3 \text{ ksi})(169 \text{ in.}^3) \]

20,700 kips \leq 862 kips

Use \( P_p = 862 \text{ kips} \)

Note: The limit on the right side of Equation J8-2 will control when \( A_2/A_1 \) exceeds 4.0

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( \phi_c P_p = 0.60(862 \text{ kips}) = 517 \text{ kips} &gt; 240 \text{ kips} )</td>
<td>( P_p/\Omega_c = 862 \text{ kips}/2.50 = 345 \text{ kips} &gt; 160 \text{ kips} )</td>
</tr>
</tbody>
</table>

Calculate the pressure under the bearing plate and determine the required thickness

For a rectangular HSS, the distance \( m \) or \( n \) is determined using 0.95 times the depth and width of the HSS.

Note: When the HSS is large compared to the edge extension, a quick and conservative approach is to include in the bearing area only the surface within the distance \( m \) or \( n \) of the HSS wall within the shape.

\[ m = n = \frac{N - 0.95(\text{outside dimension})}{2} \]

\[ = \frac{13.0 \text{ in.} - 0.95(6.00 \text{ in.})}{2} = 3.65 \text{ in.} > \left[ \frac{6 - 2(\frac{1}{2})}{2} \right] = 2.50 \text{ in.} \], therefore, the full area of the baseplate inside the HSS is effective in bearing.

The critical bending moment is the cantilever moment outside the HSS perimeter.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{pu} = \frac{P_p}{A_{off}} \frac{240 \text{ kips}}{169 \text{ in.}^2} = 1.42 \text{ ksi} )</td>
<td>( f_{pu} = \frac{P_p}{A_{off}} \frac{160 \text{ kips}}{169 \text{ in.}^2} = 0.947 \text{ ksi} )</td>
</tr>
<tr>
<td>( M_n = \frac{f_{pu} l^2}{2} )</td>
<td>( M_n = \frac{f_{pu} l^2}{2} )</td>
</tr>
<tr>
<td>( Z = \frac{t_p^2}{4} )</td>
<td>( Z = \frac{t_p^2}{4} )</td>
</tr>
<tr>
<td>( \phi_b = 0.90 )</td>
<td>( \Omega_b = 1.67 )</td>
</tr>
<tr>
<td>( M_n = M_p = F_y Z )</td>
<td>( M_n = M_p = F_y Z )</td>
</tr>
</tbody>
</table>

Equating:

\[ M_n = \phi_b M_n \text{ and solving for } t_p \text{ gives:} \]

\[ t_{p(adj)} = \frac{2 f_{pu} l^2}{\phi_b F_y} = \sqrt{\frac{2(1.42 \text{ ksi})(3.65 \text{ in.}^2)}{0.90(36 \text{ ksi})}} = 1.08 \text{ in.} \]

Therefore, use a 1\( \frac{1}{4} \) in. thick plate.
Example K.13  Rectangular HSS Strut End Plate.

Given:
Determine weld leg size, end plate thickness, and the size of ASTM A325 bolts required to resist nominal forces of 16 kips from dead load and 50 kips from live load on an ASTM A500 Gr. B HSS 4×4×¼" section. The end plate is ASTM A36.

Solution:

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strut</td>
<td>ASTM A500 Gr. B</td>
<td>46</td>
<td>58</td>
<td>Manual Table 2-3</td>
</tr>
<tr>
<td>End Plate</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
<td>Manual Table 2-3</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Geometric Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Strut</td>
<td>HSS 4×4×¼&quot;</td>
<td></td>
</tr>
<tr>
<td>End Plate</td>
<td></td>
<td>$t = 0.233$ in.</td>
</tr>
</tbody>
</table>

Calculate the required tensile strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_u$</td>
<td>1.2(16.0 kips) + 1.6(50.0 kips) = 99.2 kips</td>
<td>$P_u = 16.0$ kips + 50.0 kips = 66.0 kips</td>
</tr>
</tbody>
</table>

Preliminary size of the (4) ASTM A325 bolts

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$r_u$</td>
<td>$\frac{P_u}{n} = \frac{99.2}{4} = 24.8$ kips</td>
<td>$r_u = \frac{P_u}{n} = \frac{66.0}{4} = 16.5$ kips</td>
</tr>
</tbody>
</table>

Try 5/8-in. diameter ASTM A325 bolts

$\phi r_u = 29.8$ kips

Try 3/4-in. diameter ASTM A325 bolts

$\frac{r_u}{\Omega} = 19.9$ kips
Calculate required end-plate thickness with consideration of prying action

\[ a' = a + \frac{d^2}{2} \leq 1.25b + \frac{d^2}{2} \]
\[ = 1.50 \text{ in.} + \frac{b'}{2} \leq 1.25(1.50 \text{ in.}) + \frac{b'}{2} \]
\[ = 1.88 \text{ in.} \leq 2.25 \text{ in.} \quad \text{o.k.} \]

\[ b' = b - \frac{d^2}{2} = 1.5 \text{ in.} - \frac{b'}{2} = 1.12 \text{ in.} \]
\[ \rho = \frac{b'}{a'} = \frac{1.12}{1.88} = 0.596 \]
\[ \delta = 1 - \frac{d'}{p} = 1 - \frac{\left(\frac{b}{a} + \frac{b'}{a'}\right)}{4.0} = 0.781 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \beta = \frac{1}{\rho} \left( \frac{r_v}{r_u} - 1 \right) = \frac{1}{0.596} \left( \frac{29.8}{24.8} - 1 \right) = 0.338 ]</td>
<td>[ \beta = \frac{1}{\rho} \left( \frac{r_v}{r_u} - 1 \right) = \frac{1}{0.596} \left( \frac{19.9}{16.5} - 1 \right) = 0.346 ]</td>
</tr>
<tr>
<td>[ \alpha' = \frac{1}{\delta (1 - \beta)} = \frac{1}{0.781 (1 - 0.338)} = 0.654 \leq 1.0 ]</td>
<td>[ \alpha' = \frac{1}{\delta (1 - \beta)} = \frac{1}{0.781 (1 - 0.346)} = 0.677 \leq 1.0 ]</td>
</tr>
</tbody>
</table>

\[ t_{req} = \sqrt{\frac{4.44 r_v b'}{p F_w (1 + 0.5 \sin 1.5(\theta))}} \]
\[ = \frac{4.44(24.8 \text{ kips})(1.12 \text{ in.})}{4.00 \text{ in.}(58 \text{ ksi})(1 + 0.781(0.654))} \]
\[ = 0.593 \text{ in.} \]

Use \( \frac{b}{a} \) in. end plate, \( t_i > 0.593 \), further bolt check for prying not required.

Use (4) \( \frac{b}{a} \) in. diameter A325 bolts

Calculate the weld size required

\[ F_w = 0.60 F_{ew}(1.0 + 0.5 \sin 1.5(\theta)) = 0.60(70 \text{ ksi})(1.0 + 0.5\sin 1.5(90)) = 63.0 \text{ ksi} \]

\[ I = 4(4.00 \text{ in.}) = 16.0 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ w \geq \frac{P_u}{\phi F_w (0.707)l} \]
\[ \geq \frac{99.2 \text{ kips}}{0.75(63.0 \text{ ksi})(0.707)(16.0 \text{ in.})} \]
\[ = 0.186 \text{ in.} \] | \[ w \geq \frac{\Omega P_u}{F_w (0.707)l} \]
\[ \geq \frac{2.00(66.0 \text{ kips})}{(63.0 \text{ ksi})(0.707)(16.0 \text{ in.})} \]
\[ = 0.185 \text{ in.} \] |
Check minimum weld size requirements
For \( t = \frac{3}{8} \text{ in.} \) minimum weld = \( \frac{1}{4} \text{ in.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.250 in. &gt; 0.186 in.</td>
<td>0.250 in. &gt; 0.185 in.</td>
</tr>
<tr>
<td>Use ( \frac{3}{8} \text{ in.} ) weld leg size</td>
<td>Use ( \frac{1}{4} \text{ in.} ) weld leg size</td>
</tr>
</tbody>
</table>

Results
Use \( \frac{1}{4} \text{ in.} \) weld with \( \frac{3}{8} \text{ in.} \) end plate and (4) \( \frac{3}{4} \)-in. diameter ASTM A325 bolts.
Chapter IIA
Simple Shear Connections

The design of simple shear connections is covered in Part 10 of the AISC Steel Construction Manual.
Example II.A-1  All-Bolted Double-Angle Connection

Given:

Select an all-bolted double-angle connection between a W36×231 beam and a W14×90 column flange to support the following beam end reactions:

\[ R_D = 37.5 \text{ kips} \]
\[ R_L = 112.5 \text{ kips} \]

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W36×231</td>
<td>ASTM</td>
<td>A992</td>
<td>( F_y = 50 \text{ ksi} )</td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>ASTM</td>
<td>A992</td>
<td>( F_y = 50 \text{ ksi} )</td>
</tr>
<tr>
<td>Angles</td>
<td>2L5×3½</td>
<td>ASTM</td>
<td>A36</td>
<td>( F_y = 36 \text{ ksi} )</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W36×231</td>
<td>( t_w = 0.760 \text{ in.} )</td>
<td>Table 1-1</td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>( t_f = 0.710 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td>2L5×3½ SLBB</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(37.5 \text{ kips}) + 1.6(112.5 \text{ kips})$</td>
<td>$R_u = 37.5 \text{ kips} + 112.5 \text{ kips}$</td>
</tr>
<tr>
<td>= 225 kips</td>
<td>= 150 kips</td>
</tr>
</tbody>
</table>

Manual Table 10-1 includes checks for the limit states of bearing, shear yielding, shear rupture, and block shear rupture on the angles, and shear on the bolts.

Try 8 rows of bolts and $\frac{3}{16}$-in. angle thickness.

$\phi R_u = 247 \text{ kips} > 225 \text{ kips}$ o.k.

Check the beam web for bolt bearing.

Block shear rupture, shear yielding and shear rupture will not control, since the beam is uncoped.

Uncoped, $L_{ch} = 1\frac{3}{4}$ in.

$\phi R_u = (702 \text{ kips/in.})(0.760 \text{ in.})$

= 534 kips $> 225 \text{ kips}$ o.k.

$R_u / \Omega = (468 \text{ kips/in.})(0.760 \text{ in.})$

= 356 kips $> 150 \text{ kips}$ o.k.

Check supporting member flange for bolt bearing

$\phi R_u = (1400 \text{ kips/in.})(0.710 \text{ in.})$

= 994 kips $> 225 \text{ kips}$ o.k.

$R_u / \Omega = (936 \text{ kips/in.})(0.710 \text{ in.})$

= 665 kips $> 150 \text{ kips}$ o.k.

See Example II.A-2 for a bolted/welded double angle connection.
Example II.A-2  Bolted/Welded Double-Angle Connection

Given:
Use Manual Table 10-2 to substitute welds for bolts in the support legs of the double-angle connection (welds B). Use 70 ksi electrodes.

Material Properties:
- Beam  W36×231  ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi
- Column W14×90  ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi
- Angles  2L4×3½  ASTM A36  $F_y = 36$ ksi  $F_u = 58$ ksi

Geometric Properties:
- Beam  W36×231  $t_w = 0.760$ in.
- Column W14×90  $t_f = 0.710$ in.
- Angles  2L4×3½  SLBB
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(37.5 \text{ kips}) + 1.6(112.5 \text{ kips})$</td>
<td>$R_u = 37.5 \text{ kips} + 112.5 \text{ kips}$</td>
</tr>
<tr>
<td>= 225 kips</td>
<td>= 150 kips</td>
</tr>
</tbody>
</table>

*Design welds (welds B)*

Try $\frac{3}{16}$-in. weld size, $L = 23\frac{1}{2}$ in.

$t_{f_{\text{min}}} = 0.238 \text{ in.} < 0.710 \text{ in.}$

φ

$R_a = 279 \text{ kips} > 225 \text{ kips}$

φ

$R_a / \Omega = 186 \text{ kips} > 150 \text{ kips}$

Check minimum angle thickness

$t_{\text{min}} = w + \frac{1}{16}-\text{in.} \geq \frac{3}{8} \text{ in.}$

= $\frac{3}{8} \text{ in.} + \frac{1}{16} \text{ in.}$

= $\frac{3}{8} \text{ in.} > \frac{3}{8} \text{ in.}$

Use 2L4×3\(\frac{1}{2}\)×\(\frac{3}{4}\)

Check bolt shear. Check angles for bolt bearing, shear yielding, shear rupture, and block shear rupture

Check 8 rows of bolts and $\frac{3}{8}$ in. angle thickness.

$R_a / \Omega = 170 \text{ kips} > 150 \text{ kips}$

Check beam web for bolt bearing.

Uncoped, $L_{ch} = 1\frac{3}{4}$ in.

$R_a = (702 \text{ kips/in.})(0.760 \text{ in.})$

= 534 kips > 225 kips

Use 2L4×3\(\frac{1}{2}\)×\(\frac{3}{4}\)

Check supporting member flange for bolt bearing

$R_a = (1400 \text{ kips/in.})(0.710 \text{ in.})$

= 994 kips > 225 kips

Note: In this example, because of the relative size of the cope to the overall beam size, the coped section does not control. When this can not be determined by inspection, see Manual Part 9 for the design of the coped section.

See Example II.A-1 for an all-bolted double-angle connection (beam-to-column flange).
Example II.A-3  All-Welded Double-Angle Connection

Given:

Use Manual Table 10-3 to design an all-welded double-angle connection between a W36×231 beam and a W14×90 column flange.
Use 70 ksi electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>Material</th>
<th>Fy (ksi)</th>
<th>Fu (ksi)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W36×231</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
<td>Manual Table 2-3</td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
<td>Manual Table 2-3</td>
</tr>
<tr>
<td>Angles</td>
<td>2L4×3⅜</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
<td>Manual Table 2-3</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>Twist (in)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W36×231</td>
<td>0.760</td>
<td>Manual Tables 1-1 and 1-15</td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>0.710</td>
<td>Manual Tables 1-1 and 1-15</td>
</tr>
<tr>
<td>Angles</td>
<td>2L4×3⅜</td>
<td></td>
<td>Manual Tables 1-1 and 1-15</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(37.5 \text{ kips}) + 1.6(112.5 \text{ kips}) )</td>
<td>( R_u = 37.5 \text{ kips} + 112.5 \text{ kips} )</td>
</tr>
<tr>
<td>( = 225 \text{ kips} )</td>
<td>( = 150 \text{ kips} )</td>
</tr>
<tr>
<td>Design the weld between the beam-web and the angle leg (welds A)</td>
<td>Design the weld between the beam-web and the angle leg (welds A)</td>
</tr>
<tr>
<td>Try ( \frac{3}{4} ) in. weld size, ( L = 24 ) in.</td>
<td>Try ( \frac{3}{4} ) in. weld size, ( L = 24 ) in.</td>
</tr>
<tr>
<td>( t_{w_{\min}} = 0.381 \text{ in.} &lt; 0.760 \text{ in.} ) o.k.</td>
<td>( t_{w_{\min}} = 0.381 \text{ in.} &lt; 0.760 \text{ in.} ) o.k.</td>
</tr>
<tr>
<td>( \phi R_u = 343 \text{ kips} &gt; 225 \text{ kips} ) o.k.</td>
<td>( R_u / \Omega = 229 \text{ kips} &gt; 150 \text{ kips} ) o.k.</td>
</tr>
<tr>
<td>Design the welds between support and the angle leg (welds B)</td>
<td>Design the welds between support and the angle leg (welds B)</td>
</tr>
<tr>
<td>Try ( \frac{3}{4} ) in. weld size</td>
<td>Try ( \frac{3}{4} ) in. weld size</td>
</tr>
<tr>
<td>( t_{f_{\min}} = 0.190 \text{ in.} &lt; 0.710 \text{ in.} ) o.k.</td>
<td>( t_{f_{\min}} = 0.190 \text{ in.} &lt; 0.710 \text{ in.} ) o.k.</td>
</tr>
<tr>
<td>( \phi R_u = 229 \text{ kips} &gt; 225 \text{ kips} ) o.k.</td>
<td>( R_u / \Omega = 153 \text{ kips} &gt; 150 \text{ kips} ) o.k.</td>
</tr>
<tr>
<td>Check the minimum angle thickness</td>
<td>Check the minimum angle thickness</td>
</tr>
<tr>
<td>( t_{\min} = w + \frac{1}{16} \text{ in.} \geq \frac{1}{8} \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{3}{4} \text{ in.} + \frac{1}{16} \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{5}{16} \text{ in.} \geq \frac{1}{8} \text{ in.} ) o.k.</td>
<td></td>
</tr>
<tr>
<td>Use 2L4×3×( \frac{5}{16} )</td>
<td>Use 2L4×3×( \frac{5}{16} )</td>
</tr>
</tbody>
</table>

Comment:

See Example II.A-1 for an all-bolted double-angle connection and Example II.A-2 for a bolted/welded double-angle connection.
Example II.A-4  All-Bolted Double-Angle Connection

Given:

Use Manual Table 10-1 to select an all-bolted double-angle connection between a W18×50 beam and a W21×62 girder web to support the following beam end reactions:

\[ R_D = 10 \text{ kips} \]
\[ R_L = 30 \text{ kips} \]

The beam top flange is coped 2-in. deep by 4-in. long, \( L_{ev} = 1\frac{1}{2} \text{ in.}, \) \( L_{eh} = 1\frac{1}{2} \text{ in.} \) (assumed to be 1½ in. for calculation purposes to account for possible underrun in beam length)

Use 3⁄4-in. diameter ASTM A325-N bolts in standard holes.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>( F_y ) 50 ksi</th>
<th>( F_u ) 65 ksi</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×50</td>
<td>ASTM A992</td>
<td>( F_y ) 50 ksi</td>
<td>( F_u ) 65 ksi</td>
</tr>
<tr>
<td>Girder W21×62</td>
<td>ASTM A992</td>
<td>( F_y ) 50 ksi</td>
<td>( F_u ) 65 ksi</td>
</tr>
<tr>
<td>Angles 2L4×3½</td>
<td>ASTM A36</td>
<td>( F_y ) 36 ksi</td>
<td>( F_u ) 58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>( d ) 18.0 in.</th>
<th>( t_w ) 0.355 in.</th>
<th>( S_{net} ) 23.4 in.</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×50</td>
<td>( c ) 4 in.</td>
<td>( d_c ) 2 in.</td>
<td>( e ) 4 in. + ½ in. = 4½-in.</td>
<td>Table 2-3</td>
</tr>
<tr>
<td>Cope</td>
<td>( h_0 ) 16.0 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girder W21×62</td>
<td>( t_w ) 0.400 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles 2L4×3½ SLBB</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual Table 10-1</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(10\text{ kips}) + 1.6(30\text{ kips}) = 60\text{ kips}$</td>
<td>$R_u = 10\text{ kips} + 30\text{ kips} = 40\text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>$\phi R_u = 76.4\text{ kips} &gt; 60\text{ kips}$</td>
<td>$\phi R_u = 50.9\text{ kips} &gt; 40\text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\phi R_u = (200\text{ kips/in.})(0.355\text{ in.}) = 71.0\text{ kips} &gt; 60\text{ kips}$</td>
<td>$\phi R_u = (133\text{ kips/in.})(0.355\text{ in.}) = 47.2\text{ kips} &gt; 40\text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$\phi R_u = (526\text{ kips/in.})(0.400) = 210\text{ kips} &gt; 60\text{ kips}$</td>
<td>$\phi R_u = (351\text{ kips/in.})(0.400\text{ in.}) = 140\text{ kips} &gt; 40\text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td>$U_{bs} = 1.0$</td>
<td>$U_{bs} = 1.0$</td>
<td></td>
</tr>
<tr>
<td>$\phi R_u = F_u A_m U_{bs} + \min\left(\phi 0.6 F_y A_{gy}, \phi F_u A_{nv}\right)$</td>
<td>$\phi R_u = F_u A_m U_{bs} + \min\left(0.6 F_y A_{gy}, \frac{F_u A_{nv}}{\Omega}\right)$</td>
<td></td>
</tr>
<tr>
<td>Tension Component</td>
<td>Tension Component</td>
<td></td>
</tr>
<tr>
<td>$\phi F_u A_{ut} = 51.8\text{ kips/in.}(0.355\text{ in.})$</td>
<td>$F_u A_{ut} / \Omega = 34.5\text{ kips/in.}(0.355\text{ in.})$</td>
<td></td>
</tr>
<tr>
<td>Shear Yielding Component</td>
<td>Shear Yielding Component</td>
<td></td>
</tr>
<tr>
<td>$\phi 0.6 F_y A_{gy} = 163\text{ kips/in.}(0.355\text{ in.})$</td>
<td>$0.6 F_y A_{gy} / \Omega = 109\text{ kips/in.}(0.355\text{ in.})$</td>
<td></td>
</tr>
<tr>
<td>Shear Rupture Component</td>
<td>Shear Rupture Component</td>
<td></td>
</tr>
<tr>
<td>$\phi 0.6 F_u A_{nv} = 148\text{ kips/in.}(0.355\text{ in.})$</td>
<td>$0.6 F_u A_{nv} / \Omega = 98.7\text{ kips/in.}(0.355\text{ in.})$</td>
<td></td>
</tr>
</tbody>
</table>

Check bolt shear. Check angles for bolt bearing, shear yielding, shear rupture and block shear rupture

Try 3 rows of bolts and ¼ in. angle thickness

Top flange coped, $L_{ev} = 1\frac{1}{4}\text{ in.}$, $L_{eh} = 1\frac{1}{4}\text{ in}$

Check beam web for bolt bearing, block shear rupture, shear yielding and shear rupture

Check beam web for bolt bearing

Check supporting member web for bolt bearing

Check block shear rupture

Since one vertical row of bolts is used, the tension stress can be taken as uniform, therefore

$U_{bs} = 1.0$

$\phi R_u = F_u A_m U_{bs} + \min\left(\phi 0.6 F_y A_{gy}, \phi F_u A_{nv}\right)$

Shear Yielding Component

Shear Rupture Component

Commentary

Section J4.3

Eqn. J4-5

Manual Tables 9-3a

Manual Table 9-3b

Manual Table 9-3c
\[
\phi R_n = (148 \text{ kips/in.} + 51.8 \text{ kips/in.})(0.355 \text{ in.}) \\
= 70.9 \text{ kips} < 76.3 \text{ kips} \\
\text{70.9 kips > 60 kips } \text{ o.k.}
\]

\[
\frac{R_n}{\Omega} = (98.7 \text{ kips/in.} + 34.5 \text{ kips/in.})(0.355 \text{ in.}) \\
= 47.3 \text{ kips} < 50.9 \text{ kips} \\
\text{47.3 kips > 40 kips } \text{ o.k.}
\]

Note: The middle portion of Manual Table 10-1 includes checks of the limit-state of bolt bearing on the beam web and the limit-state of block shear rupture on coped beams. Manual Tables 9-3a, 9-3b and 9-3c are required for values of \( L_{ev} \) and \( L_{eb} \) beyond the limits of Table 10-1. For coped members, the limit states of flexural rupture and local buckling must be checked independently, per Part 9.

**Check flexural rupture on the coped section**

\[
S_{net} = 23.4 \text{ in}^3
\]

\[
R_n = \frac{F_s S_{net}}{e} = \frac{(65 \text{ ksi})(23.4 \text{ in.}^3)}{4\frac{1}{2} \text{ in.}} = 338 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(338 \text{ kips}) )</td>
<td>( \frac{R_n}{\Omega} = \frac{338 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>= 254 kips &gt; 60 kips \text{ o.k.}</td>
<td>= 169 kips &gt; 40 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>

**Check local web buckling at the coped section**

Verify \( c \leq 2d \) and \( d_c \leq \frac{d}{2} \)

\[
c = 4.0 \text{ in.} < 2(18.0 \text{ in.}) = 36.0 \text{ in. } \text{o.k.}
\]

\[
d_c = 2.0 \text{ in.} < \frac{18.0 \text{ in.}}{2} = 9.0 \text{ in. } \text{o.k.}
\]

\[
\frac{c}{d} = \frac{4 \text{ in.}}{18.0 \text{ in.}} = 0.222, \quad \frac{c}{h_0} = \frac{4 \text{ in.}}{16.0 \text{ in.}} = 0.250
\]

Since \( \frac{c}{d} \leq 1.0 \),

\[
f = 2\left(\frac{c}{d}\right) = 2(0.222) = 0.444
\]
Since \( \frac{c}{h_0} \leq 1.0, \)

\[
k = 2.2 \left( \frac{h_0}{c} \right)^{1.65} = 2.2 \left( \frac{16.0}{4.00} \right)^{1.65} = 21.7
\]

\[
F_{y} = 26.210 \left( \frac{t_w}{h_0} \right)^2 f'k = 26.210 \left( \frac{0.355 \text{ in.}}{16.0 \text{ in.}} \right)^2 (0.444)(21.7) = 124 \text{ ksi} \leq F_y
\]

\[
R_n = \frac{F_{y} S_{net}}{e} = \frac{(50 \text{ ksi})(23.4 \text{ in.}^3)}{4 \frac{1}{2} \text{ in.}} = 260 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(260 \text{ kips}) )</td>
<td>( R_n / \Omega = \frac{260 \text{ kips}}{1.67} )</td>
</tr>
<tr>
<td>= 234 kips &gt; 60 kips</td>
<td>o.k. = 156 kips &gt; 40 kips</td>
</tr>
</tbody>
</table>

**Check shear yielding on beam web**

\[
R_n = 0.6 F_y A_g = 0.6(50 \text{ ksi})(0.355 \text{ in})(16.0 \text{ in}) = 170 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(170 \text{ kips}) )</td>
<td>( R_n / \Omega = \frac{170 \text{ kips}}{1.50} )</td>
</tr>
<tr>
<td>= 170 kips &gt; 60 kips</td>
<td>o.k. = 113 kips &gt; 40 kips</td>
</tr>
</tbody>
</table>

**Check shear rupture on beam web**

\[
A_{sv} = t_w [h_o - 3(0.875 \text{ in.})] = (0.355 \text{ in.})(16.0 \text{ in.} - 2.63 \text{ in.}) = 4.75 \text{ in}^2
\]

\[
R_n = 0.6 F_u A_{sv} = 0.6(65 \text{ ksi})(4.75 \text{ in}^2) = 185 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.75(185 \text{ kips}) )</td>
<td>( R_n / \Omega = \frac{185 \text{ kips}}{2.00} )</td>
</tr>
<tr>
<td>= 139 kips &gt; 60 kips</td>
<td>o.k. = 92.6 kips &gt; 40 kips</td>
</tr>
</tbody>
</table>

Note: see **Example II.A-5** for a bolted/welded double-angle connection.
Example II.A-5  Bolted/Welded Double-Angle Connection (beam-to-girder web).

Given:

Use Manual Table 10-2 to substitute welds for bolts in the supported-beam-web legs of the double-angle connection (welds A).

Use 70 ksi electrodes.
Use ¼-in. diameter ASTM A325-N bolts in standard holes.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Type</th>
<th>Property</th>
<th>Value</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×50</td>
<td>ASTM A992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Girder</td>
<td>W21×62</td>
<td>ASTM A992</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Angles</td>
<td>2L4×3½</td>
<td>ASTM A36</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Property</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>d</td>
<td>18.0 in.</td>
</tr>
<tr>
<td></td>
<td>tc</td>
<td>0.355 in.</td>
</tr>
<tr>
<td></td>
<td>Snet</td>
<td>23.4 in.³</td>
</tr>
<tr>
<td>Girder</td>
<td>h0</td>
<td>16.0 in.</td>
</tr>
<tr>
<td>Angles</td>
<td>tw</td>
<td>0.400 in.</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips}) = 60.0 \text{ kips}$</td>
<td>$R_u = 10 \text{ kips} + 30 \text{ kips} = 40.0 \text{ kips}$</td>
<td>Table 10-2</td>
</tr>
<tr>
<td><strong>Design welds (welds A)</strong></td>
<td><strong>Design welds (welds A)</strong></td>
<td></td>
</tr>
<tr>
<td>Try $\frac{3}{16}$-in. weld size, $L = 8\frac{1}{2}$ in.</td>
<td>Try $\frac{3}{16}$-in. weld size, $L = 8\frac{1}{2}$ in.</td>
<td></td>
</tr>
<tr>
<td>$t_{w_{\text{min}}} = 0.286 \text{ in.} &lt; 0.355 \text{ in.}$</td>
<td>$t_{w_{\text{min}}} = 0.286 \text{ in.} &lt; 0.355 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
<td></td>
</tr>
<tr>
<td>$\phi R_u = 110 \text{ kips} &gt; 60 \text{ kips}$</td>
<td>$R_u / \Omega = 73.4 \text{ kips} &gt; 40 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Check minimum angle thickness**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w = \text{ weld size}$</td>
<td>$w = \text{ weld size}$</td>
<td></td>
</tr>
<tr>
<td>$t_{\text{min}} = w + \frac{1}{16}$-in. $\geq \frac{1}{4}$ in.</td>
<td>$t_{\text{min}} = w + \frac{1}{16}$-in. $\frac{1}{4}$ in.</td>
<td></td>
</tr>
<tr>
<td>$= \frac{3}{16}$ in. + $\frac{1}{16}$ in.</td>
<td>$= \frac{3}{16}$ in. + $\frac{1}{16}$ in.</td>
<td></td>
</tr>
<tr>
<td>$= \frac{1}{4}$ in. $\geq \frac{1}{4}$ in.</td>
<td>$= \frac{1}{4}$ in. $\geq \frac{1}{4}$ in.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: see **Example II.A-4** for an all-bolted double-angle connection (beam-to-girder web).
Example II.A-6  Beam End Coped at the Top Flange Only

Given:

For a W21×62 coped 8-in. deep by 9-in. long at the top flange only:

A. calculate the available strength of the beam end, considering the limit states of flexural rupture and local buckling. Assume \( e = 9\frac{1}{2} \) in.

B. choose an alternate W21 shape to eliminate the need for stiffening for an end reaction of \( R_D = 16.5 \) kips and \( R_L = 47 \) kips.

C. determine the size of doubler plate needed to stiffen the W21×62 for the given end reaction.

D. determine the size of longitudinal stiffeners needed to stiffen the W21×62 for the given end reaction.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>ASTM</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W21×62</td>
<td>A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Plate</td>
<td>A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>( d )</th>
<th>( t_w )</th>
<th>( S_{net} )</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W21×62</td>
<td>21.0 in.</td>
<td>0.400 in.</td>
<td>17.8 in.³</td>
<td>1-1 and 9-2</td>
</tr>
<tr>
<td>Cope</td>
<td>9 in.</td>
<td>8 in.</td>
<td>9 in. + ½ in. = 9½ in.</td>
<td></td>
</tr>
</tbody>
</table>
Solution A:

Check flexural rupture strength of the coped section

\[ S_{net} = 17.8 \text{ in}^3 \]

\[ R_u = \frac{F_c S_{net}}{e} = \frac{(65 \text{ ksi})(17.8 \text{ in}^3)}{9\frac{3}{4} \text{ in.}} = 122 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_u = 0.75(122 \text{kips}) = 91.3 \text{kips} )</td>
<td>( R_u / \Omega = \frac{122 \text{kips}}{2.00} = 61 \text{kips} )</td>
</tr>
</tbody>
</table>

Check local buckling

Verify parameters

\( c \leq 2d \)

\( c = 9.0 \text{ in.} < 2(21.0 \text{ in.}) = 42.0 \text{ in.} \) o.k.

\( d_c < d / 2 \)

\( d_c = 8.0 \text{ in.} < \frac{21.0 \text{ in.}}{2} = 10.5 \text{ in.} \) o.k.

\[ \frac{c}{d} = \frac{9 \text{ in.}}{21.0 \text{ in.}} = 0.429; \frac{c}{h_0} = \frac{9 \text{ in.}}{13.0 \text{ in.}} = 0.693 \]

Since \( \frac{c}{d} \leq 1.0, \)

\[ f = 2\left(\frac{c}{d}\right) = 2(0.429) = 0.858 \]

Since \( \frac{c}{h_0} \leq 1.0, \)

\[ k = 2.2\left(\frac{h_0}{c}\right)^{1.65} = 2.2 \left(\frac{13.0 \text{ in.}}{9.0 \text{ in.}}\right)^{1.65} = 4.03 \]

For a top cope only, the critical buckling stress is

\[ F_{cr} = 26,210 \left(\frac{t}{h_0}\right)^2 f k \leq F_y \]

\[ = 26,210 \left(\frac{0.400 \text{ in.}}{13.0 \text{ in.}}\right)^2 (0.858)(4.03) \leq 50 \text{ ksi} \]

\[ = 85.8 \text{ ksi} \leq 50 \text{ ksi} \]
Use $F_{cr} = 50$ ksi

$$R_n = \frac{F_{cr} S_{net}}{e} = \frac{(50 \text{ ksi})(17.8 \text{ in}^2)}{9\frac{1}{2} \text{ in.}} = 93.7 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_n = 0.90(93.7 \text{ kips}) = 84.3 \text{ kips}$</td>
<td>$R_n / \Omega = \frac{93.7 \text{ kips}}{1.67} = 56.1 \text{ kips}$</td>
</tr>
</tbody>
</table>

Section F1

*Check shear yielding on beam web*

$$R_n = 0.6 F_y A_g = 0.6(50 \text{ ksi})(0.400 \text{ in})(13.0 \text{ in}) = 156 \text{ kips}$$

Eqn J4-3

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_n = 1.00(156 \text{ kips}) = 156 \text{ kips}$</td>
<td>$R_n / \Omega = \frac{156 \text{ kips}}{1.50} = 104 \text{ kips}$</td>
</tr>
</tbody>
</table>

Sect J4.2

*Check shear rupture on beam web*

$$R_n = 0.6 F_u A_{nv} = 0.6(65 \text{ ksi})(0.400 \text{ in})(13.0 \text{ in}) = 203 \text{ kips}$$

Eqn J4-4

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(203 \text{ kips}) = 203 \text{ kips}$</td>
<td>$R_n / \Omega = \frac{203 \text{ kips}}{2.00} = 101 \text{ kips}$</td>
</tr>
</tbody>
</table>

Section J4.2

Thus, the available strength is controlled by local bucking, with

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 84.3 \text{ kips.}$</td>
<td>$R_n / \Omega = 56.1 \text{ kips.}$</td>
</tr>
</tbody>
</table>
Solution B:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(16.5 \text{ kips}) + 1.6(47 \text{ kips}) = 95 \text{ kips}$</td>
<td>$R_u = 16.5 \text{ kips} + 47 \text{ kips} = 63.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Calculate required section modulus based on flexural rupture</td>
<td>Calculate required section modulus based on flexural rupture</td>
</tr>
<tr>
<td>$S_{req} = \frac{R_u e}{F_y} = \frac{(95 \text{ kips})(9\frac{1}{2} \text{ in.})}{0.75(65 \text{ ksi})}$</td>
<td>$S_{req} = \frac{R_u e \Omega}{F_u} = \frac{63.5 \text{ kips}(9\frac{1}{2} \text{ in.})(2.00)}{65 \text{ ksi}}$</td>
</tr>
<tr>
<td>$= 18.6 \text{ in.}^3$</td>
<td>$= 18.6 \text{ in.}^3$</td>
</tr>
</tbody>
</table>

Try W21×73 with an 8-in. deep cope

$S_{net} = 21.0 \text{ in.}^3 > 18.6 \text{ in.}^3 \quad \text{o.k.}$

Check local buckling

Similarly as determined in Solution A for the W21×62, the available critical stress due to local buckling for a W21×73 with an 8-in. deep cope is limited to the yield stress.

Therefore,

$$R_u = \frac{F_u S_{net}}{e} = \frac{(50 \text{ ksi})(21.0 \text{ in.}^3)}{9\frac{1}{2} \text{ in.}} = 111 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_u = 0.90(111 \text{ kips}) = 100 \text{ kips}$</td>
<td>$R_u / \Omega = \frac{111 \text{ kips}}{1.67} = 66.5 \text{ kips}$</td>
</tr>
<tr>
<td>$&gt; 95.0 \text{ kips} \quad \text{o.k.}$</td>
<td>$&gt; 63.5 \text{ kips} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>


Solution C:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual Part 9</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design doubler plate</strong></td>
<td><strong>Design doubler plate</strong></td>
<td></td>
</tr>
<tr>
<td>Doubler plate must provide a required strength of 95 kips − 84.3 kips = 10.7 kips.</td>
<td>Doubler plate must provide a required strength of 63.5 kips − 56.1 kips = 7.40 kips.</td>
<td></td>
</tr>
<tr>
<td>$S_{req} = \frac{(R_y - \phi R_{y,beam})e}{\phi F_y}$</td>
<td>$S_{req} = \frac{(R_y - R_{y,beam} / \Omega)e \Omega}{F_y}$</td>
<td></td>
</tr>
<tr>
<td>$= \frac{(95 \text{ kips} - 84.3 \text{ kips})(9 \frac{1}{2} \text{ in.})}{0.90(50 \text{ ksi})}$</td>
<td>$= \frac{(63.5 \text{ kips} - 56.1 \text{ kips})(9 \frac{1}{2} \text{ in.})(1.67)}{50 \text{ ksi}}$</td>
<td></td>
</tr>
<tr>
<td>$= 2.26 \text{ in.}^3$</td>
<td>$= 2.35 \text{ in.}^3$</td>
<td></td>
</tr>
<tr>
<td>For an 8-in. deep plate,</td>
<td>For an 8-in. deep plate,</td>
<td></td>
</tr>
<tr>
<td>$t_{req} = \frac{6S_{req}}{d^2} = \frac{6(2.26 \text{ in.}^3)}{(8 \text{ in.})^2} = 0.212 \text{ in.}$</td>
<td>$t_{req} = \frac{6S_{req}}{d^2} = \frac{6(2.35 \text{ in.}^3)}{(8 \text{ in.})^2} = 0.220 \text{ in.}$</td>
<td></td>
</tr>
</tbody>
</table>

Note: ASTM A572 grade 50 plate is recommended in order to match the beam yield strength.

Thus, since the doubler plate must extend at least $d_c$ beyond the cope, use a PL $\frac{3}{16}$ in. * 8 in. * 1’-5” with $\frac{1}{4}$ in. welds.
Solution D:

*Design longitudinal stiffeners*

Try PL ¼ in.×4 in. slotted to fit over the beam web with $F_y = 50$ ksi. From section property calculations for the neutral axis and moment of inertia, the neutral axis is located 4.40 in. from the bottom flange (8.84 in. from the top of the stiffener) and the elastic section modulus of the reinforced section is as follows:

<table>
<thead>
<tr>
<th></th>
<th>$I_0$ (in.⁴)</th>
<th>$Ad^2$ (in.⁴)</th>
<th>$I_0 + Ad^2$ (in.⁴)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Stiffener</td>
<td>0.00521</td>
<td>76.0</td>
<td>76.0</td>
</tr>
<tr>
<td>W21×62 web</td>
<td>63.2</td>
<td>28.6</td>
<td>91.8</td>
</tr>
<tr>
<td>W21×62 bottom flange</td>
<td>0.160</td>
<td>84.9</td>
<td>85.1</td>
</tr>
</tbody>
</table>

\[ \Sigma I_x = 253 \text{ in.}^4 \]

\[ S_{net} = \frac{I_c}{e} = \frac{253 \text{ in.}^4}{8.84 \text{ in.}} = 28.6 \text{ in.}^3 \]

and the nominal strength of the reinforced section is

\[ R_n = \frac{F_y S_{net}}{e} = \frac{(50 \text{ ksi})(28.6 \text{ in.}^3)}{9.5 \text{ in.}} = 151 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculate design strength</strong></td>
<td><strong>Calculate allowable strength</strong></td>
</tr>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi R_n = 0.90(151 \text{ kips}) = 136 \text{ kips} &gt; 95 \text{ kips}$</td>
<td>$R_n / \Omega = \frac{151 \text{ kips}}{1.67} = 90.4 \text{ kips} &gt; 63.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

*Plate dimensions*

Since the longitudinal stiffening must extend at least $d_c$ beyond cope,

Use PL ¼ in.×4 in.×1'-5" with ¼ in. welds
Example II.A-7  Beam End Coped at the Top and Bottom Flanges.

Given:

A W21×62 is coped 3-in. deep by 7-in. long at the top flange and 4-in. deep by 7-in. long at the bottom flange. Calculate the available strength of the beam end considering the limit states of flexural rupture, local buckling, shear yielding, and shear rupture. Assume $e = 7\frac{1}{2}$ in.

Material Properties:

Beam W21×62  ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi  Manual Table 2-3

Geometric Properties:

Beam W21×62  $d = 21.0$ in.  $t_w = 0.400$ in.  Manual Tables 1-1 and 9-2a
Cope  $c = 7$ in.  $d_c = 4$ in.  $e = 7\frac{1}{2}$ in.  $h_0 = 14.0$ in.

Solution:

Check flexural rupture on the coped section

$$S_{net} = \frac{t_w h_0^2}{6} = \frac{(0.400 \text{ in.})(14.0 \text{ in.})^2}{6} = 13.1 \text{ in.}^3$$

$$R_n = \frac{F_u S_{net}}{e} = \frac{(65 \text{ ksi})(13.1 \text{ in.}^3)}{7\frac{1}{2} \text{ in.}} = 114 \text{ kips}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(114 \text{ kips}) = 85.1 \text{ kips}$</td>
<td>$R_n / \Omega = \frac{114 \text{ kips}}{2.00} = 57.0 \text{ kips}$</td>
</tr>
</tbody>
</table>
Check local buckling

Verify parameters

\( c \leq 2d \)

\[ c = 7.0 \text{ in.} < 2(21.0 \text{ in.}) = 42.0 \text{ in.} \quad \text{o.k.} \]

\( d_c \leq 0.2d \)

\[ d_c = 3.0 \text{ in.} < 0.2(21.0) = 4.20 \text{ in.} \quad \text{o.k.} \]

\[ f_p = 3.5 - 7.5 \left( \frac{d}{d} \right) = 3.5 - 7.5 \left( \frac{3.0 \text{ in.}}{21.0 \text{ in.}} \right) = 2.43 \]

For the doubly-coped beam when \( c \leq 2d \) and \( d_c \leq 0.2d \), the critical buckling stress is

\[ F_{cr} = 0.62\pi E \left( \frac{t}{h_0} \right) f_p \leq F_y \]

\[ F_{cr} = 0.62\pi(29,000) \left( \frac{0.400 \text{ in.}^2}{(7 \text{ in.})(14.0 \text{ in.})} \right)(2.43) = 224 \text{ ksi} \leq 50 \text{ ksi} \]

Use \( F_{cr} = 50 \text{ ksi} \)

\[ R_n = \frac{F_{cr} \cdot S_{net}}{e} = \frac{(50 \text{ ksi})(13.1 \text{ in.}^3)}{7\frac{1}{2} \text{ in.}} = 87.3 \text{ kips} \quad \text{(same as yielding)} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(87.3 \text{ kips}) = 78.6 \text{ kips} )</td>
<td>( R_n / \Omega = \frac{(87.3 \text{ kips})}{1.67} = 52.3 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check shear yielding on beam web

\[ R_n = 0.6 F_y A_g = 0.6(50 \text{ ksi})(0.400 \text{ in.})(14.0 \text{ in.}) = 168 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 1.00(168 \text{ kips}) = 168 \text{ kips} )</td>
<td>( R_n / \Omega = \frac{168 \text{ kips}}{1.50} = 112 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check shear rupture on beam web

\[ R_n = 0.6 F_y A_{nv} = 0.6(65 \text{ ksi})(0.400 \text{ in.})(14.0 \text{ in.}) = 218 \text{ kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = 0.90(218 \text{ kips}) = 196 \text{ kips} )</td>
<td>( R_n / \Omega = \frac{196 \text{ kips}}{1.50} = 131 \text{ kips} )</td>
</tr>
<tr>
<td>LRFD</td>
<td>ASD</td>
</tr>
<tr>
<td>------</td>
<td>-----</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_n = 0.75(218 \text{ kips}) = 164 \text{ kips}$</td>
<td>$R_n/\Omega = \frac{218 \text{ kips}}{2.00} = 109 \text{ kips}$</td>
</tr>
</tbody>
</table>

Thus, the available strength is controlled by local buckling, with:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 78.6 \text{ kips}$</td>
<td>$R_n/\Omega = 52.3 \text{ kips}$</td>
</tr>
</tbody>
</table>
Example II.A-8  All-Bolted Double-Angle Connections (beams-to-girder web).

Given:

Design the all-bolted double-angle connections between the $W_{12}\times40$ beam (A) and $W_{21}\times50$ beam (B) and the $W_{30}\times99$ girder-web to support the following beam end reactions:

<table>
<thead>
<tr>
<th>Beam A</th>
<th>Beam B</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{DA} = 4.17$ kips</td>
<td>$R_{DB} = 18.3$ kips</td>
</tr>
<tr>
<td>$R_{LA} = 12.5$ kips</td>
<td>$R_{LB} = 55.0$ kips</td>
</tr>
</tbody>
</table>

Use $\frac{3}{8}$-in. diameter ASTM A325-N bolts in standard holes.

Material Properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>Grade</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_{12}\times40$</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>$W_{21}\times50$</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>$W_{30}\times99$</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Angle</td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>$t_w$ (in.)</th>
<th>$d_c$ (in.)</th>
<th>$c$ (in.)</th>
<th>$h_o$ (in.)</th>
<th>$S_{net}$ (in.$^3$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam $W_{12}\times40$ top flange cope</td>
<td>0.295</td>
<td>2</td>
<td>5</td>
<td>9.9</td>
<td>8.03</td>
</tr>
<tr>
<td>Beam $W_{21}\times50$ top flange cope</td>
<td>0.380</td>
<td>2</td>
<td>5</td>
<td>18.8</td>
<td>32.5</td>
</tr>
<tr>
<td>Girder $W_{30}\times99$</td>
<td>0.520</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Manual Table 2-3 and Manual Table 9-2
**Solution:**

**Beam A:**

<table>
<thead>
<tr>
<th>IIA-24</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$R_{fu} = (1.2)(4.17 \text{kips}) + (1.6)(12.5 \text{kips})$</td>
<td>$R_{fu} = 4.17 \text{kips} + 12.5 \text{kips}$</td>
</tr>
<tr>
<td></td>
<td>= 25 kips</td>
<td>= 16.7 kips</td>
</tr>
</tbody>
</table>

Check bolt shear, check angles for bolt bearing, shear yielding, shear rupture, and block shear rupture for the W12×40 (beam A)

Try two rows of bolts and ¼-in. angle thickness

$\phi R_{n} = 48.9 \text{kips} > 25 \text{kips}$ \text{o.k.}

Check beam web for bolt bearing, block shear rupture, shear yielding, and shear rupture (beam A)

From Table 10-1, for two rows of bolts and $L_{cv} = 1\frac{1}{4} \text{ in.}$ and $L_{ch} = 1\frac{1}{2} \text{ in.}$

$\phi R_{n} = (126 \text{kips/in.})(0.295 \text{ in.})$

$= 37.2 \text{kips} > 25 \text{kips}$ \text{o.k.}

Check flexural rupture of the coped section (beam A)

$S_{net} = 8.03 \text{ in}^3$

$\phi = 0.75$

$\phi R_{n} = \frac{\phi F_{u} S_{net}}{e} = \frac{0.75(65 \text{ ksi})(8.03 \text{ in}^3)}{(5 \text{ in.} + \frac{1}{2} \text{ in.})}$

$= 71.2 \text{kips} > 25 \text{kips}$ \text{o.k.}

Check coped section

$S_{net} = 8.03 \text{ in}^3$

Verify parameters

$c \leq 2d$

5 in. $\leq 2(11.9 \text{ in.}) = 23.8 \text{ in.}$ \text{o.k.}

$d_{c} \leq d/2$

2 in. $\leq 11.9 \text{ in.} / 2 = 5.95 \text{ in.}$ \text{o.k.}
\[
\frac{c}{d} = \frac{5 \text{ in.}}{11.9 \text{ in.}} = 0.420 \leq 1.0; \quad \frac{c}{h_o} = \frac{5 \text{ in.}}{9.9 \text{ in.}} = 0.505 \leq 1.0
\]

Calculate plate buckling model adjustment factor

\[
f = 2\left(\frac{c}{d}\right) = 2(0.420) = 0.840
\]

Calculate plate buckling coefficient

\[
k = 2.2\left(\frac{h_o}{c}\right)^{1.65} = 2.2\left(\frac{9.90}{5.00}\right)^{1.65} = 6.79
\]

\[
F_{cr} = 26,210\left(\frac{h_o}{c}\right)^2 f_k \leq F_v
\]

\[
= 26,210\left(\frac{0.250 \text{ in.}^2}{9.9 \text{ in.}}\right)^2 (0.840)(6.79) = 95.3 \text{ ksi} \leq 50 \text{ ksi}
\]

Use \(F_{cr} = 50 \text{ ksi}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi R = \frac{\phi F_{cr} S_{net}}{e} = \frac{0.90 (50 \text{ ksi})(8.03 \text{ in.}^3)}{(5 \tfrac{\frac{1}{2}}{2} \text{ in.})} = 65.7 \text{ kips} &gt; 25 \text{ kips} \quad \text{o.k.} )</td>
<td>(R_n / \Omega = \frac{(F_{cr} / \Omega) S_{net}}{e} = \frac{(50 \text{ ksi})(8.03 \text{ in.}^3)}{1.67(5 \tfrac{1}{2} \text{ in.})} = 43.7 \text{ kips} &gt; 16.7 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Check shear yielding on beam web

\(R_n = 0.6 F_y A_g = 0.6(50 \text{ ksi})(0.295 \text{ in})(9.90 \text{ in.}) = 87.6 \text{ kips} \quad \text{Eqn J4-3}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 1.00)</td>
<td>(\Omega = 1.50)</td>
</tr>
<tr>
<td>(\phi R_n = 1.00(87.6 \text{ kips}) )</td>
<td>(R_n / \Omega = \frac{87.6 \text{ kips}}{1.50} = 58.4 \text{kips} &gt; 16.7 \text{kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Check shear rupture on beam web

\(A_{nv} = t_w [(h_o - 2(0.875 \text{ in.})] = 0.295 \text{ in.} (9.9 \text{ in.} - 1.75 \text{ in.}) = 2.40 \text{ in}^2 \)

\(R_n = 0.6 F_y A_{nv} = 0.6(65 \text{ ksi})(2.40 \text{ in}^2) = 93.8 \text{ kips} \quad \text{Eqn J4-4}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.75)</td>
<td>(\Omega = 2.00)</td>
</tr>
<tr>
<td>(\phi R_n = 0.75(93.8 \text{ kips}) = 70.3 \text{kips} )</td>
<td>(R_n / \Omega = \frac{93.8 \text{ kips}}{2.00} = 46.9 \text{kips} &gt; 16.7 \text{kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>
Beam B:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{Bu} = (1.2)(18.3\text{kips}) + (1.6)(55.0 \text{kips})$</td>
<td>$R_{Bu} = 18.3 \text{kips} + 55.0 \text{kips}$</td>
</tr>
<tr>
<td>= 110 kips</td>
<td>= 73.3 kips</td>
</tr>
</tbody>
</table>

*Check bolt shear, check angles for bolt bearing, shear yielding, shear rupture, and block shear rupture for beam B (W21×50)*

Try five rows of bolts and $\frac{3}{8}$ in. angle thickness.

$\phi R_n = 125 \text{kips} > 110 \text{kips}$ \ o.k.

*Check beam web for bolt bearing, block shear rupture, shear yielding, and shear rupture (beam B)*

From tables, for five rows of bolts and $L_{ev} = 1\frac{1}{4} \text{ in.}$ and $L_{eh} = 1\frac{1}{2} \text{ in.}$

$\phi R_n = (312 \text{kips/in.})(0.380 \text{ in.})$

= 119 kips > 110 kips \ o.k.

*Check flexural rupture of the coped section (beam B)*

$S_{net} = 32.5 \text{ in}^3$

$\phi = 0.75$

$\phi R_n = \frac{\phi F_a S_{net}}{e} = \frac{0.75(65 \text{ ksi})(32.5 \text{ in}^3)}{(5\frac{1}{2} \text{ in.})}$

= 288 kips > 110 kips \ o.k.

*Check flexural rupture of the coped section (beam B)*

$S_{net} = 32.5 \text{ in}^3$

$\Omega = 2.00$

$R_n/\Omega = (208 \text{kips/in.})(0.380 \text{ in.})$

= 79.0 kips > 73.3 kips \ o.k.

*Check coped section*

$S_{net} = 32.5 \text{ in}^3$

*Verify parameters*

$c \leq 2d$

5 in. $\leq 2(20.8 \text{ in.}) = 41.6 \text{ in.}$ \ o.k.

$d_c \leq d/2$

2 in. $\leq 20.8 \text{ in.} / 2 = 10.4 \text{ in.}$ \ o.k.

$$\frac{c}{d} = \frac{5 \text{ in.}}{20.8 \text{ in.}} = 0.240 \leq 1.0; \quad \frac{c}{h_0} = \frac{5 \text{ in.}}{18.8 \text{ in.}} = 0.266 \leq 1.0$$
Calculate plate buckling model adjustment factor

\[ f = 2 \left( \frac{c}{a} \right) = 2(0.240) = 0.480 \]

Calculate plate buckling coefficient

\[ k = 2.2 \left( \frac{h_o}{c} \right)^{1.65} = 2.2 \left( \frac{18.8}{5.00} \right)^{1.65} = 19.6 \]

\[ F_{cr} = 26,210 \left( \frac{t_w}{h_o} \right)^2 f_k \leq F_y \]
\[ = 26,210 \left( \frac{0.380 \text{ in.}}{18.8 \text{ in.}} \right)^2 (0.480)(19.6) = 101 \text{ ksi} \leq 50 \text{ ksi} \]

Use \( F_{cr} = 50 \text{ ksi} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 ) ( \Omega = 1.67 )</td>
<td>( \phi_R = \frac{\phi F_{cr} S_{net}}{e} = \frac{0.90(50 \text{ ksi}) (32.5 \text{ in.}^3)}{(5 \frac{1}{2} \text{ in.})} )</td>
</tr>
<tr>
<td>266 kips &gt; 110 kips o.k.</td>
<td>117 kips &gt; 73.3 kips o.k.</td>
</tr>
</tbody>
</table>

Check shear yielding on beam web

\( R_n = 0.6 F_y A_g = 0.6(50 \text{ ksi})(0.380 \text{ in.})(18.8 \text{ in.}) = 214 \text{ kips} \) \[ \text{Eqn J4-3} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 ) ( \Omega = 1.50 )</td>
<td>( \phi_R = 1.00(214 \text{ kips}) ) ( R_n / \Omega = \frac{214 \text{ kips}}{1.50} )</td>
</tr>
<tr>
<td>214 kips &gt; 110 kips o.k.</td>
<td>143 kips &gt; 73.3 kips o.k.</td>
</tr>
</tbody>
</table>

Check shear rupture on beam web

\( A_{sv} = t_w [h_o - (5)(0.875 \text{ in.})] = 0.380 \text{ in.} (18.8 \text{ in.} - 4.38 \text{ in.}) = 5.48 \text{ in.}^2 \)

\( R_n = 0.6 F_u A_{sv} = 0.6(65 \text{ ksi})(5.48 \text{ in.}^2) = 213 \text{ kips} \) \[ \text{Eqn J4-4} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 ) ( \Omega = 2.00 )</td>
<td>( \phi_R = 0.75(213 \text{ kips}) = 160 \text{ kips} ) ( R_n / \Omega = \frac{213 \text{ kips}}{2.00} = 107 \text{ kips} )</td>
</tr>
<tr>
<td>160 kips &gt; 110 kips o.k.</td>
<td>107 kips &gt; 73.3 kips o.k.</td>
</tr>
</tbody>
</table>
Supporting Girder

Check the supporting girder web

The required bearing strength per bolt is greatest for the bolts that are loaded by both connections. Thus, for the design of these 4 critical bolts, the required strength is determined as

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From Beam A, each bolt must support one-fourth of 25 kips or 6.25 kips/bolt.</td>
<td>From Beam A, each bolt must support one-fourth of 16.7 kips or 4.18 kips/bolt.</td>
</tr>
<tr>
<td>From Beam B, each bolt must support one-tenth of 110 kips or 11.0 kips/bolt.</td>
<td>From Beam B, each bolt must support one-tenth of 73.3 kips or 7.33 kips/bolt.</td>
</tr>
<tr>
<td>Thus,</td>
<td>Thus,</td>
</tr>
<tr>
<td>$R_u = 6.25 \text{kips/bolt} + 11.0 \text{kips/bolt}$</td>
<td>$R_u = 4.18 \text{kips/bolt} + 7.33 \text{kips/bolt}$</td>
</tr>
<tr>
<td>$= 17.3 \text{kips/bolt}$</td>
<td>$= 11.5 \text{kips/bolt}$</td>
</tr>
<tr>
<td>The design bearing strength per bolt is</td>
<td>The allowable bearing strength per bolt is</td>
</tr>
<tr>
<td>$\phi_r = (87.8 \text{kips/in})(0.520 \text{in.})$</td>
<td>$r_u / \Omega = (58.5 \text{kips/in.})(0.520 \text{in.})$</td>
</tr>
<tr>
<td>$= 45.7 \text{kips/bolt} \gt 17.3 \text{kips/bolt} \text{o.k.}$</td>
<td>$= 30.4 \text{kips/bolt} \gt 11.5 \text{kips/bolt} \text{o.k.}$</td>
</tr>
</tbody>
</table>

Although not required for design, the tabulated values may be verified by hand calculations, as follows

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_r = \phi(1.2L_c t F_u) \leq \phi(2.4d t F_u)$</td>
<td>$r_u / \Omega = (1.2L_c t F_u) / \Omega \leq (2.4d t F_u) / \Omega$</td>
</tr>
<tr>
<td>$\phi 1.2L_c t F_u =$</td>
<td>$1.2L_c t F_u / \Omega =$</td>
</tr>
<tr>
<td>$(0.75)(1.2)(3 \text{ in.} - \frac{7}{8} \text{ in.})(0.520 \text{ in.})(65 \text{ ksi})$</td>
<td>$(1.2)(3 \text{ in.} - \frac{7}{8} \text{ in.})(0.520 \text{ in.})(65 \text{ ksi})/(2.00)$</td>
</tr>
<tr>
<td>$= 64.7 \text{kips}$</td>
<td>$= 43.1 \text{kips}$</td>
</tr>
<tr>
<td>$\phi(2.4d t F_u) =$</td>
<td>$(2.4d t F_u) / \Omega =$</td>
</tr>
<tr>
<td>$(0.75)(2.4)(\frac{3}{4} \text{ in.})(0.520 \text{ in.})(65 \text{ ksi})$</td>
<td>$(2.4)(\frac{3}{4} \text{ in.})(0.520 \text{ in.})(65 \text{ ksi})/(2.00)$</td>
</tr>
<tr>
<td>$= 45.6 \text{kips} &lt; 64.6 \text{kips}$</td>
<td>$= 30.4 \text{kips} &lt; 43.1 \text{kips}$</td>
</tr>
<tr>
<td>$\phi_r = 45.6 \text{kips/bolt} \gt 17.3 \text{kips/bolt} \text{o.k.}$</td>
<td>$r_u / \Omega = 30.4 \text{kips/bolt} \gt 11.5 \text{kips/bolt} \text{o.k.}$</td>
</tr>
</tbody>
</table>

Manual Table 7-5

Eqn. J3-6a
Example II.A-9  Offset All-Bolted Double-Angle Connections  
(beams-to-girder web)

Given:

Two all-bolted double-angle connections are made back-to-back with offset beams. Design the connections to accommodate an offset of 6 in.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>ASTM</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>W18×50</td>
<td>A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Beam</td>
<td>W16×45</td>
<td>A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Angle</td>
<td></td>
<td>A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>$t_w$ (in)</th>
<th>$d$ (in)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Girder</td>
<td>W18×50</td>
<td>0.355</td>
<td>18.0</td>
</tr>
<tr>
<td>Beam</td>
<td>W16×45</td>
<td>0.345</td>
<td>16.1</td>
</tr>
</tbody>
</table>

Solution:

Modify the 2L4×3½×¼ connection designed in Example II.A-4 to work in the configuration shown above. The offset dimension (6 in.) is approximately equal to the gage on the support from the previous example (6 ¼ in.) and, therefore, is not recalculated below.

Thus, the bearing strength of the middle vertical row of bolts (through both connections), which now carry a portion of the reaction for both connections, must be verified for this new configuration.

For each beam,

$R_D = 10$ kips
$R_L = 30$ kips
The required bearing strength per bolt is

\[ r_u = \frac{(2 \text{ connections})(60 \text{ kips/connection})}{6 \text{ bolts}} = 20.0 \text{ kips/bolt} \]

Check supporting girder web

The design strength per bolt is

\[ \phi r_u = (87.8 \text{ kips/in})(0.355 \text{ in.}) \]

\[ = 31.2 \text{ kips/bolt} > 20.0 \text{ kips/bolt} \text{ o.k.} \]

The required bearing strength per bolt is

\[ r_u = \frac{(2 \text{ connections})(40 \text{ kips/connection})}{6 \text{ bolts}} = 13.3 \text{ kips/bolt} \]

Check supporting girder web

The allowable strength per bolt is

\[ r_u / \Omega = (58.5 \text{ kips/in.})(0.355 \text{ in.}) \]

\[ = 20.8 \text{ kips/bolt} > 13.3 \text{ kips/bolt} \text{ o.k.} \]

Note: If the bolts are not spaced equally from the supported beam web, the force in each column of bolts should be determined by using a simple beam analogy between the bolts, and applying the laws of statics.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = (1.2)(10 \text{ kips}) + (1.6)(30 \text{ kips}) = 60 \text{ kips} )</td>
<td>( R_u = 10 \text{ kips} + 30 \text{ kips} = 40 \text{ kips} )</td>
</tr>
<tr>
<td>The required bearing strength per bolt is</td>
<td>The required bearing strength per bolt is</td>
</tr>
<tr>
<td>( r_u = \frac{(2 \text{ connections})(60 \text{ kips/connection})}{6 \text{ bolts}} = 20.0 \text{ kips/bolt} )</td>
<td>( r_u = \frac{(2 \text{ connections})(40 \text{ kips/connection})}{6 \text{ bolts}} = 13.3 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>Check supporting girder web</td>
<td>Check supporting girder web</td>
</tr>
<tr>
<td>The design strength per bolt is</td>
<td>The allowable strength per bolt is</td>
</tr>
<tr>
<td>( \phi r_u = (87.8 \text{ kips/in})(0.355 \text{ in.}) )</td>
<td>( r_u / \Omega = (58.5 \text{ kips/in.})(0.355 \text{ in.}) )</td>
</tr>
<tr>
<td>[ = 31.2 \text{ kips/bolt} &gt; 20.0 \text{ kips/bolt} \text{ o.k.} ]</td>
<td>[ = 20.8 \text{ kips/bolt} &gt; 13.3 \text{ kips/bolt} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>
Example II.A-10  Skewed Double Bent-Plate Connection (beam-to-girder web).

Given:

Design the skewed double bent-plate connection between the $W_{16\times77}$ beam to $W_{27\times94}$ girder-web to support the following beam end reactions:

\[ R_D = 13.3 \text{ kips} \]
\[ R_L = 40.0 \text{ kips} \]

Use $\frac{3}{8}\text{-in.}$ diameter ASTM A325-N bolts in standard holes through the support. Use 70 ksi electrode welds to the supported beam.
Material Properties:

- **W16×77**  
  ASTM A992  
  $F_y = 50$ ksi  
  $F_u = 65$ ksi  
  Manual

- **W27×94**  
  ASTM A992  
  $F_y = 50$ ksi  
  $F_u = 65$ ksi  
  Tables 2-3

- **Plate Material**  
  ASTM A36  
  $F_y = 36$ ksi  
  $F_u = 58$ ksi  
  and 2-4

Geometric Properties:

- **W16×77**  
  $t_w = 0.455$ in.  
  $d = 16.5$ in.  
  Manual

- **W27×94**  
  $t_w = 0.490$ in.  
  Table 1-1

Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2 (13.3 \text{ kips}) + 1.6 (40.0 \text{ kips}) = 80.0 \text{ kips}$</td>
<td>$13.3 \text{ kips} + 40.0 \text{ kips} = 53.3 \text{ kips}$</td>
</tr>
</tbody>
</table>

See the scaled layout (c) of the connection. Assign load to each vertical row of bolts by assuming a simple beam analogy between bolts and applying the laws of statics.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>Required strength of bent plate A</td>
<td>Required strength for bent plate A</td>
</tr>
<tr>
<td></td>
<td>$R_u = \frac{80.0 \text{ kips} (2.25 \text{ in.})}{6.0 \text{ in.}} = 30.0 \text{ kips}$</td>
<td>$R_u = \frac{53.3 \text{ kips} (2.25 \text{ in.})}{6.0 \text{ in.}} = 20.0 \text{ kips}$</td>
</tr>
<tr>
<td>$R_u$</td>
<td>Required strength for bent plate B</td>
<td>Required strength for bent plate B</td>
</tr>
<tr>
<td></td>
<td>$R_u = 80.0 \text{ kips} - 30.0 \text{ kips} = 50.0 \text{ kips}$</td>
<td>$R_u = 53.3 \text{ kips} - 20.0 \text{ kips} = 33.3 \text{ kips}$</td>
</tr>
</tbody>
</table>

Assume that the welds across the top and bottom of the plates will be $2\frac{1}{2}$ in. long, and that the load acts at the intersection of the beam centerline and the support face.

While the welds do not coincide on opposite faces of the beam web and the weld groups are offset, the locations of the weld groups will be averaged and considered identical see Figure (d).

**Design welds**

Assume plate length of $8\frac{1}{2}$ in.

$$k = \frac{kl}{I} = \frac{2\frac{1}{2} \text{ in.}}{8\frac{1}{2} \text{ in.}} = 0.294 \text{ in.}$$

From tables, with $\theta = 0^\circ$ and $k = 0.294$  

Manual Table 8-8

$$x_l = \frac{2.5(1.25)(2)}{2.5(2) + 8.5} = 0.463 \text{ in.}$$
$$x = \frac{0.463}{8.5} = 0.054$$
Thus,

\[ a = \frac{(a_l + x_l) - x_l}{l} = \frac{3\frac{3}{8} \text{ in} - 0.463 \text{ in.}}{8\frac{1}{2} \text{ in.}} = 0.372 \]

Interpolating from tables, with \( \theta = 0^\circ \), \( a = 0.372 \), and \( k = 0.294 \),

\[ C = 2.52 \]

The required weld size for two such welds is

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( D_{req} = \frac{R_u}{\phi CC_l} )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( D_{req} = \frac{R_u}{\phi CC_l} )</td>
<td>( = \frac{(50 \text{ kips})}{0.75(2.52)(1)(8\frac{1}{2} \text{ in.})} )</td>
<td>( D_{req} = \frac{\Omega R_u}{CC_l} )</td>
</tr>
<tr>
<td></td>
<td>( = 3.11 \rightarrow 4 \text{ sixteenths} )</td>
<td>( = \frac{2.0(33.3 \text{ kips})}{(2.52)(1)(8\frac{1}{2} \text{ in.})} )</td>
</tr>
<tr>
<td></td>
<td>( = 3.11 \rightarrow 4 \text{ sixteenths} )</td>
<td>( = 3.11 \rightarrow 4 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

Use \( \frac{3}{4} \)-in. fillet welds.

Check beam web thickness

According to Part 9 of the Manual, with \( F_{EXX} = 70 \text{ ksi} \) on both sides of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is

\[ t_{min} = \frac{6.19D}{F_u} = \frac{(6.19)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.381 \text{ in.} < 0.455 \text{ in.} \text{ o.k.} \]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design bolts</td>
<td>Design bolts</td>
<td></td>
</tr>
<tr>
<td>Maximum shear to bent plate = 50 kips</td>
<td>Maximum shear to bent plate = 33.3 kips</td>
<td></td>
</tr>
<tr>
<td>Use 3 rows of 7/8 in. diameter ASTM A325-N bolts.</td>
<td>Use 3 rows of 7/8 in. diameter ASTM A325-N bolts.</td>
<td></td>
</tr>
<tr>
<td>Check shear on bolts</td>
<td>Check shear on bolts</td>
<td></td>
</tr>
<tr>
<td>( \phi R_u = n(\phi r_u) )</td>
<td>( R_u / \Omega = n(r_u / \Omega) )</td>
<td></td>
</tr>
<tr>
<td>( = (3 \text{ bolts})(21.6 \text{ kips/bolt}) )</td>
<td>( = (3 \text{ bolts})(14.4 \text{ kips/bolt}) )</td>
<td></td>
</tr>
<tr>
<td>( = 64.8 \text{ kips} &gt; 50 \text{ kips} \text{ o.k.} )</td>
<td>( = 43.2 \text{ kips} &gt; 33.3 \text{ kips} \text{ o.k.} )</td>
<td></td>
</tr>
<tr>
<td>Check bearing on support</td>
<td>Check bearing on support</td>
<td></td>
</tr>
<tr>
<td>( \phi r_u = 102 \text{ kips/in.} )</td>
<td>( R_u / \Omega = 68.3 \text{ kips/in.} )</td>
<td></td>
</tr>
<tr>
<td>( = (102 \text{ kips/in})(0.490 \text{ in.})(3 \text{ bolts}) )</td>
<td>( = (68.3 \text{ kips/in})(0.490 \text{ in.})(3 \text{ bolts}) )</td>
<td></td>
</tr>
<tr>
<td>( = 150 \text{ kips} &gt; 50 \text{ kips} \text{ o.k.} )</td>
<td>( = 100 \text{ kips} &gt; 33.3 \text{ kips} \text{ o.k.} )</td>
<td></td>
</tr>
</tbody>
</table>
### Design bent plates

Try a PL $\frac{3}{16}$ in.

#### Check bearing on plates

- $\phi r_{ni} = 91.4$ kips/in.
- $\phi r_{no} = 40.8$ kips/in.

\[
\phi R_n = \left[ (91.4 \text{ kips/in}) (2 \text{ bolts}) + (40.8 \text{ kips/in}) (1 \text{ bolt}) \right] \left( \frac{3}{16} \text{ in.} \right)
\]

\[= 70 \text{ kips} > 50 \text{ kips} \quad \text{o.k.}\]

#### Check gross shear yielding of plates

- $\phi = 1.00$

\[\phi R_n = \phi \left( 0.6F_y \right) A_g
\]

\[= (1.00)(0.6)(36 \text{ ksi})(8\frac{1}{2} \text{ in.})(\frac{3}{16} \text{ in.})
\]

\[= 57.4 \text{ kips} > 50 \text{ kips} \quad \text{o.k.}\]

#### Check shear rupture of the plates

- $\phi = 0.75$

\[A_g = \left[ (8\frac{1}{2} \text{ in.})-(3)(1.0 \text{ in.}) \right] \left( \frac{3}{16} \text{ in.} \right)
\]

\[= 1.72 \text{ in.}^2
\]

\[\phi R_n = \phi \left( 0.6F_y \right) A_n
\]

\[= (0.75)(0.6)(58 \text{ ksi})(1.72 \text{ in.}^2)
\]

\[= 44.9 \text{ kips} < 50 \text{ kips} \quad \text{n.g.}\]

Increase the plate thickness to $\frac{3}{8}$ in.

\[A_g = \left[ (8\frac{1}{2} \text{ in.})-(3)(1.0 \text{ in.}) \right] \left( \frac{3}{8} \text{ in.} \right)
\]

\[= 2.06 \text{ in.}^2
\]

\[\phi R_n = (0.75)(0.6)(58 \text{ ksi})(2.06 \text{ in.}^2)
\]

\[= 53.8 \text{ kips} > 50 \text{ kips} \quad \text{o.k.}\]

Check block shear rupture of the plate

with $n = 3, L_{cv} = L_{ch} = 1\frac{1}{4}$ in.,

\[\phi R_n = \phi F_y A_n U_{bs} + \min \left( \phi 0.6F_y A_g, \phi F_u A_m \right)
\]

### Design bent plates

Try a PL $\frac{3}{16}$ in.

#### Check bearing on plates

- $r_{ni}/\Omega_v = 60.9$ kips/in.
- $r_{no}/\Omega_v = 27.2$ kips/in.

\[= \left[ (60.9 \text{ kips/in}) (2 \text{ bolts}) + (27.2 \text{ kips/in}) (1 \text{ bolt}) \right] \left( \frac{3}{16} \text{ in.} \right)
\]

\[= 46 \text{ kips} > 33.3 \text{ kips} \quad \text{o.k.}\]

#### Check gross shear yielding of plates

\[\Omega = 1.50
\]

\[R_u/\Omega = \left( 0.6F_y \right) A_g / \Omega
\]

\[= (0.6)(36 \text{ ksi})(8\frac{1}{2} \text{ in.})(\frac{3}{16} \text{ in.})/1.50
\]

\[= 38.3 \text{ kips} > 33.3 \text{ kips} \quad \text{o.k.}\]

#### Check shear rupture of the plates

\[\Omega = 2.00
\]

\[A_g = \left[ (8\frac{1}{2} \text{ in.})-(3)(1.0 \text{ in.}) \right] \left( \frac{3}{16} \text{ in.} \right)
\]

\[= 1.72 \text{ in.}^2
\]

\[R_u/\Omega = (0.6F_u) A_n / \Omega
\]

\[= (0.6)(58 \text{ ksi})(1.72 \text{ in.}^2)/2.00
\]

\[= 29.9 \text{ kips} < 33.3 \text{ kips} \quad \text{n.g.}\]

Increase the plate thickness to $\frac{3}{8}$ in.

\[A_g = \left[ (8\frac{1}{2} \text{ in.})-(3)(1.0 \text{ in.}) \right] \left( \frac{3}{8} \text{ in.} \right)
\]

\[= 2.06 \text{ in.}^2
\]

\[R_u/\Omega = (0.6)(58 \text{ ksi})(2.06 \text{ in.}^2)/2.00
\]

\[= 35.8 \text{ kips} > 33.3 \text{ kips} \quad \text{o.k.}\]

Check block shear rupture of the plate

with $n = 3, L_{cv} = L_{ch} = 1\frac{1}{4}$ in.,

\[R_u/\Omega = F_y A_n U_{bs} + \min \left( \frac{0.6F_y A_g}{\Omega}, \frac{F_u A_n}{\Omega} \right)
\]
Thus, the configuration shown in Figure II.A-10 can be supported using \( \frac{3}{8} \)-in. bent plates, and \( \frac{1}{4} \)-in. fillet welds.
Example II.A-11  Shear End-Plate Connection (beam to girder web).

Given:

Design a shear end-plate connection to connect a W18×50 beam to W21×62 girder web, to support the following beam end reactions:

\[ R_D = 10 \text{ kips} \]
\[ R_L = 30 \text{ kips} \]

Use \( \frac{3}{4} \)-in. diameter ASTM A325-N bolts in standard holes and 70 ksi electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Specification</th>
<th>Yield</th>
<th>Tensile</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×50</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Girder</td>
<td>W21×62</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Plate</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Specification</th>
<th>( d )</th>
<th>( t_w )</th>
<th>( S_{net} )</th>
<th>( h_0 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×50</td>
<td>18 in.</td>
<td>0.355 in.</td>
<td>23.4 in.(^3)</td>
<td>16.0 in.</td>
</tr>
<tr>
<td>Cope</td>
<td></td>
<td>4 ( \frac{1}{2} ) in.</td>
<td>2 in.</td>
<td>4( \frac{1}{2} ) in.</td>
<td></td>
</tr>
<tr>
<td>Girder</td>
<td>W21×62</td>
<td></td>
<td>0.400 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>60 kips</td>
<td>40 kips</td>
</tr>
<tr>
<td>( R_u ) \text{ LRFD}</td>
<td>Check bolt shear. Check end plate for bolt bearing, shear yielding, shear rupture and block shear rupture.</td>
<td>Check bolt shear. Check end plate for bolt bearing, shear yielding, shear rupture and block shear rupture.</td>
</tr>
<tr>
<td>( R_u ) \text{ ASD}</td>
<td>Try 3 rows of bolts and ( \frac{1}{4} ) in. plate thickness</td>
<td>Try 3 rows of bolts and ( \frac{1}{4} ) in. plate thickness</td>
</tr>
<tr>
<td>( \phi R_u )</td>
<td>o.k. 76.4 kips &gt; 60 kips</td>
<td>o.k. 50.9 kips &gt; 40 kips</td>
</tr>
</tbody>
</table>

Manual Tables 2-3 and 2-4

Manual Tables 1-1 and 9-2

Manual Table 10-4
Check weld shear, check beam web shear rupture

Try ⅛-in. weld.

\[ t_{\text{tw}_{\text{min}}} = 0.381 \text{ in.} > 0.355 \text{ in.} \quad \text{Proration req'd} \]

\[
\phi R_n = \frac{89.1 \text{ kips}}{(0.355 \text{ in.})} \left(\frac{0.381 \text{ in.}}{0.355 \text{ in.}}\right) \\
= 83.0 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}
\]

Check supporting member web or flange for bolt bearing

\[
\phi R_n = (526 \text{ kips/in.})(0.400 \text{ in.}) \\
= 210 \text{ kips} > 60 \text{ kips} \quad \text{o.k.}
\]

Check coped section

As was shown in Example II.A-4, coped section does not control design. \text{o.k.}

Check web shear

As was shown in Example II.A-4, web shear does not control design. \text{o.k.}

Note: See Example II.A-4 for an all-bolted double-angle connection and Example II.A-5 for a bolted/welded double-angle connection.
Example II.A-12  All-Bolted Unstiffened Seated Connection (beam-to-column web).

Given:

Design an all-bolted unstiffened seated connection between a W16×50 beam and W14×90 column web to support the following end reactions:

\[ R_D = 9 \text{ kips} \]
\[ R_L = 27.5 \text{ kips} \]

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes.

Material Properties:

- **Beam** W16×50  
  ASTM A992  
  \( F_y = 50 \text{ ksi} \)  
  \( F_u = 65 \text{ ksi} \)  
  Manual

- **Column** W14×90  
  ASTM A992  
  \( F_y = 50 \text{ ksi} \)  
  \( F_u = 65 \text{ ksi} \)  
  Table 2-3

- **Angles**  
  ASTM A36  
  \( F_y = 36 \text{ ksi} \)  
  \( F_u = 58 \text{ ksi} \)  
  Table 2-4

Geometric Properties:

- **Beam** W16×50  
  \( t_w = 0.380 \text{ in.} \)  
  \( d = 16.3 \text{ in.} \)  
  \( t_f = 0.630 \text{ in.} \)  
  \( k = 1.03 \text{ in.} \)  
  Manual

- **Column** W14×90  
  \( t_w = 0.440 \text{ in.} \)  
  Table 1-1
<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.2(9 \text{kips}) + 1.6(27.5 \text{kips}) = 55 \text{kips} )</td>
<td>( R_u = 9 \text{kips} + 27.5 \text{kips} = 36.5 \text{kips} )</td>
<td>Section J10 Manual Table 9-4</td>
</tr>
<tr>
<td><strong>Solution:</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Check beam web</td>
<td></td>
<td></td>
</tr>
<tr>
<td>For local web yielding,</td>
<td>( N_{\text{min}} = \frac{R_u - \phi R_i}{\phi R_i} \geq k )</td>
<td>For local web yielding,</td>
</tr>
<tr>
<td></td>
<td>( = \frac{55 \text{kips} - 49.0 \text{kips}}{19.0 \text{kips}} \geq 1.03 )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 0.316 \text{ in.} &lt; 1.03 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = 1.03 \text{ in.} )</td>
<td></td>
</tr>
<tr>
<td>For web crippling,</td>
<td></td>
<td></td>
</tr>
<tr>
<td>When ( \frac{N}{d} \leq 0.2 )</td>
<td>( N_{\text{min}} = \frac{R_u - \phi R_i}{\phi R_i} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{55 \text{kips} - 67.2 \text{kips}}{5.81 \text{kips}} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td></td>
</tr>
<tr>
<td>When ( \frac{N}{d} &gt; 0.2 )</td>
<td>( N_{\text{min}} = \frac{R_u - \phi R_i}{\phi R_i} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>( = \frac{55 \text{kips} - 60.9 \text{kips}}{7.74 \text{kips/in.}} )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>which results in a negative quantity.</td>
<td></td>
</tr>
<tr>
<td>Thus, ( N_{\text{min}} = 1.03 \text{ in.} )</td>
<td></td>
<td>Thus, ( N_{\text{min}} = 1.03 \text{ in.} )</td>
</tr>
</tbody>
</table>
Check shear yielding and flexural yielding of angle. Check local yielding and crippling of beam web

Try an 8 in. angle length with a $\frac{3}{4}$ in. thickness and a $3\frac{1}{2}$ in. minimum outstanding leg.

$\phi R_n = 117 \text{ kips} > 55 \text{ kips} \quad \text{o.k.}$

Try L6×4×$\frac{3}{4}$ (4-in. OSL), 8-in. long with 5½-in. bolt gage, connection type B (four bolts).

For $\frac{3}{8}$-in. diameter ASTM A325-N bolts, $\phi R_n = \phi r_a n = 86.6 \text{ kips} > 55 \text{ kips} \quad \text{o.k.}$

Check bolt bearing on the angle

Bolt single shear strength = 21.6 kips

$\phi R_n = \phi (2.4 t F_u)$

$= 0.75(2.4)(7/8 \text{ in.})(3/4 \text{ in.})(58 \text{ ksi})$

$= 68.5 \text{ kips} > 21.6 \text{ kips} \quad \text{o.k.}$

Check supporting column

Bolt single shear strength = 21.6 kips

$\phi R_n = \phi (2.4 t F_u)$

$= 0.75(2.4)(7/8 \text{ in.})(0.440 \text{ in.})(65 \text{ ksi})$

$= 45.0 \text{ kips} > 27.1 \text{ kips} \quad \text{o.k.}$

Check bolt bearing on the angle

Bolt single shear strength = 14.4 kips

$R_n / \Omega = \frac{(2.4 t F_u)}{\Omega}$

$= \frac{(2.4)(7/8 \text{ in.})(3/4 \text{ in.})(58 \text{ ksi})}{2.00}$

$= 45.7 \text{ kips} > 14.4 \text{ kips} \quad \text{o.k.}$

Check supporting column

Bolt single shear strength = 14.4 kips

$R_n / \Omega = \frac{(2.4 t F_u)}{\Omega}$

$= \frac{(2.4)(7/8 \text{ in.})(0.440 \text{ in.})(65 \text{ ksi})}{2.00}$

$= 30.0 \text{ kips} > 14.4 \text{ kips} \quad \text{o.k.}$

Select top angle and bolts

Use an L4×4×$\frac{3}{4}$ with two $\frac{3}{8}$-in. diameter ASTM A325-N through each leg.
Example II.A-13  Bolted/Welded Unstiffened Seated Connection
(beam-to-column flange)

Given:

Design an unstiffened seated connection between a W21×62 beam and a W14×61 column
flange, to support the following beam end reactions:

\[ R_D = 9 \text{ kips} \]
\[ R_L = 27.5 \text{ kips} \]

Use \( \frac{3}{4} \)-in. diameter ASTM A325-N bolts in standard holes to connect the supported beam to
the seat and top angles. Use 70 ksi electrode welds to connect the seat and top angles to the
column flange.

Note: For calculation purposes, assume setback is equal to \( \frac{3}{4} \) in. to account for possible
beam underrun.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
<th>Grade</th>
<th>( F_y )</th>
<th>( F_u )</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W21×62</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual</td>
</tr>
<tr>
<td>Column</td>
<td>W14×61</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Table 2-3</td>
</tr>
<tr>
<td>Angles</td>
<td></td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Size</th>
<th>( t_w )</th>
<th>( d )</th>
<th>( t_f )</th>
<th>( k )</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W21×62</td>
<td>0.400 in.</td>
<td>21.0 in.</td>
<td>0.615 in.</td>
<td>1.12 in.</td>
<td>Manual</td>
</tr>
<tr>
<td>Column</td>
<td>W14×61</td>
<td>0.645 in.</td>
<td></td>
<td></td>
<td></td>
<td>Table 1-1</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(9 \text{ kips}) + 1.6(27.5 \text{ kips}) = 55 \text{ kips}$</td>
<td>$R_u = 9 \text{ kips} + 27.5 \text{ kips} = 36.5 \text{ kips}$</td>
</tr>
<tr>
<td><em>Check the strength of the beam web</em></td>
<td><em>Check the strength of the beam web</em></td>
</tr>
<tr>
<td>For local web yielding</td>
<td>For local web yielding</td>
</tr>
<tr>
<td>$N_{\text{min}} = \frac{R_u - \phi R_s}{\phi R_s} \geq k$</td>
<td>$N_{\text{min}} = \frac{R_u - R_s / \Omega}{R_s / \Omega} \geq k$</td>
</tr>
<tr>
<td>$= \frac{55 \text{ kips} - 55.8 \text{ kips}}{20 \text{ kips}} \geq 1.12 \text{ in.}$</td>
<td>$= \frac{36.5 \text{ kips} - 37.2 \text{ kips}}{13.3 \text{ kips}} \geq 1.12 \text{ in.}$</td>
</tr>
<tr>
<td>$N_{\text{min}} = 1.12 \text{ in.}$</td>
<td>$N_{\text{min}} = 1.12 \text{ in.}$</td>
</tr>
<tr>
<td>For web crippling,</td>
<td>For web crippling,</td>
</tr>
<tr>
<td>$\left( \frac{N}{d} \right)_{\text{max}} = \frac{3.25}{21} = 0.16 &lt; 0.2$</td>
<td>$\left( \frac{N}{d} \right)_{\text{max}} = \frac{3.25}{21} = 0.16 &lt; 0.2$</td>
</tr>
<tr>
<td>When $\frac{N}{d} \leq 0.2$</td>
<td>When $\frac{N}{d} \leq 0.2$</td>
</tr>
<tr>
<td>$N_{\text{min}} = \frac{R_u - \phi R_s}{\phi R_s}$</td>
<td>$N_{\text{min}} = \frac{R_u - R_s / \Omega}{R_s / \Omega}$</td>
</tr>
<tr>
<td>$= \frac{55 \text{ kips} - 71.7 \text{ kips}}{5.37 \text{ kips}}$</td>
<td>$= \frac{36.5 \text{ kips} - 47.8 \text{ kips}}{3.58 \text{ kips}}$</td>
</tr>
<tr>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity.</td>
</tr>
<tr>
<td>Therefore, $N_{\text{min}} = 1.12 \text{ in.}$</td>
<td>Therefore, $N_{\text{min}} = 1.12 \text{ in.}$</td>
</tr>
</tbody>
</table>

*Check shear yielding and flexural yielding of angle. Check local yielding and crippling of beam web*

Try an 8 in. angle length with a 5/8-in. thickness and a 3 1/2 in. minimum outstanding leg.

$N_{\text{req}} > 11/6 \text{ in.}$

For $N = 1 1/6 \text{ in.}$

$\phi R_u = 81.0 \text{ kips} > 55 \text{ kips}$ \textbf{o.k.}

$R_u / \Omega = 53.9 \text{ kips} > 36.5 \text{ kips}$ \textbf{o.k.}

Manual Table 9-4

Manual Table 10-6
<table>
<thead>
<tr>
<th>Try an L8×4×3⁄8 (4 in. OSL), 8 in. long with 3⁄16 in. fillet welds.</th>
<th>Try an L8×4×3⁄8 (4 in. OSL), 8 in. long with 3⁄16 in. fillet welds.</th>
<th>Manual Table 10-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_a = 66.7$ kips &gt; 55 kips</td>
<td>$R_u / \Omega = 44.5$ kips &gt; 36.5 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>Use two 3⁄4-in. diameter ASTM A325-N bolts to connect the beam to the seat angle.</td>
<td>Use two 3⁄4-in. diameter ASTM A325-N bolts to connect the beam to the seat angle.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

The strength of the bolts must be verified if horizontal forces are added to the connection.

**Select top angle, bolts, and welds**

Use L4×4×1⁄2 with two 3⁄4 in. diameter ASTM A325-N bolts through the supported-beam leg of the angle. Use a 3⁄16-in. fillet weld along the toe of the angle to the column flange. See the discussion in Manual Part 10.

Note: See Example II.A-12 for an all-bolted unstiffened seat connection.
Example II.A-14  Stiffened Seated Connection (beam-to-column flange).

Given:

Design a stiffened seated connection between a $W_{21} \times 68$ beam and a $W_{14} \times 90$ column flange, to support the following end reactions:

$$R_D = 21 \text{ kips}$$
$$R_L = 62.5 \text{ kips}$$

Use $\frac{3}{4}$ in. diameter ASTM A325-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70 ksi electrode welds to connect the stiffener and top angle to the column flange.

Note: For calculation purposes, assume setback is equal to $\frac{3}{4}$ in. to account for possible beam underrun.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>$F_y$</th>
<th>$F_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Column</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Angles and plates</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>$t_w$</th>
<th>$d$</th>
<th>$t_f$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>0.430 in.</td>
<td>21.1 in.</td>
<td>0.685 in.</td>
</tr>
<tr>
<td>Column</td>
<td>0.710 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(21 \text{ kips}) + 1.6(62.5 \text{ kips}) = 125 \text{ kips}$</td>
<td>$R_u = 21 \text{ kips} + 62.5 \text{ kips} = 83.5 \text{ kips}$</td>
</tr>
<tr>
<td><strong>Determine stiffener width $W$ required</strong></td>
<td><strong>Determine stiffener width $W$ required</strong></td>
</tr>
<tr>
<td>For web crippling, assume $N/d &gt; 0.2$</td>
<td>For web crippling, assume $N/d &gt; 0.2$</td>
</tr>
<tr>
<td>$W_{min} = \frac{R_u - \phi R_s}{\phi R_s} + \text{setback}$</td>
<td>$W_{min} = \frac{R_u - \phi R_s}{\phi R_s} + \text{setback}$</td>
</tr>
<tr>
<td>[= \frac{125 \text{ kips} - 75.9 \text{ kips}}{7.94 \text{ kips/in.}} + \frac{\gamma}{4} \text{ in.}]</td>
<td>[= \frac{83.5 \text{ kips} - 50.6 \text{ kips}}{5.29 \text{ kips/in.}} + \frac{\gamma}{4} \text{ in.}]</td>
</tr>
<tr>
<td>[= 6.93 \text{ in.}]</td>
<td>[= 6.97 \text{ in.}]</td>
</tr>
<tr>
<td>For local web yielding,</td>
<td>For local web yielding,</td>
</tr>
<tr>
<td>$W_{min} = \frac{R_u - \phi R_s}{\phi R_s} + \text{setback}$</td>
<td>$W_{min} = \frac{R_u - \phi R_s}{\phi R_s} + \text{setback}$</td>
</tr>
<tr>
<td>[= \frac{125 \text{ kips} - 63.7 \text{ kips}}{21.5 \text{ kips/in.}} + \frac{\gamma}{4} \text{ in.}]</td>
<td>[= \frac{83.5 \text{ kips} - 42.5 \text{ kips}}{14.3 \text{ kips/in.}} + \frac{\gamma}{4} \text{ in.}]</td>
</tr>
<tr>
<td>[= 3.60 \text{ in.} &lt; 6.93 \text{ in.}]</td>
<td>[= 3.62 \text{ in.} &lt; 6.97 \text{ in.}]</td>
</tr>
<tr>
<td>Use $W = 7 \text{ in.}$</td>
<td>Use $W = 7 \text{ in.}$</td>
</tr>
<tr>
<td><strong>Check assumption</strong></td>
<td><strong>Check assumption</strong></td>
</tr>
<tr>
<td>$N = \frac{6.93 \text{ in.} - 0.75 \text{ in.}}{21.1 \text{ in.}}$</td>
<td>$N = \frac{6.97 \text{ in.} - 0.75 \text{ in.}}{21.1 \text{ in.}}$</td>
</tr>
<tr>
<td>[= 0.293 &gt; 0.20 \text{ o.k.}]</td>
<td>[= 0.295 &gt; 0.20 \text{ o.k.}]</td>
</tr>
<tr>
<td><strong>Determine stiffener length $L$ and stiffener to column flange weld size</strong></td>
<td><strong>Determine stiffener length $L$ and stiffener to column flange weld size</strong></td>
</tr>
<tr>
<td>Try a stiffener with $L = 15 \text{ in.}$ and $\frac{3}{8}\text{ in.}$ weld</td>
<td>Try a stiffener with $L = 15 \text{ in.}$ and $\frac{3}{8}\text{ in.}$ weld</td>
</tr>
<tr>
<td>$\phi R_s = 139 \text{ kips} &gt; 125 \text{ kips}$</td>
<td>$R_u / \Omega = 93.0 \text{ kips} &gt; 83.5 \text{kips}$</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Determine weld requirements for seat plate**

Use $\frac{3}{8}\text{ in.}$ fillet welds on each side of the stiffener. Minimum length of seat-plate-to-column flange weld is $0.2(L) = 3 \text{ in.}$ If the weld between the seat plate and stiffener plate is required to be stronger than the weld between the seat plate and the column flange, use $\frac{3}{8}\text{ in.}$ fillet welds on each side of the stiffener to the seat plate, length of weld $= 7 \text{ in.} > 3 \text{ in.}$ **o.k.**
Determine the seat plate dimensions

A width of 8 in. is adequate to accommodate two \( \frac{3}{4} \)-in. diameter ASTM A325-N bolts on a 5\( \frac{1}{2} \) in. gage connecting the beam flange to the seat plate.

Use a PL\( \frac{3}{8} \) in. \( \times \) 7 in. \( \times \) 8 in. for the seat plate.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the stiffener plate thickness</td>
<td>Determine the stiffener plate thickness</td>
</tr>
<tr>
<td>Determine minimum plate thickness to develop the stiffener-to-seat-plate weld.</td>
<td>Determine minimum plate thickness to develop the stiffener-to-seat-plate weld.</td>
</tr>
<tr>
<td>( t_{\text{min}} = 2w = 2(\frac{3}{16} \text{ in.}) = \frac{3}{8} \text{ in.} )</td>
<td>( t_{\text{min}} = 2w = 2(\frac{3}{16} \text{ in.}) = \frac{3}{8} \text{ in.} )</td>
</tr>
<tr>
<td>Determine minimum plate thickness for a stiffener with ( F_y = 36 \text{ ksi} ) and beam with ( F_y = 50 \text{ ksi} ).</td>
<td>Determine minimum plate thickness for a stiffener with ( F_y = 36 \text{ ksi} ) and beam with ( F_y = 50 \text{ ksi} ).</td>
</tr>
<tr>
<td>( t_{\text{min}} = \frac{50}{36} t_w = \frac{50}{36} (0.430 \text{ in.}) )</td>
<td>( t_{\text{min}} = \frac{50}{36} t_w = \frac{50}{36} (0.430 \text{ in.}) )</td>
</tr>
<tr>
<td>= 0.597 in. &lt; ( \frac{3}{8} ) in.</td>
<td>= 0.597 in. &lt; ( \frac{3}{8} ) in.</td>
</tr>
<tr>
<td>Use a PL ( \frac{3}{8} ) in. ( \times ) 7 in. ( \times ) 15 in.</td>
<td>Use a PL ( \frac{3}{8} ) in. ( \times ) 7 in. ( \times ) 15 in.</td>
</tr>
</tbody>
</table>

Check column web thickness

\[
 t_{w \text{ min}} = \frac{3.09D}{F_u} = \frac{3.09(5)(2)}{65} = 0.475 \text{ in.}
\]

\( t_w \) for W14\( \times \)90 = 0.44 in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = 139 \text{ kips} \left( \frac{0.440 \text{ in.}}{0.475 \text{ in.}} \right) )</td>
<td>( \frac{R_n}{\Omega} = 93.0 \text{ kips} \left( \frac{0.440 \text{ in.}}{0.475 \text{ in.}} \right) )</td>
</tr>
<tr>
<td>= 129 kips &gt; 125 kips \text{ o.k.}</td>
<td>= 86.1 kips &gt; 83.5 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>

Select top angle, bolts, and welds

Use a L4\( \times \)4\( \times \)4 with two \( \frac{3}{4} \) in. diameter ASTM A325-N bolts through the supported-beam leg of the angle. Use a \( \frac{3}{8} \) in. fillet weld along the toe of the supported leg of the angle.
Example II.A-15  Stiffened Seated Connection (beam-to-column web).

Given:

Design a stiffened seated connection between a $W_{21\times68}$ beam and a $W_{14\times90}$ column web to support the following beam end reactions:

- $R_D = 21$ kips
- $R_D = 62.5$ kips

Use 3/4 in. diameter ASTM A325-N bolts in standard holes to connect the supported beam to the seat plate and top angle. Use 70 ksi electrode welds to connect the stiffener and top angle to the column web.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Material</th>
<th>$F_y$</th>
<th>$F_u$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam $W_{21\times68}$</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual, Tables 2-3</td>
</tr>
<tr>
<td>Column $W_{14\times90}$</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual, Tables 2-3</td>
</tr>
<tr>
<td>Angles and plates</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
<td>Manual, Tables 2-3</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>$t_w$</th>
<th>$d$</th>
<th>$t_f$</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam $W_{21\times68}$</td>
<td>0.430 in.</td>
<td>21.1 in.</td>
<td>0.685 in.</td>
<td>Manual, Table 1-1</td>
</tr>
<tr>
<td>Column $W_{14\times90}$</td>
<td>0.440 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

** $B_{max} = W/2 \geq 2\%$ in.**
Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(21 \text{ kips}) + 1.6(62.5 \text{ kips}) = 125 \text{ kips}$</td>
<td>$R_u = 21 \text{ kips} + 62.5 \text{ kips} = 83.5 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>Determine stiffener width $W$ required</td>
<td>Determine stiffener width $W$ required</td>
<td></td>
</tr>
<tr>
<td>As previously calculated in Example II.A-14, use $W = 7 \text{ in.}$</td>
<td>As previously calculated in Example II.A-14, use $W = 7 \text{ in.}$</td>
<td></td>
</tr>
<tr>
<td>Determine stiffener length $L$ and stiffener to column web weld size</td>
<td>Determine stiffener length $L$ and stiffener to column web weld size</td>
<td></td>
</tr>
<tr>
<td>As previously calculated in Example II.A-14, use $L = 15 \text{ in.}$ and $\frac{3}{16} \text{ in.}$ weld size.</td>
<td>As previously calculated in Example II.A-14, use $L = 15 \text{ in.}$ and $\frac{3}{16} \text{ in.}$ weld size.</td>
<td></td>
</tr>
</tbody>
</table>

Determine weld requirements for seat plate

As previously calculated in Example II.A-14, use 3 in. of $\frac{3}{16} \text{ in.}$ weld on both sides of the seat plate for the seat-plate-to-column-web welds and for the seat-plate-to-stiffener welds.

Determine seat plate dimensions

For a column-web support, the maximum distance from the face of the support to the line of the bolts between the beam flange and seat plate is $3\frac{1}{2}$ in. The PL $\frac{3}{8} \text{ in.} \times 7 \text{ in.} \times 8 \text{ in.}$ previously selected in Example II.A-14 will accommodate these bolts.

Determine stiffener plate thickness

As previously calculated in Example II.A-14, use a PL $\frac{3}{8} \text{ in.} \times 7 \text{ in.} \times 15 \text{ in.}$

Select top angle, bolts, and welds

Use L4×4×$\frac{3}{4}$ with two $\frac{3}{4}$ in. diameter ASTM A325-N bolts through the supported-beam leg of the angle. Use a $\frac{3}{16} \text{ in.}$ fillet weld along the toe of the supported leg of the angle.

Check the strength of the column web

If only one side of the column web has a stiffened seated connection, then

$$ t_{w_{min}} = \frac{3.09D}{F_u} = \frac{3.09(5)}{65} = 0.24 \text{ in.} $$

If both sides of the column web have a stiffened seated connection, then

$$ t_{w_{min}} = \frac{6.19D}{F_u} = \frac{6.18(5)}{65} = 0.48 \text{ in.} $$

Column $t_w = 0.44 \text{ in.}$, which is sufficient for the one-sided stiffened seated connection shown.

Note: Additional detailing considerations for stiffened-seated connections are given on page 10-94 of the Manual.
**Example II.A-16  Offset Unstiffened Seated Connection**  
*(beam-to-column flange).*

**Given:**

Determine the seat angle and weld size required for the unstiffened seated connection between a W14×48 beam and a W12×65 column-flange connection with an offset of 5½ in, to support the following beam end reactions:

\[ R_D = 5 \text{ kips} \]
\[ R_L = 15 \text{ kips} \]

Use 70 ksi electrode welds to connect the seat angle to the column flange.

**Material Properties:**

<table>
<thead>
<tr>
<th>Material</th>
<th>ASTM</th>
<th>( F_y )</th>
<th>( F_u )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>W14×48</td>
<td>A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual, Tables 2-3 and 2-4</td>
</tr>
<tr>
<td>W12×65</td>
<td>A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td></td>
</tr>
<tr>
<td>Angle</td>
<td>A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
<td></td>
</tr>
</tbody>
</table>

**Geometric Properties:**

<table>
<thead>
<tr>
<th>Material</th>
<th>( t_w )</th>
<th>( d )</th>
<th>( t_f )</th>
<th>( k )</th>
<th>Notes</th>
</tr>
</thead>
<tbody>
<tr>
<td>W14×48</td>
<td>0.340 in.</td>
<td>13.8 in.</td>
<td>0.595 in.</td>
<td>1.19 in.</td>
<td>Manual, Table 1-1</td>
</tr>
<tr>
<td>W12×65</td>
<td>0.605 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = (1.2)(5 \text{ kips}) + (1.6)(15 \text{ kips}) = 30 \text{ kips}$</td>
<td>$R_u = 5 \text{ kips} + 15 \text{ kips} = 20 \text{ kips}$</td>
</tr>
<tr>
<td><strong>Design the seat angle and welds</strong></td>
<td><strong>Design the seat angle and welds</strong></td>
</tr>
<tr>
<td>The required strength for the right-hand weld can be determined by summing moments about the left-hand weld.</td>
<td>The required strength for the right-hand weld can be determined by summing moments about the left-hand weld.</td>
</tr>
<tr>
<td>$R_{ul} = \frac{(30 \text{ kips})(3 \text{ in.})}{3\frac{1}{2} \text{ in.}} = 25.7 \text{ kips}$</td>
<td>$R_{ul} = \frac{(20 \text{ kips})(3 \text{ in.})}{3\frac{1}{2} \text{ in.}} = 17.1 \text{ kips}$</td>
</tr>
<tr>
<td>Selecting the welds on both sides of the seat to resist this force, the total required strength would be $R_u = 51.4 \text{ kips}$</td>
<td>Selecting the welds on both sides of the seat to resist this force, the total required strength would be $R_u = 34.2 \text{ kips}$</td>
</tr>
<tr>
<td>For local web yielding,</td>
<td>For local web yielding,</td>
</tr>
<tr>
<td>$N_{\min} = \frac{R_u - \phi R_n}{\phi R_u} \geq k$</td>
<td>$N_{\min} = \frac{R_u - \left(\frac{R_u}{\Omega}\right)}{\left(\frac{R_u}{\Omega}\right)} \geq k$</td>
</tr>
<tr>
<td>$= \frac{(51.4 \text{ kips}) - (50.4 \text{ kips})}{17.0 \text{ kips/in.}} &gt; 1.19 \text{ in.}$</td>
<td>$= \frac{(34.2 \text{ kips}) - (33.6 \text{ kips})}{11.3 \text{ kips/in.}} &gt; 1.19 \text{ in.}$</td>
</tr>
<tr>
<td>$= 0.06 \text{ in.} &gt; 1.19 \text{ in.}$</td>
<td>$= 0.05 \text{ in.} &gt; 1.19 \text{ in.}$</td>
</tr>
<tr>
<td>$N_{\min} = 1.19 \text{ in.}$</td>
<td>$N_{\min} = 1.19 \text{ in.}$</td>
</tr>
<tr>
<td>For web crippling,</td>
<td>For web crippling,</td>
</tr>
<tr>
<td>When $\frac{N}{d} \leq 0.2$</td>
<td>When $\frac{N}{d} \leq 0.2$</td>
</tr>
<tr>
<td>$N_{\min} = \frac{R_u - \phi R_n}{\phi R_u}$</td>
<td>$N_{\min} = \frac{R_u - \left(\frac{R_u}{\Omega}\right)}{\left(\frac{R_u}{\Omega}\right)}$</td>
</tr>
<tr>
<td>$= \frac{(51.4 \text{ kips}) - (55.2 \text{ kips})}{5.19 \text{ kips/in.}}$</td>
<td>$= \frac{(34.2 \text{ kips}) - (36.8 \text{ kips})}{3.46 \text{ kips/in.}}$</td>
</tr>
<tr>
<td>which results in a negative quantity.</td>
<td>which results in a negative quantity</td>
</tr>
<tr>
<td>Thus, $N_{req} = 1\frac{1}{2}\text{in.}$</td>
<td>Thus, $N_{req} = 1\frac{1}{2}\text{in.}$</td>
</tr>
<tr>
<td>A 6 in. angle length with a $\frac{1}{8}$-in. thickness provides</td>
<td>A 6 in. angle length with a $\frac{1}{8}$-in. thickness provides</td>
</tr>
<tr>
<td>$\phi R_u = 55.2 \text{ kips} &gt; 51.4 \text{ kips}$ <strong>o.k.</strong></td>
<td>$\frac{R_u}{\Omega} = 36.7 \text{ kips} &gt; 34.2 \text{ kips}$ <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Manual Table 9-4

Manual Table 10-6
With a L7×4 (OSL) angle and $\frac{3}{16}$-in. fillet welds, the weld strength from the tables is

$\phi R_n = 66.7 \text{ kips} > 51.4 \text{ kips} \quad \text{o.k.}$

Use L7×4× $\frac{3}{16}$×6 in. for the seat angle. Use two $\frac{3}{8}$ in. diameter ASTM A325-N bolts to connect the beam to the seat angle and weld the angle to the column with $\frac{3}{16}$-in. fillet welds.

*Select top angle, bolts, and welds*

Use L4×4×$\frac{3}{8}$ with two $\frac{3}{8}$-in. diameter ASTM A325-N bolts through the outstanding leg of the angle.

Use a $\frac{3}{16}$-in. fillet weld along the toe of the angle to the column flange (maximum size permitted by the Specification).

With a L7×4 (OSL) angle and $\frac{3}{16}$-in. fillet welds, the weld strength from the tables is

$R_n/\Omega = 44.5 \text{ kips} > 34.2 \text{ kips} \quad \text{o.k.}$

Use L7×4×$\frac{3}{16}$×6 in. for the seat angle. Use two $\frac{3}{8}$ in. diameter ASTM A325-N bolts to connect the beam to the seat angle and weld the angle to the column with $\frac{3}{16}$-in. fillet welds.

*Select top angle, bolts, and welds*

Use L4×4×$\frac{3}{8}$ with two $\frac{3}{8}$-in. diameter ASTM A325-N bolts through the outstanding leg of the angle.

Use a $\frac{3}{16}$-in. fillet weld along the toe of the angle to the column flange (maximum size permitted by the Specification).
Example II.A-17  Single-Plate Connection  
(conventional – beam-to-column flange)

Given:

Design a single-plate connection between a W16×50 beam and a W14×90 column flange to support the following beam end reactions:

\[ R_D = 8 \text{ kips} \]
\[ R_L = 25 \text{ kips} \]

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes and 70 ksi electrode welds.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W16×50</th>
<th>Column W14×90</th>
<th>Plate</th>
<th>ASTM A36</th>
<th>( F_y ) = 50 ksi</th>
<th>( F_u ) = 65 ksi</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ASTM A992</td>
<td>ASTM A992</td>
<td>ASTM A36</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W16×50</th>
<th>Column W14×90</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( t_w = 0.380 \text{ in.} )</td>
<td>( t_j = 0.710 \text{ in.} )</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(8 \text{ kips}) + 1.6(25 \text{ kips}) = 50 \text{ kips}$</td>
<td>$R_u = 8 \text{ kips} + 25 \text{ kips} = 33 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>Check bolt shear. Check plate for shear yielding, shear rupture, and block shear rupture</td>
<td>Check bolt shear. Check plate for shear yielding, shear rupture, and block shear rupture</td>
<td></td>
</tr>
<tr>
<td>Try four rows of bolts, $\frac{1}{4}$ in. single plate thickness, and $\frac{3}{16}$ in. fillet weld size.</td>
<td>Try four rows of bolts, $\frac{1}{4}$ in. single plate thickness, and $\frac{3}{16}$ in. fillet weld size.</td>
<td></td>
</tr>
<tr>
<td>$\phi R_u = 52.2 \text{ kips} &gt; 50 \text{ kips}$</td>
<td>$R_u / \Omega = 34.8 \text{ kips} &gt; 33 \text{ kips}$</td>
<td></td>
</tr>
<tr>
<td>Check minimum fillet weld size</td>
<td>Check minimum fillet weld size</td>
<td></td>
</tr>
<tr>
<td>Weld size = $\frac{3}{16}$ in.</td>
<td>Weld size = $\frac{3}{16}$ in.</td>
<td></td>
</tr>
<tr>
<td>Check beam web for bolt bearing. Block shear rupture, shear yielding and shear rupture will not control for an uncoped section.</td>
<td>Check beam web for bolt bearing. Block shear rupture, shear yielding and shear rupture will not control for an uncoped section.</td>
<td></td>
</tr>
<tr>
<td>For an uncoped section,</td>
<td>For an uncoped section,</td>
<td></td>
</tr>
<tr>
<td>$\phi R_u = (351 \text{ kips/in.})(0.380 \text{ in.})$</td>
<td>$R_u / \Omega = (234 \text{ kips/in.})(0.380 \text{ in.})$</td>
<td></td>
</tr>
<tr>
<td>$= 133 \text{ kips} &gt; 50 \text{ kips}$</td>
<td>$= 88.9 \text{ kips} &gt; 33 \text{ kips}$</td>
<td></td>
</tr>
</tbody>
</table>

Manual Table 10-9
Section J2.2b
Manual Table 10-1
Example II.A-18  Single-Plate Connection (beam-to-girder web)

Given:

Design a single-plate connection between a W18×35 beam to a W21×62 girder web to support the following beam end reactions:

\[ R_d = 6.5 \text{ kips} \]
\[ R_l = 20 \text{ kips} \]

Top flange coped 2-in. deep by 4-in. long, \( L_{co} = 1\frac{1}{2} \text{ in.} \), \( L_{eh} = 1\frac{1}{2} \text{ in.} \) (assumed to be 1¼ in. for calculation purposes to account for possible underrun in beam length),

Use \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts in standard holes and 70 ksi electrode welds.

![Diagram](image.png)

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>ASTM Material</th>
<th>( F_y ) (ksi)</th>
<th>( F_u ) (ksi)</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×35</td>
<td>A992</td>
<td>50</td>
<td>65</td>
<td>Manual</td>
<td>Tables 2-3 and 2-4</td>
</tr>
<tr>
<td>Girder W21×62</td>
<td>A992</td>
<td>50</td>
<td>65</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plate</td>
<td>A36</td>
<td>36</td>
<td>58</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>( t_w ) (in.)</th>
<th>( d ) (in.)</th>
<th>( t_f ) (in.)</th>
<th>( d_c ) (in.)</th>
<th>( e ) (in.)</th>
<th>( h_0 ) (in.)</th>
<th>Table</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×35</td>
<td></td>
<td>0.300</td>
<td>17.7</td>
<td>0.425</td>
<td></td>
<td></td>
<td></td>
<td>Manual</td>
</tr>
<tr>
<td>Cope</td>
<td></td>
<td>4</td>
<td></td>
<td></td>
<td>2</td>
<td>4 ( \frac{1}{2} )</td>
<td>15.7</td>
<td>Table 1-1</td>
</tr>
<tr>
<td>Girder W21×62</td>
<td></td>
<td>0.400</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

Note: The connection plate dimensions for a ⅛ in. diameter bolt in Manual Table 10-9 are based on $L_{cb} = 1\frac{1}{2}$ in. min. and $L_{ev}$ (top & bottom) = 1¼ in. min.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(6.5 \text{kips}) + 1.6(20 \text{kips}) = 40 \text{kips}$</td>
<td>$R_u = 6.5 \text{kips} + 20 \text{kips} = 26.5 \text{kips}$</td>
</tr>
<tr>
<td>Check bolt shear. Check plate for bolt bearing, shear yielding, shear rupture, and block shear rupture</td>
<td>Check bolt shear. Check plate for bolt bearing, shear yielding, shear rupture, and block shear rupture</td>
</tr>
<tr>
<td>Try four rows of bolts, ⅛ in. single plate thickness, and ⅛ in. fillet weld size.</td>
<td>Try four rows of bolts, ⅛ in. single plate thickness, and ⅛ in. fillet weld size.</td>
</tr>
<tr>
<td>$\phi R_u = 52.2 \text{kips} &gt; 40 \text{kips}$ <strong>o.k.</strong></td>
<td>$R_u / \Omega = 34.8 \text{kips} &gt; 26.5 \text{kips}$ <strong>o.k.</strong></td>
</tr>
<tr>
<td>Check minimum fillet weld size</td>
<td>Check minimum fillet weld size</td>
</tr>
<tr>
<td>Weld size = ⅛ in. <strong>o.k.</strong></td>
<td>Weld size = ⅛ in. <strong>o.k.</strong></td>
</tr>
<tr>
<td>Check beam web for bolt bearing, block shear rupture, shear yielding and shear rupture</td>
<td>Check beam web for bolt bearing, block shear rupture, shear yielding and shear rupture</td>
</tr>
<tr>
<td>For coped section, $n=4$, $L_{ev} = L_{cb} = 1\frac{1}{2}$ in.</td>
<td>For coped section, $n=4$, $L_{ev} = L_{cb} = 1\frac{1}{2}$ in.</td>
</tr>
<tr>
<td>$\phi R_u = (257 \text{kips/in.})(0.300 \text{ in.})$</td>
<td>$R_u / \Omega = (171 \text{kips/in.})(0.300 \text{ in.})$</td>
</tr>
<tr>
<td>$= 77.1 \text{kips} &gt; 40 \text{kips}$ <strong>o.k.</strong></td>
<td>$= 51.3 \text{kips} &gt; 26.5 \text{kips}$ <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

Note: For coped beam sections, the limit states of flexural rupture and local buckling should be checked independently per Part 9. However, for the shallow cope in this example, flexural yielding and local buckling do not govern. For an illustration of these checks, see **Example II.A-4.**
Example II.A-19  Extended Single-Plate Connection (beam-to-column web)

Given:

Design the connection between a W16×36 beam and the web of a W14×90 column, to support the following beam end reactions:

\[ R_D = 6 \text{ kips} \]
\[ R_L = 18 \text{ kips} \]

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes. The beam is braced by the floor diaphragm.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>ASTM A992</th>
<th>( F_y = 50 \text{ ksi} )</th>
<th>( F_u = 65 \text{ ksi} )</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×36</td>
<td>ASTM A992</td>
<td>( F_y = 50 \text{ ksi} )</td>
<td>( F_u = 65 \text{ ksi} )</td>
<td>Tables 2-3</td>
</tr>
<tr>
<td>W14×90</td>
<td>ASTM A36</td>
<td>( F_y = 36 \text{ ksi} )</td>
<td>( F_u = 58 \text{ ksi} )</td>
<td>and 2-4</td>
</tr>
<tr>
<td>Plate Material</td>
<td>ASTM A36</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>( t_w = 0.295 \text{ in.} )</th>
<th>( d = 16.0 \text{ in.} )</th>
<th>( b_f = 14.5 \text{ in.} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×36</td>
<td>Manual</td>
<td></td>
<td></td>
</tr>
<tr>
<td>W14×90</td>
<td>Manual</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Table 1-1</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Solution:

Determine the required strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(6 \text{ kips}) + 1.6(18 \text{ kips}) = 36 \text{ kips} )</td>
<td>( R_u = 6 \text{ kips} + 18 \text{ kips} = 24 \text{ kips} )</td>
</tr>
</tbody>
</table>
Determine the distance from the support to the first line of bolts

\[ a = 9 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Determine the bearing strength of one bolt on the beam web</td>
<td>Determine the bearing strength of one bolt on the beam web</td>
</tr>
<tr>
<td>( \phi r_n = 2.4 F_w t_n d_b )</td>
<td>( r_n = \frac{2.4 F_w t_n d_b}{\Omega} )</td>
</tr>
<tr>
<td>[ = 0.75(2.4)(65)(0.295)(0.875)(2.33) ]</td>
<td>[ = 2.4(65)(0.295)(0.875)(2.33)/2.00 ]</td>
</tr>
<tr>
<td>[ = 30.2 \text{ kips} ]</td>
<td>[ = 20.1 \text{ kips} ]</td>
</tr>
</tbody>
</table>

Determine the shear strength of one bolt

\( \phi r_n = 21.6 \text{ kips} \)

Therefore, shear controls over bearing

Determine the strength of the bolt group

For \( e = 10.5 \text{ in.}, C = 2.33 \)

\( \phi R_n = C\phi r_n = 2.33(21.6 \text{ kips}) \)

\[ = 50.3 \text{ kips} > 36 \text{ kips} \quad \text{o.k.} \]

Therefore, shear controls over bearing

Determine the strength of the bolt group

For \( e = 10.5 \text{ in.}, C = 2.33 \)

\( \frac{R_n}{\Omega} = \frac{C r_n}{\Omega} = 2.33(14.4 \text{ kips}) \)

\[ = 33.6 \text{ kips} > 24 \text{ kips} \quad \text{o.k.} \]

Determine the maximum plate thickness such that the plate will yield before the bolts shear.

\[ M_{\text{max}} = 1.25 F_{nv} A_b C' \]

\( 1.25 F_{nv} = 1.25(48 \text{ ksi}) = 60 \text{ ksi} \)

\( A_b = 0.601 \text{ in.}^2 \)

\( C' = 26 \text{ in.} \)

\( M_{\text{max}} = (60 \text{ ksi})(0.601 \text{ in.}^2)(26 \text{ in.}) \)

\[ = 938 \text{ kip-in.} \]

\[ t_{\text{max}} = \frac{6 M_{\text{max}}}{F_y d^2} = \frac{6(938 \text{ kip-in.})}{36 \text{ ksi}(12 \text{ in.})^2} = 1.09 \text{ in.} \]

Manual Table 7-5

Manual Table 7-8
Try a plate thickness of ½ in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check flexural strength of the plate</strong></td>
<td><strong>Check flexural strength of the plate</strong></td>
</tr>
<tr>
<td>The required strength is</td>
<td>The required strength is</td>
</tr>
<tr>
<td>[ M_a = R_a e = (36 \text{ kips})(9 \text{ in.}) ]</td>
<td>[ M_a = R_a e = (24 \text{ kips})(9 \text{ in.}) ]</td>
</tr>
<tr>
<td>= 324 in.-kips</td>
<td>= 216 in.-kips</td>
</tr>
<tr>
<td><strong>Check shear yielding of the plate</strong></td>
<td><strong>Check shear yielding of the plate</strong></td>
</tr>
<tr>
<td>[ \phi R_n = \phi 0.6F_y A_g ]</td>
<td>[ R_n / \Omega = 0.6F_y A_g / \Omega ]</td>
</tr>
<tr>
<td>[ \phi = 1.00 ]</td>
<td>[ \Omega = 1.50 ]</td>
</tr>
<tr>
<td>= (1.00)(0.6)(36 ksi)(12 in.)(½ in.)</td>
<td>= (0.6)(36 ksi)(12 in.)(½ in.)/1.50</td>
</tr>
<tr>
<td>= 130 kips &gt; 36 kips <strong>o.k.</strong></td>
<td>= 86.4 kips &gt; 24 kips <strong>o.k.</strong></td>
</tr>
<tr>
<td><strong>Determine critical flexural stress in presence of shear stress, ( f_v )</strong></td>
<td><strong>Determine critical flexural stress in presence of shear stress, ( f_v )</strong></td>
</tr>
<tr>
<td>( f_v = \frac{36 \text{ kips}}{(\frac{1}{2} \text{ in.})(12 \text{ in.})} = 6 \text{ksi} )</td>
<td>( f_v = \frac{24 \text{ kips}}{(\frac{1}{2} \text{ in.})(12 \text{ in.})} = 4 \text{ksi} )</td>
</tr>
<tr>
<td>[ \phi F_{cr} = \sqrt{\left(\phi F_y\right)^2 - 3f_v^2} ]</td>
<td>[ F_{cr} = \frac{\sqrt{\left(F_v\right)^2 - 3f_v^2}}{\Omega} ]</td>
</tr>
<tr>
<td>[ = \sqrt{(0.9 \times 36 \text{ksi})^2 - 3(6 \text{ksi})^2} ]</td>
<td>[ = \sqrt{\left(\frac{36 \text{ksi}}{1.67}\right)^2 - 3(4 \text{ksi})^2} ]</td>
</tr>
<tr>
<td>= 30.7 ksi</td>
<td>= 20.4 ksi</td>
</tr>
<tr>
<td>[ \phi M_n = \phi F_{cr} Z = \left(30.7 \text{ksi}\right)\left(\frac{1}{2} \text{in.}\right)\left(12 \text{in.}\right)^2 / 4 ]</td>
<td>[ M_n / \Omega = F_{cr} Z = \left(20.4 \text{ksi}\right)\left(\frac{1}{2} \text{in.}\right)\left(12 \text{in.}\right)^2 / 4 ]</td>
</tr>
<tr>
<td>= 503 kip-in. &gt; 324 kip-in. <strong>o.k.</strong></td>
<td>= 367 kip-in. &gt; 216 kip-in. <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

\[ Z_{net} = \frac{f_v d^2}{4} \left[1 - \frac{d_x + 0.125}{3}\right] = \frac{0.5(12)^2}{4} \left[1 - \frac{0.875 + 0.125}{3}\right] = 12.0 \text{in.}^3 \]
For flexural rupture,\n\[ \phi M_n = \phi F_u Z_{net} \]
\[ \phi = 0.75 \]
\[ \phi F_u Z_{net} = 0.75(58\text{ ksi})(12.0\text{ in.}^3) \]
\[ = 522\text{ in.-kips} > 324\text{ in.-kips} \text{ o.k.} \]

For flexural rupture,\n\[ M_n / \Omega = F_u Z_{net} / \Omega \]
\[ \Omega = 2.00 \]
\[ F_u Z_{net} / \Omega = 348\text{ in.-kips} > 216\text{ in.-kips} \text{ o.k.} \]

Check shear rupture of the plate
\[ A_n = \left( t_s \right) \left[ d - n \left( d_s + 0.125 \text{ in.} \right) \right] \]
\[ = \left( \frac{\sqrt{3}}{2} \text{ in.} \right) \left[ (12 \text{ in.} ) - (4)(0.875 \text{ in.} + 0.125 \text{ in.}) \right] = 4\text{ in.}^2 \]

Check block shear rupture of the plate
\[ n = 4, L_{eh} = 1\frac{1}{2} \text{ in.}, L_{ch} = 1\frac{3}{4} \text{ in.}, \]
\[ \phi R_n = \phi F_u A_n U_{bs} + \min \left( \phi 0.6 F_u A_{gyv}, \phi F_u A_{gy} \right) \]

Tension rupture component
\[ \phi F_u A_{nt} = 35.4 \text{ kips/in.} \left( \frac{\sqrt{3}}{2} \text{ in.} \right) \]

Shear yielding component
\[ \phi 0.6 F_u A_{gy} = 170 \text{ kips/in.} \left( \frac{\sqrt{3}}{2} \text{ in.} \right) \]

Shear component
\[ \phi 0.6 F_u A_{sw} = 194 \text{ kips/in.} \left( \frac{\sqrt{3}}{2} \text{ in.} \right) \]
\[ \phi R_n = (170 \text{ kips/in.} + 35.4 \text{kips/in.}) \left( \frac{\sqrt{3}}{2} \text{ in.} \right) \]
\[ = 103 \text{ kips} > 36 \text{ kips} \text{ o.k.} \]
Check local buckling of the plate

This check is analogous to the local buckling check for doubly coped beams as illustrated previously in the Manual Part 9 where $c = 6$ in. and $d_c = 1\frac{1}{2}$ in. at both the top and bottom flanges

\[ F_{cr} = F_y Q \]

\[ \lambda = \frac{h_y \sqrt{F_y}}{10 t_w \sqrt{475 + 280 \left( \frac{h_y}{c} \right)^2}} \]

\[ \lambda = \frac{(12 \text{ in.}) \sqrt{36 \text{ ksi}}}{10 \left( \frac{1}{2} \sqrt{475 + 280 \left( \frac{12.0 \text{ in.}}{9 \text{ in.}} \right)^2} \right)} = 0.462 \]

\[ \lambda < 0.7, \text{ therefore, } Q = 1 \]

\[ F_{cr} = F_y \]

Therefore, plate buckling does not control.
Example II.A-20   All-Bolted Single-Plate Shear Splice

Given:

Design an all-bolted single-plate shear splice between a W24×55 beam and a W24×68 beam.

\[ R_D = 10 \text{ kips} \]
\[ R_L = 30 \text{ kips} \]

Use \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts with 5 in. between vertical bolt rows.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Grade</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W24×55</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>W24×68</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Splice Plate</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Manual Tables 2-3 and 2-4

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>( t_w )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W24×55</td>
<td>0.395 in.</td>
</tr>
<tr>
<td>Beam W24×68</td>
<td>0.415 in.</td>
</tr>
</tbody>
</table>

Manual Table 1-1
Solution:

*Design the bolt groups*

Note: When the splice is symmetrical, the eccentricity of the shear to the center of gravity of either bolt group is equal to half the distance between the centroids of the bolt groups. Therefore, each bolt group can be designed for the shear, \( R_u \) or \( R_a \), and one-half the eccentric moment, \( R_e \) or \( R_ae \).

Using a symmetrical splice, each bolt group will carry one-half the eccentric moment. Thus, the eccentricity on each bolt group \( e/2 = 2.5 \text{ in.} \).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \( R_u = 1.2(10 \text{ kips}) + 1.6(30 \text{ kips}) \)  
  \( = 60 \text{ kips} \) | \( R_u = 10 \text{ kips} + 30 \text{ kips} \)  
  \( = 40 \text{ kips} \) |
| For bolt shear, \( \phi r_u = 21.6 \text{ kips/bolt} \) | For bolt shear, \( \phi r_u / \Omega = 14.4 \text{ kips/bolt} \)  
  (Manual Table 7-1)  
  (Manual Table 7-6) |
| For bearing on the web of the W24×55, \( \phi r_u = (102 \text{ kips/in.})(0.395 \text{ in.}) \)  
  \( = 40.3 \text{ kips/bolt} \) | For bearing on the web of the W24×55, \( \phi r_u / \Omega = (68.3 \text{ kips/in.})(0.395 \text{ in.}) \)  
  \( = 27.0 \text{ kips/bolt} \) |
| Since bolt shear is more critical, \( C_{\text{min}} = \frac{R_u}{\phi r_u} = \frac{60 \text{ kips}}{21.6 \text{ kips/bolt}} = 2.78 \)  
  with \( \theta = 0^\circ \) and \( e = 2.5 \text{ in.} \), a four-bolt connection provides \( C = 3.07 > 2.78 \)  
  o.k. | Since bolt shear is more critical, \( C_{\text{min}} = \frac{R_u}{\phi r_u / \Omega} = \frac{40 \text{ kips}}{14.4 \text{ kips/bolt}} = 2.78 \)  
  with \( \theta = 0^\circ \) and \( e = 2.5 \text{ in.} \), a four-bolt connection provides \( C = 3.07 > 2.78 \)  
  o.k.  
  (Manual Table 7-7) |

*Design splice plate*

Try PL \( \frac{3}{8} \text{ in.} \times 8 \text{ in.} \times 1^\prime-0^\prime \).

**Check flexure of the plate**

\[ M_e = \frac{R_e e}{2} = \frac{(60 \text{ kips})(5 \text{ in.})}{2} = 150 \text{ in.-kips} \]

For flexural yielding,

\[ \phi M_e = \phi F_Y Z_x \]

\[ \phi = 0.90 \]

\[ = (0.90)(36 \text{ ksi}) \left( \frac{\left( \frac{3}{8} \text{ in.} \right)(12 \text{ in.})^2}{4} \right) \]

\[ = 437 \text{ in.-kips} > 150 \text{ in.-kips} \]

Manual Part. 15

**Design splice plate**

Try PL \( \frac{3}{8} \text{ in.} \times 8 \text{ in.} \times 1^\prime-0^\prime \).

**Check flexure of the plate**

\[ M_e = \frac{R_e e}{2} = \frac{(40 \text{ kips})(5 \text{ in.})}{2} = 100 \text{ in.-kips} \]

For flexural yielding,

\[ M_e / \Omega = F_Y Z_x / \Omega \]

\[ \Omega = 1.67 \]

\[ = (36 \text{ ksi}) \left( \frac{\left( \frac{3}{8} \text{ in.} \right)(12 \text{ in.})^2}{4} \right) / 1.67 \]

\[ = 291 \text{ in.-kips} > 100 \text{ in.-kips} \]
For flexure rupture,

\[ Z_{net} = \frac{t}{4} \left[ d^2 - s^2 n \left( n^2 - 1 \right) \left( d + \frac{1}{n} \text{ in.} \right) \right] \]

\[ \approx \frac{\gamma_{\text{in.}}}{4} \left[ 12 \text{ in.}^2 - \left( 3 \text{ in.} \right)^2 \left( 4^2 - 1 \right) \left( \frac{1}{n} \text{ in.} + \frac{1}{n} \text{ in.} \right) \right] = 9.28 \text{ in.}^3 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi M_n = \phi F_u Z_{net} )</td>
<td>( M_n/\Omega = F_u Z_{net}/\Omega )</td>
</tr>
<tr>
<td>( = (0.75)(58 \text{ ksi})(9.28 \text{ in}^3) )</td>
<td>( = (58 \text{ ksi})(9.28 \text{ in}^3) / 2.00 )</td>
</tr>
<tr>
<td>( = 404 \text{ in.-kips} &gt; 150 \text{ in.-kips} )</td>
<td>( = 269 \text{ in.-kips} &gt; 100 \text{ in.-kips} ) ( \text{o.k.} )</td>
</tr>
<tr>
<td><strong>Check shear yielding of the plate</strong></td>
<td><strong>Check shear yielding of the plate</strong></td>
</tr>
<tr>
<td>( \phi R_n = \phi \left( 0.6 F_u A_y \right) )</td>
<td>( R_n / \Omega = \left( 0.6 F_u A_y \right) / \Omega )</td>
</tr>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( = 1.00 \left( 0.6 \left( 36 \text{ ksi} \right) \left( 12 \text{ in.} \right) \left( \gamma_{\text{in.}} \text{ in.} \right) \right) )</td>
<td>( = \left( 0.6 \left( 36 \text{ ksi} \right) \left( 12 \text{ in.} \right) \left( \gamma_{\text{in.}} \text{ in.} \right) \right) / 1.50 )</td>
</tr>
<tr>
<td>( = 97.2 \text{ kips} &gt; 60 \text{ kips} )</td>
<td>( = 64.8 \text{ kips} &gt; 40 \text{ kips} ) ( \text{o.k.} )</td>
</tr>
<tr>
<td><strong>Check shear rupture of the plate</strong></td>
<td><strong>Check shear rupture of the plate</strong></td>
</tr>
<tr>
<td>( \phi R_n = \phi \left( 0.6 F_u A_y \right) )</td>
<td>( R_n / \Omega = \left( 0.6 F_u A_y \right) / \Omega )</td>
</tr>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( = 0.75 \left( 0.6 \right) \left( 58 \text{ ksi} \right) \left[ 12 \text{ in.} - 4 \left( 1 \text{ in.} \right) \right] \left( \gamma_{\text{in.}} \text{ in.} \right) )</td>
<td>( = 0.6 \left( 58 \text{ ksi} \right) \left[ 12 \text{ in.} - 4 \left( 1 \text{ in.} \right) \right] \left( \gamma_{\text{in.}} \text{ in.} \right) / 2.00 )</td>
</tr>
<tr>
<td>( = 78.3 \text{ kips} &gt; 60 \text{ kips} )</td>
<td>( = 52.2 \text{ kips} &gt; 40 \text{ kips} ) ( \text{o.k.} )</td>
</tr>
<tr>
<td><strong>Check block shear rupture of the plate</strong></td>
<td><strong>Check block shear rupture of the plate</strong></td>
</tr>
<tr>
<td>( L_{eh} = L_{ev} = 1\frac{1}{2} \text{ in.} , )</td>
<td>( L_{eh} = L_{ev} = 1\frac{1}{2} \text{ in.} , )</td>
</tr>
<tr>
<td>( \phi R_n = \phi F_u A_m U_{bs} + \min \left( \phi \left( 0.6 F_y A_{gy} \right), \phi F_u A_{nv} \right) )</td>
<td>( R_n / \Omega = \frac{F_u A_m U_{bs}}{\Omega} + \min \left( \frac{0.6 F_y A_{gy}}{\Omega}, \frac{F_u A_{nv}}{\Omega} \right) )</td>
</tr>
<tr>
<td>Because maximum shear stress occurs at one row of bolts</td>
<td>Because maximum shear stress occurs at one row of bolts</td>
</tr>
<tr>
<td>( U_{bs} = 1.00 )</td>
<td>( U_{bs} = 1.00 )</td>
</tr>
</tbody>
</table>

Commentary

Section J4.3
<table>
<thead>
<tr>
<th>Tension rupture component</th>
<th>Shear yielding component</th>
<th>Shear rupture component</th>
<th>Shear rupture component</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi F_y A_w = 43.5 \text{kips/in.}(% \text{ in.})$</td>
<td>$F_y A_w / \Omega = 29.0 \text{kips/in.}(% \text{ in.})$</td>
<td>$0.6 F_y A_y / \Omega = 170 \text{kips/in.}(% \text{ in.})$</td>
<td>$0.6 F_y A_y / \Omega = 113 \text{kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td>Shear yielding component</td>
<td>$\phi 0.6 F_y A_y = 170 \text{kips/in.}(% \text{ in.})$</td>
<td>$0.6 F_y A_y / \Omega = 122 \text{kips/in.}(% \text{ in.})$</td>
<td>$0.6 F_y A_y / \Omega = 183 \text{kips/in.}(% \text{ in.})$</td>
</tr>
<tr>
<td>$\phi R_n = (170 \text{kips/in.} + 43.5 \text{kips/in.})(% \text{ in.})$</td>
<td>$R_n / \Omega = (113 \text{kips/in.} + 29.0 \text{kips/in.})(% \text{ in.})$</td>
<td>$60 \text{kips} &gt; 80.1 \text{kips}$</td>
<td>$60 \text{kips} &gt; 53.3 \text{kips}$</td>
</tr>
<tr>
<td>$\phi R_n = 80.1 \text{kips} &gt; 60 \text{kips}$</td>
<td>$\phi R_n = 53.3 \text{kips} &gt; 40 \text{kips}$</td>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
| Use PL $\frac{3}{8} \text{ in.} \times 8 \text{ in.} \times 1' - 0$ | Use PL $\frac{3}{8} \text{ in.} \times 8 \text{ in.} \times 1' - 0$ | Manual Table 9-3a | Manual Table 9-3b | Manual Table 9-3c
Example II.A-21  Bolted/Welded Single-Plate Shear Splice

Given:

Design a single-plate shear splice between a W16×31 beam and W16×50 beam (not illustrated) to support the following beam end reactions:

$$R_{D} = 8.0 \text{ kips}$$
$$R_{L} = 24.0 \text{ kips}$$

Use ¾-in. diameter ASTM A325-N bolts through the web of the W16×50 and 70 ksi electrode welds to the web of the W16×31.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th></th>
<th>$F_{y}$</th>
<th>$F_{u}$</th>
<th>Manual/Tables</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×31</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual Tables 2-3 and 2-4</td>
</tr>
<tr>
<td>W16×50</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual Tables 2-3 and 2-4</td>
</tr>
<tr>
<td>Splice Plate</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
<td>Manual Tables 2-3 and 2-4</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>$t_{w}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×31</td>
<td>0.275 in.</td>
</tr>
<tr>
<td>W16×50</td>
<td>0.380 in.</td>
</tr>
</tbody>
</table>

Solution:

$$R_{u} = 1.2(8 \text{ kips}) + 1.6(24 \text{ kips}) = 48 \text{ kips}$$

$$R_{a} = 8 \text{ kips} + 24 \text{ kips} = 32 \text{ kips}$$

Design the weld group

Since the splice is unsymmetrical and the weld group is more rigid, it will be designed for the full moment from the eccentric shear.

Assume PL ¾ in.×8 in.×1'-0
\[
k = \frac{kl}{l} = \frac{3\frac{1}{2}}{12} \text{ in.} = 0.292
\]

By interpolation, with \( \theta = 0^\circ \), \( x = 0.0538 \) and \( x_l = 0.646 \) in.

\[al = 6\frac{1}{2} \text{ in.} - 0.646 \text{ in.} = 5.85 \text{ in.}, \quad a = \frac{al}{l} = \frac{5.85}{12} \text{ in.} = 0.488\]

By interpolation, \( C = 2.14 \) and the required weld size is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{req} = \frac{P_a}{\phi CC_l} = \frac{(48 \text{ kips})}{0.75(2.14)(1.0)(12 \text{ in.})} = 2.49 \rightarrow 3 \text{ sixteenths} )</td>
<td>( D_{req} = \frac{P_a \Omega}{CC_l} = \frac{(32 \text{ kips})(2.0)}{(2.14)(1.0)(12 \text{ in.})} = 2.49 \rightarrow 3 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

The minimum weld size is \( \frac{1}{6} \text{ in.} \).  
Use a \( \frac{1}{6} \)-in. weld size.

The minimum weld size is \( \frac{1}{6} \text{ in.} \).  
Use a \( \frac{1}{6} \)-in. weld size.

Check shear rupture of beam web at the weld (W16×31)

For fillet welds with \( F_{ex} = 70 \text{ ksi} \) on one side of the connection, the minimum thickness required to match the available shear rupture strength of the connection element to the available shear rupture strength of the base metal is:

\[
t_{min} = \frac{3.09D}{F_v} = \frac{(3.09)(3 \text{ sixteenths})}{65 \text{ ksi}} = 0.143 < 0.275 \text{ in.} \quad \text{o.k.}
\]

Design the bolt group

Since the weld group was designed for the full eccentric moment, the bolt group will be designed for shear only.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For bolt shear ( \phi r_a = 15.9 \text{ kips/bolt} )</td>
<td>For bolt shear ( r_v/\Omega = 10.6 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>For bearing on the ( \frac{1}{6} \text{-in.} ) thick single plate, conservatively use the design values provided for ( L_e = 1 \frac{1}{2} \text{ in.} ) ( \phi r_a = (44.0 \text{ kips/in./bolt})\left(\frac{3}{6} \text{ in.}\right) )</td>
<td>For bearing on the ( \frac{1}{6} \text{-in.} ) thick single plate, conservatively use the design values provided for ( L_e = 1 \frac{1}{2} \text{ in.} ) ( r_v/\Omega = (29.4 \text{ kips/in./bolt})\left(\frac{3}{6} \text{ in.}\right) )</td>
</tr>
<tr>
<td>( \phi r_a = 16.5 \text{ kips/bolt} )</td>
<td>( r_v/\Omega = 11.0 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>
Since bolt shear is more critical than bearing,

\[
\begin{align*}
\eta_{\text{min}} &= \frac{R_s}{\phi_\tau} \\
&= \frac{48 \text{ kips}}{15.9 \text{ kips/bolt}} \\
&= 3.02 \rightarrow 4 \text{ bolts}
\end{align*}
\]

Since bolt shear is more critical than bearing,

\[
\begin{align*}
\eta_{\text{min}} &= \frac{R_s}{r_e / \Omega} \\
&= \frac{32 \text{ kips}}{10.6 \text{ kips/bolt}} \\
&= 3.02 \rightarrow 4 \text{ bolts}
\end{align*}
\]

**Design the single plate**

As before, try a PL \( \frac{3}{8} \) in. \( \times \) 8 in. \( \times \) 1’-0

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check flexure of the plate</strong></td>
<td><strong>Check flexure of the plate</strong></td>
</tr>
<tr>
<td>( M_u = R_s e = (48 \text{ kips})(5.85 \text{ in.}) = 281 \text{ in.-kips} )</td>
<td>( M_u = R_s e = (32 \text{ kips})(5.85 \text{ in.}) = 187 \text{ in.-kips} )</td>
</tr>
<tr>
<td>For flexural yielding</td>
<td>For flexural yielding</td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi M_u = \phi F_c Z_x )</td>
<td>( M_u / \Omega = F_c Z_x / \Omega )</td>
</tr>
<tr>
<td>( = (0.9)(36 \text{ ksi}) \left( \frac{\frac{3}{8} \text{ in.}(12 \text{ in.})^2}{4} \right) )</td>
<td>( = (36 \text{ ksi}) \left( \frac{\frac{3}{8} \text{ in.}(12 \text{ in.})^2}{4} \right) /1.67 )</td>
</tr>
<tr>
<td>( = 437 \text{ in.-kips} &gt; 281 \text{ in.-kips} \text{ o.k.} )</td>
<td>( = 291 \text{ in.-kips} &gt; 187 \text{ in.-kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>

| **Check flexural rupture** | **Check flexural rupture** |
| \( Z_{\text{net}} = Z_e = \frac{t_p d^2}{4} \) | \( Z_{\text{net}} = Z_e = \frac{t_p d^2}{4} \) |
| \( = \left( \frac{\frac{3}{8} \text{ in.}}{4} \right)(12 \text{ in.})^2 = 13.5 \text{ in}^3 \) | \( = \left( \frac{\frac{3}{8} \text{ in.}}{4} \right)(12 \text{ in.})^2 = 13.5 \text{ in}^3 \) |
| \( \phi M_u = \phi F_c Z_{\text{net}} \) | \( M_u / \Omega = \frac{F_c Z_{\text{net}}}{\Omega} \) |
| \( \phi = 0.75 \) | \( \Omega = 2.00 \) |
| \( = 0.75(58 \text{ ksi})(13.5 \text{ in.}^3) \) | \( = \frac{(58 \text{ ksi})(13.5 \text{ in.}^3)}{2.00} \) |
| \( = 587 \text{ in.-kips} > 281 \text{ in.-kips} \text{ o.k.} \) | \( = 392 \text{ in.-kips} > 187 \text{ in.-kips} \text{ o.k.} \) |
**Check shear yielding of the plate**

\[
\phi R_y = \phi \left(0.6 F_y A_y \right)
\]

\[
\phi = 1.00
\]

\[
= 1.00 \left[0.6 (36 \text{ ksi}) (12 \text{ in.}) \left(\frac{\text{in.}}{\text{ksi}}\right)\right]
\]

\[
= 97.2 \text{ kips} > 48 \text{ kips} \quad \text{o.k.}
\]

**Check shear rupture of the plate**

\[
\phi R_u = \phi \left(0.6 F_u A_u \right)
\]

\[
\phi = 0.75
\]

\[
= 0.75 (0.6) (58 \text{ ksi}) \left[12 \text{ in.} - 4 \left(\frac{\text{in.}}{\text{ksi}}\right)\right] \left(\frac{\text{ksi}}{\text{in.}}\right)
\]

\[
= 83.2 \text{ kips} > 48 \text{ kips} \quad \text{o.k.}
\]

**Check block shear rupture of the plate**

\[L_{eh} = L_{ev} = 1\frac{1}{2} \text{ in.,}\]

Thus,

\[
\phi R_y = \phi F_u A_m U_{ba} + \min \left(0.6 F_y A_{gy}, \phi F_u A_{nv}\right)
\]

\[
U_{ba} = 1.0
\]

Tension rupture component

\[\phi F_u A_m = 46.2 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

Shear yielding component

\[0.6 F_y A_{gy} = 170 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

Shear rupture component

\[0.6 F_y A_{rv} = 194 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

\[
\phi R_y = (170 \text{ kips/in.} + 46.2 \text{ kips/in.}) \left(\frac{\text{ksi}}{\text{in.}}\right)
\]

\[
= 81.1 \text{ kips} > 48 \text{ kips} \quad \text{o.k.}
\]

Use PL \(\frac{3}{8}\) in. x 8 in. x 1'-0

---

**Check shear yielding of the plate**

\[
R_u / \Omega = \left(0.6 F_y A_y \right) / \Omega
\]

\[
\Omega = 1.50
\]

\[
= \left[0.6 (36 \text{ ksi}) (12 \text{ in.}) \left(\frac{\text{ksi}}{\text{in.}}\right)\right] / 1.50
\]

\[
= 64.8 \text{ kips} > 32 \text{ kips} \quad \text{o.k.}
\]

**Check shear rupture of the plate**

\[
R_u / \Omega = \left(0.6 F_u A_u \right) / \Omega
\]

\[
\Omega = 2.00
\]

\[
= 0.6 (58 \text{ ksi}) \left[12 \text{ in.} - 4 \left(\frac{\text{ksi}}{\text{in.}}\right)\right] \left(\frac{\text{ksi}}{\text{in.}}\right) / 2.0
\]

\[
= 55.5 \text{ kips} > 32 \text{ kips} \quad \text{o.k.}
\]

**Check block shear rupture of the plate**

\[L_{eh} = L_{ev} = 1\frac{1}{2} \text{ in.,}\]

Thus,

\[
\phi R_y = \phi F_u A_m U_{ba} + \min \left(0.6 F_y A_{gy}, \phi F_u A_{nv}\right)
\]

\[
U_{ba} = 1.0
\]

Tension rupture component

\[\phi F_u A_m = 30.8 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

Shear yielding component

\[0.6 F_y A_{gy} / \Omega = 113 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

Shear rupture component

\[0.6 F_y A_{rv} / \Omega = 129 \text{ kips/in.} \left(\frac{\text{ksi}}{\text{in.}}\right)\]

\[
R_y / \Omega = (113 \text{ kips/in.} + 30.8 \text{ kips/in.})(\frac{\text{ksi}}{\text{in.}})
\]

\[
= 53.9 \text{ kips} > 32 \text{ kips} \quad \text{o.k.}
\]

Use PL \(\frac{3}{8}\) in. x 8 in. x 1'-0
### Example II.A-22  Bolted Bracket Plate Design

Given:

Design a bracket plate to support the following loads:

\[
\begin{align*}
P_D &= 9 \text{ kips} \\
P_L &= 27 \text{ kips}
\end{align*}
\]

Use 3/8-in. diameter ASTM A325-N bolts in standard holes

![ Bracket Plate Diagram ]

Material Properties:

| Plate Material | ASTM A36 | \( F_y = 36 \text{ ksi} \) | \( F_u = 58 \text{ ksi} \) | Manual Table 2-4 |

Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips}) = 54 \text{ kips} )</td>
<td>( R_u = 9 \text{ kips} + 27 \text{ kips} = 36 \text{ kips} )</td>
</tr>
<tr>
<td>Design bolts</td>
<td>( C_{\text{min}} = \frac{R_u}{\phi r_s} )</td>
<td>( C_{\text{min}} = \frac{R_u}{r_s / \Omega} )</td>
</tr>
<tr>
<td></td>
<td>= \frac{54 \text{ kips}}{21.6 \text{ kips/bolt}}</td>
<td>= \frac{36 \text{ kips}}{14.4 \text{ kips/bolt}}</td>
</tr>
<tr>
<td></td>
<td>= 2.50</td>
<td>= 2.50</td>
</tr>
<tr>
<td></td>
<td>For ( \theta = 0^\circ ), a 5\frac{1}{2} \text{ in.} gage with ( s = 3 \text{ in.}, e_s = 12 \text{ in.}, ) and ( n = 6 )</td>
<td>For ( \theta = 0^\circ ), a 5\frac{1}{2} \text{ in.} gage with ( s = 3 \text{ in.}, e_s = 12 \text{ in.}, ) and ( n = 6 )</td>
</tr>
<tr>
<td></td>
<td>( C = 4.53 &gt; 2.50 )</td>
<td>( C = 4.53 &gt; 2.50 )</td>
</tr>
</tbody>
</table>

\[ \text{o.k.} \]
**Check bolt bearing**

Try\( PL \times 20 \text{ in.}, L_e \geq 2 \frac{1}{4} \text{ in.} \)

\[ \phi r_e = (91.4 \text{ kips/bolt})(\frac{3}{8} \text{ in.}) = 34.3 \text{ kips/bolt} \]

Since this is greater than the single-shear strength of one bolt, bolt bearing is not critical.

**Check bolt bearing**

Try\( PL \times 20 \text{ in.}, L_e \geq 2 \frac{1}{4} \text{ in.} \)

\[ r_e / \Omega = (60.9 \text{ kips/bolt})(\frac{3}{8} \text{ in.}) = 22.8 \text{ kips/bolt} \]

Since this is greater than the single-shear strength of one bolt, bolt bearing is not critical.

**Check flexure in the bracket plate**

On line K, the required strength \( M_a \) is

\[ M_a = P_a e_b = (54 \text{ kips})(12 \text{ in.} - 2\frac{1}{4} \text{ in.}) \]

\[ = 500 \text{ in.-kips} \]

For flexural yielding on line K,

\[ \phi = 0.90 \]

\[ \phi M_a = \phi F_y Z_s \]

\[ = 0.90(36 \text{ ksi})(\frac{\frac{3}{8} \text{ in.}}{4}\left(20 \text{ in.}\right)^2) \]

\[ = 1220 \text{ in.-kips} > 500 \text{ in.-kips} \quad \text{o.k.} \]

For flexural rupture on line K,

\[ Z_{net} = \frac{1}{6} \text{ in.} \left[ \frac{(20 \text{ in.})^2}{4} - 2(1.5 + 4.5 + 7.5)(1 \text{ in.}) \right] \]

\[ = 27.4 \text{ in.}^3 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi M_a = \phi F_y Z_{net} = 0.75(58 \text{ ksi})(27.4 \text{ in.}^3) )</td>
<td>( M_a / \Omega = \frac{F_y Z_{net}}{\Omega} = \frac{(58 \text{ ksi})(27.4 \text{ in.}^3)}{2.00} )</td>
</tr>
<tr>
<td>= 1190 \text{ in.-kips} &gt; 500 \text{ in.-kips} \quad \text{o.k.}</td>
<td>= 795 \text{ in.-kips} &gt; 333 \text{ in.-kips} \quad \text{o.k.}</td>
</tr>
</tbody>
</table>
For flexural yielding on the free edge of the triangular plate,
\[
z = 1.39 - 2.2\left(\frac{b}{a}\right) + 1.27\left(\frac{b}{a}\right)^2 - 0.25\left(\frac{b}{a}\right)^3
\]
\[
= 1.39 - 2.2\left(\frac{15.75}{20}\right) + 1.27\left(\frac{15.75}{20}\right)^2 - 0.25\left(\frac{15.75}{20}\right)^3 = 0.340
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi = 0.90)</td>
<td>(\Omega = 1.67)</td>
</tr>
<tr>
<td>(\phi P_a = \phi F_s z b t)</td>
<td>(P_a / \Omega = \frac{F_s z b t}{\Omega})</td>
</tr>
<tr>
<td>(= 0.90(36 \text{ ksi})(0.340)(15.25 \text{ in.})(0.375 \text{ in.}))</td>
<td>(= \frac{(36 \text{ ksi})(0.286)(15.25 \text{ in.})(0.375 \text{ in.})}{1.67})</td>
</tr>
<tr>
<td>(= 63 \text{ kips} &gt; 54.0 \text{ kips} \quad \text{\textit{o.k.}})</td>
<td>(= 41.9 \text{ kips} &gt; 36.0 \text{ kips} \quad \text{\textit{o.k.}})</td>
</tr>
</tbody>
</table>

Check local buckling of the bracket plate

\[
b = 15.75 \text{ in.} = 0.763 < 1.0
\]

Since \(0.5 \leq \frac{b}{a} < 1.0\),

\[
t_{\text{min}} = b\left(\frac{\sqrt{F_s}}{250}\right) = (15.75 \text{ in.})\left(\frac{\sqrt{36 \text{ ksi}}}{250}\right) = 0.366 \text{ in.} < \frac{3}{8} \text{ in.} \quad \text{o.k.}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check shear yielding of the bracket plate</td>
<td>Check shear yielding of the bracket plate</td>
</tr>
<tr>
<td>(\phi = 1.00)</td>
<td>(\Omega = 1.50)</td>
</tr>
<tr>
<td>(\phi R_s = \phi \left(0.6 F_s\right) A_g)</td>
<td>(R_s / \Omega = \frac{\left(0.6 F_s\right) A_g}{\Omega})</td>
</tr>
<tr>
<td>(= 1.0(0.6)(36 \text{ ksi})(20 \text{ in.})(\frac{3}{8} \text{ in.}))</td>
<td>(= \frac{0.6(36 \text{ ksi})(20 \text{ in.})(\frac{3}{8} \text{ in.})}{1.50})</td>
</tr>
<tr>
<td>(= 162 \text{ kips} &gt; 54.0 \text{ kips} \quad \text{\textit{o.k.}})</td>
<td>(= 108 \text{ kips} &gt; 36.0 \text{ kips} \quad \text{\textit{o.k.}})</td>
</tr>
</tbody>
</table>

Check shear rupture of the bracket plate

\[
\phi = 0.75
\]

\[
= 0.75(0.6)(58 \text{ ksi})\left[20 \text{ in.} - 6(1 \text{ in.})\right]\left(\frac{3}{8} \text{ in.}\right)
\]

\[
= 137 \text{ kips} > 54.0 \text{ kips} \quad \text{\textit{o.k.}}
\]

Check shear rupture of the bracket plate

\[
\Omega = 2.00
\]

\[
= \frac{0.6(58 \text{ ksi})\left[20 \text{ in.} - 6(1 \text{ in.})\right]\left(\frac{3}{8} \text{ in.}\right)}{2.00}
\]

\[
= 91.4 \text{ kips} > 36.0 \text{ kips} \quad \text{\textit{o.k.}}
\]
Check block shear rupture of the bracket plate (shear plane on line K, tension plane across bottom two bolts)

Since the eccentricity reduces the 12 bolts to 4.53 effective, the equivalent block shear rupture force applied concentric to the bolt group is

\[
R_u = (54.0 \text{ kips})(12/4.53) = 143 \text{ kips.}
\]

\[
R_u = (36.0 \text{ kips})(12/4.53) = 95.4 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u )</td>
<td>( R_u )</td>
</tr>
<tr>
<td>(54.0 kips)(12/4.53) = 143 kips</td>
<td>(36.0 kips)(12/4.53) = 95.4 kips</td>
</tr>
</tbody>
</table>

\[
A_{ss} = \left[ 5 \frac{\sqrt{2}}{2} \text{ in.} + 2 \frac{\sqrt{2}}{2} \text{ in.} - 1 \frac{\sqrt{2}}{2} \text{ in.} \right] \left( \frac{\sqrt{2}}{2} \text{ in.} \right) = 2.34 \text{ in.}^2
\]

\[
A_{sv} = (20 \text{ in.} - 2 \frac{\sqrt{2}}{2} \text{ in.}) \cdot \left( \frac{\sqrt{2}}{2} \text{ in.} \right) = 6.56 \text{ in.}^2
\]

\[
A_{sv} = 6.56 \text{ in.}^2 - 5.5(1 \text{ in.}) \left( \frac{\sqrt{2}}{2} \text{ in.} \right) = 4.50 \text{ in.}^2
\]

Compare

\[
0.6F_u A_{ss} = 0.6(58 \text{ ksi})(4.50 \text{ in.}^2) = 157 \text{ kips}
\]

\[
0.6F_u A_{sv} = 0.6(36 \text{ ksi})(6.56 \text{ in.}^2) = 142 \text{ kips}
\]

Shear on the gross area controls, thus;

The connection has two vertical rows of bolts, therefore \( U_{br} = 0.5 \)

\[
R_u = 0.6F_u A_{sv} + U_{br} F_u A_{ss} = 142 \text{ kips} + 0.5(58 \text{ ksi}) \left( 2.34 \text{ in.}^2 \right) = 210 \text{ kips}
\]

\[
\phi R_u = 0.75(210 \text{ kips}) = 157 \text{ kips}
\]

\[
R_u / \Omega = \frac{210 \text{ kips}}{2.00} = 105 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
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</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u )</td>
<td>( R_u / \Omega )</td>
</tr>
<tr>
<td>0.75(210 kips) = 157 kips</td>
<td>( \frac{210 \text{ kips}}{2.00} = 105 \text{ kips} )</td>
</tr>
</tbody>
</table>

157 kips > 143 kips \text{ o.k.} \quad 105 kips > 95.4 kips \text{ o.k.}
Example II.A-23   Welded Bracket Plate Design.

Given:

Design a welded bracket plate, using 70 ksi electrodes, to support the following loads:

\[ P_D = 9 \text{ kips} \]
\[ P_L = 27 \text{ kips} \]

Material Properties:

Plate Material: ASTM A36

\[ F_y = 36 \text{ ksi} \]
\[ F_u = 58 \text{ ksi} \]

Solution:

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
R_u = 1.2(9 \text{ kips}) + 1.6(27 \text{ kips}) = 54 \text{ kips} & R_u = 9 \text{ kips} + 27 \text{ kips} = 36 \text{ kips} \\
\hline
\end{array}
\]

Assume \( PL \frac{1}{2} \text{ in.} \times 18 \text{ in.} \)

Try a C-shaped weld with \( kl = 3 \text{ in.} \) and \( l = 18 \text{ in.} \)

\[ k = kl/l = 3 \text{ in.} / 18 \text{ in.} = 0.167 \]

\[ x = 0.0221 \]

and

\[ al + xl = 11\frac{1}{4} \text{ in.} \]

\[ a(18 \text{ in.}) + (0.0221)(18 \text{ in.}) = 11\frac{1}{4} \text{ in.} \]
\( a = 0.603 \)

Interpolate using \( \theta = 0^\circ, k = 0.167 \), and \( a = 0.603 \)

\( C = 1.47 \)

\( C_T = 1.0 \) for E70XX electrode.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{req} = \frac{P_a}{\phi CC/l} = \frac{54 \text{ kips}}{0.75(1.47)(1.0)(18 \text{ in.})} = 2.72 \rightarrow 3 \text{ sixteenths} &lt; \frac{1}{2} \text{ in.} - \frac{1}{16} \text{ in.} = \frac{1}{16} \text{ o.k.} )</td>
<td>( D_{req} = \frac{P_a\Omega}{CC/l} = \frac{(36 \text{ kips})(2.00)}{(1.47)(1.0)(18 \text{ in.})} = 2.72 \rightarrow 3 \text{ sixteenths} &lt; \frac{1}{2} \text{ in.} - \frac{1}{16} \text{ in.} = \frac{1}{16} \text{ o.k.} )</td>
</tr>
<tr>
<td>Use a ( \frac{1}{16} )-in. fillet weld.</td>
<td>Use a ( \frac{1}{16} )-in. fillet weld.</td>
</tr>
<tr>
<td>( D_{min} = 3 \text{ o.k.} )</td>
<td>( D_{min} = 3 \text{ o.k.} )</td>
</tr>
</tbody>
</table>

Check the flexural strength of the bracket plate

Conservatively taking the required moment strength of the plate as equal to the moment strength of the weld group,

\[ M_u = P_a(al) = (54 \text{ kips})(0.603)(18 \text{ in.}) = 586 \text{ in.-kips} \]

\[ M_u = P_a(al) = (36 \text{ kips})(0.603)(18 \text{ in.}) = 391 \text{ in.-kips} \]

For flexural yielding of the plate,

\[ M_a = F_z Z_x = (36 \text{ kips}) \left( \frac{\frac{1}{2} \text{ in.}}{4} \right)^2 = 1,458 \text{ in.-kips} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(1.458 in.-kips) ( \frac{4}{\Omega} ) = 1,312 in.-kips</td>
<td>Manual Part 15</td>
</tr>
<tr>
<td>( \phi M_a = 0.90(1,458 \text{ in.-kips}) = 1,312 \text{ in.-kips} )</td>
<td>( M_a / \Omega = \frac{(1,458 \text{ in.-kips})}{1.67} = 873 \text{ in.-kips} )</td>
</tr>
</tbody>
</table>

\( 1,312 \text{ in.-kips} > 586 \text{ in.-kips} \text{ o.k.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{b}{a} = \frac{14}{18} \text{ in.} = 0.820 \text{ Manual Part 15} )</td>
<td>( \frac{P_a}{\phi} = \frac{F_z z b t}{(36 \text{ kips})(0.302)(14 \frac{1}{4} \text{ in.})(\frac{1}{2} \text{ in.})} = 80.2 \text{ kips} )</td>
</tr>
<tr>
<td>( z = 1.39 - 2.2 \left( \frac{b}{a} \right) + 1.27 \left( \frac{b}{a} \right)^2 - 0.25 \left( \frac{b}{a} \right)^3 = 1.39 - 2.2(0.820) + 1.27(0.820)^2 - 0.25(0.820)^3 = 0.302 )</td>
<td></td>
</tr>
</tbody>
</table>
Check local buckling of the bracket plate

Since \( 0.5 \leq \frac{b}{a} < 1.0 \)

\[
t_{\text{min}} = b \left( \frac{\sqrt{F_y}}{250} \right) = (14 \frac{\text{in.}}{\sqrt{ksi}}) \left( \frac{\sqrt{36 \text{ ksi}}}{250} \right) = 0.354 \text{ in.} < \frac{1}{2} \text{ in.} \quad \text{o.k.}
\]

Check shear yielding of the bracket plate

\[
R_s = 0.6 F_y A_g = 0.6 (36 \text{ ksi}) (18 \text{ in.}) (\frac{\sqrt{ksi}}{\sqrt{ksi}}) = 194 \text{ kips}
\]

### Tables

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_a = 0.90(80.2 \text{ kips}) = 72.2 \text{ kips} )</td>
<td>( \frac{P_a}{\Omega} = \frac{80.2 \text{ kips}}{1.67} = 48.0 \text{ kips} )</td>
</tr>
<tr>
<td>72.2 kips &gt; 54.0 kips ( \text{o.k.} )</td>
<td>48.0 kips &gt; 36.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_y = 1.00(194 \text{ kips}) = 194 \text{ kips} )</td>
<td>( \frac{P_y}{\Omega} = \frac{194 \text{ kips}}{1.50} = 130 \text{ kips} &gt; 36.0 \text{ kips} )</td>
</tr>
<tr>
<td>194 kips &gt; 54.0 kips ( \text{o.k.} )</td>
<td>130 kips &gt; 36.0 kips ( \text{o.k.} )</td>
</tr>
</tbody>
</table>
Example II.A-24   Eccentrically-Loaded Bolt Group (IC method)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the bolts using the instantaneous center of rotation method. Use \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts in standard holes. Assume that bolt shear controls over bearing.

Solution A:

Assume the load is vertical \( (\theta = 0^\circ) \) as illustrated below

With \( \theta = 0^\circ \), with \( s = 3 \text{ in.} \), \( e = 16 \text{ in.} \), and \( n = 6 \):

\[
C = 3.55
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_n = 21.6 \text{ kips} )</td>
<td>( r_n / \Omega = 14.4 \text{ kips} )</td>
</tr>
<tr>
<td>( \phi R_n = C\phi r_n )</td>
<td>( R_n / \Omega = C\left(\frac{r_n}{\Omega}\right) )</td>
</tr>
<tr>
<td>( = 3.55(21.6 \text{ kips}) )</td>
<td>( = 3.55(14.4 \text{ kips}) )</td>
</tr>
<tr>
<td>( = 76.7 \text{ kips} )</td>
<td>( = 51.1 \text{ kips} )</td>
</tr>
</tbody>
</table>

Thus, \( P_u \) must be less than or equal to 76.7 kips.

Note: The eccentricity of the load significantly reduces the shear strength of the bolt group.
Solution B:
Assume the load acts at an angle of $75^\circ$ with respect to vertical ($\theta = 75^\circ$) as illustrated below.

With $\theta = 75^\circ$, $s = 3$ in., $e = 16$ in., and $n = 6$:

$C = 7.90$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = C\phi r_n = 7.90(21.6 \text{ kips}) = 171 \text{ kips}$</td>
<td>$R_u / \Omega = C\phi r_n / \Omega = 7.90(14.4 \text{ kips}) = 114 \text{ kips}$</td>
</tr>
<tr>
<td>Thus, $P_u$ must be less than or equal to 171 kips.</td>
<td>Thus, $P_u$ must be less than or equal to 114 kips.</td>
</tr>
</tbody>
</table>

Manual Table 7-9
Example II.A-25  Eccentrically Loaded Bolt Group (elastic method)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the bolts using the elastic method for $\theta = 0^\circ$. Compare the result with that of the previous example. Use $\frac{3}{8}$ in. diameter ASTM A325-N bolts in standard holes. Assume that bolt shear controls over bearing. $I_p = 406$ in.$^4$ per in.$^2$

Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct shear force per bolt</strong></td>
<td><strong>Direct shear force per bolt</strong></td>
</tr>
<tr>
<td>$r_{px} = 0, \ r_{py} = \frac{P_u}{n} = \frac{P_u}{12}$</td>
<td>$r_{px} = 0, \ r_{py} = \frac{P_u}{n} = \frac{P_u}{12}$</td>
</tr>
<tr>
<td><strong>Additional shear force due to eccentricity</strong></td>
<td><strong>Additional shear force due to eccentricity</strong></td>
</tr>
<tr>
<td>$r_{mx} = \frac{P_u ec_x}{I_p} = \frac{P_u (16 \text{ in.})(7\frac{1}{2}\text{-in.})}{406 \text{ in.}^4 \text{ per in.}^2}$</td>
<td>$r_{mx} = \frac{P_u ec_x}{I_p} = \frac{P_u (16 \text{ in.})(7\frac{1}{2}\text{-in.})}{406 \text{ in.}^4 \text{ per in.}^2}$</td>
</tr>
<tr>
<td>= 0.296 $P_u$</td>
<td>= 0.296 $P_u$</td>
</tr>
<tr>
<td>$r_{my} = \frac{P_u ec_y}{I_p} = \frac{P_u (16 \text{ in.}) \left(\frac{5\frac{1}{2}\text{-in.}}{2}\right)}{406 \text{ in.}^4 \text{ per in.}^2}$</td>
<td>$r_{my} = \frac{P_u ec_y}{I_p} = \frac{P_u (16 \text{ in.}) \left(\frac{5\frac{1}{2}\text{-in.}}{2}\right)}{406 \text{ in.}^4 \text{ per in.}^2}$</td>
</tr>
<tr>
<td>= 0.108 $P_u$</td>
<td>= 0.108 $P_u$</td>
</tr>
<tr>
<td>Resultant shear force</td>
<td>Resultant shear force</td>
</tr>
<tr>
<td>-----------------------</td>
<td>-----------------------</td>
</tr>
<tr>
<td>$r_a = \sqrt{(r_{ux} + r_{uy})^2 + (r_{px} + r_{py})^2}$</td>
<td>$r_a = \sqrt{(r_{ux} + r_{uy})^2 + (r_{px} + r_{py})^2}$</td>
</tr>
<tr>
<td>$= \sqrt{(0 + 0.296P_u)^2 + \left(\frac{P}{12} + 0.108P_u\right)^2}$</td>
<td>$= \sqrt{(0 + 0.296P_a)^2 + \left(\frac{P}{12} + 0.108P_a\right)^2}$</td>
</tr>
<tr>
<td>$= 0.352P_u$</td>
<td>$= 0.352P_a$</td>
</tr>
</tbody>
</table>

Since $r_a$ must be less than or equal to the available strength,

$$P_u \leq \frac{\phi r_a}{0.352} = \frac{21.6 \text{ kips}}{0.352} = 61.3 \text{ kips}$$

$$P_a \leq \frac{r_a / \Omega}{0.352} = \frac{14.4 \text{ kips}}{0.352} = 40.9 \text{ kips}$$

Note: the elastic method, shown here, is more conservative than the instantaneous center of rotation method, shown in Example II.A-24a.
Example II.A-26  Eccentrically-Loaded Weld Group (IC method)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the weld group, using the instantaneous center of rotation method. Use a $\frac{3}{16}$-in. fillet weld and 70 ksi electrode.

Solution A. Assume that the load is vertical ($\theta = 0^\circ$) as illustrated below.

\[ l = 10 \text{ in.}, \quad k_l = 5 \text{ in.}, \text{ therefore } k = 0.5 \]

With $\theta = 0^\circ$, $x = 0.125$

\[ xl + al = 10 \text{ in.} \]

\[ 0.125(10 \text{ in.}) + a(10 \text{ in.}) = 10 \text{ in.} \]

\[ a = 0.875 \]

By interpolation, $C = 1.88$

\[
\begin{array}{c|c|c}
\text{LRFD} & \text{ASD} \\
\hline
\phi R_u = \phi CC_i DL & R_u / \Omega = \frac{CC_i DL}{\Omega} \\
0.75(1.88)(1.0)(6 \text{ sixteenths})(10 \text{ in.}) & 1.88(1.0)(6 \text{ sixteenths})(10 \text{ in.}) \\
= 84.6 \text{ kips} & \frac{84.6}{2.00} = 56.4 \text{ kips} \\
\hline
\end{array}
\]

Thus, $P_u$ must be less than or equal to 84.6 kips.

Note: The eccentricity of the load significantly reduces the shear strength of this weld group as compared to the concentrically loaded case.
Solution B. Assume that the load acts at an angle of 75° with respect to vertical (\( \theta = 75° \)) as illustrated below.

As determined in solution a,

\[ k = 0.5 \] and \[ a = 0.875, \]

By interpolation, with \( \theta = 75° \)

\[ C = 3.45 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi CDl )</td>
<td>( R_u / \Omega = \frac{CC_Dl}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(3.45)(1.0)(6 sixteenths)(10 in.)</td>
<td>= \frac{3.45(1.0)(6 \text{ sixteenths})(10 \text{ in.})}{2.00}</td>
</tr>
<tr>
<td>= 155 kips</td>
<td>= 103 kips</td>
</tr>
</tbody>
</table>

Thus, \( P_u \) must be less than or equal to 155 kips.

Thus, \( P_a \) must be less than or equal to 103 kips.
Example II.A-27  Eccentrically-Loaded Weld Group (elastic method)

Given:

Determine the largest eccentric force that can be supported by the available shear strength of the welds in the connection, using the elastic method. Compare the result with that of the previous example. Use \( \frac{3}{8} \) in. fillet welds made with E70 electrodes. \( I_p = 385 \text{ in.}^4 \text{ per in.}^2 \)

Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Direct shear force per inch of weld</strong></td>
<td><strong>Direct shear force per inch of weld</strong></td>
</tr>
<tr>
<td>( r_{px} = 0, \ r_{py} = \frac{P_a}{l} = \frac{P_u}{20} )</td>
<td>( r_{px} = 0, \ r_{py} = \frac{P_a}{l} = \frac{P_u}{20} )</td>
</tr>
<tr>
<td>( r_{mx} = \frac{P_u ec_y}{I_p} = \frac{P_u (8.75 \text{ in.})(5 \text{ in.})}{385 \text{ in.}^4 \text{ per in.}^2} = 0.114 P_u )</td>
<td>( r_{mx} = \frac{P_u ec_y}{I_p} = \frac{P_u (8.75 \text{ in.})(5 \text{ in.})}{385 \text{ in.}^4 \text{ per in.}^2} = 0.114 P_u )</td>
</tr>
<tr>
<td>( r_{my} = \frac{P_u ec_x}{I_p} = \frac{P_u (8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4 \text{ per in.}^2} = 0.0852 P_u )</td>
<td>( r_{my} = \frac{P_u ec_x}{I_p} = \frac{P_u (8.75 \text{ in.})(3.75 \text{ in.})}{385 \text{ in.}^4 \text{ per in.}^2} = 0.0852 P_u )</td>
</tr>
<tr>
<td><strong>Resultant shear force</strong></td>
<td><strong>Resultant shear force</strong></td>
</tr>
<tr>
<td>( r_u = \sqrt{(r_{px} + r_{mx})^2 + (r_{py} + r_{my})^2} )</td>
<td>( r_u = \sqrt{(r_{px} + r_{mx})^2 + (r_{py} + r_{my})^2} )</td>
</tr>
<tr>
<td>( = \sqrt{(0 + 0.114 P_u)^2 + \left( \frac{P_u}{20} + 0.0852 P_u \right)^2} )</td>
<td>( = \sqrt{(0 + 0.114 P_u)^2 + \left( \frac{P_u}{20} + 0.0852 P_u \right)^2} )</td>
</tr>
<tr>
<td>( = 0.177 P_u )</td>
<td>( = 0.177 P_u )</td>
</tr>
</tbody>
</table>

Manual Part 8
Since \( r_u \) must be less than or equal to the available strength,

\[
P_u \leq \frac{\phi r_u}{0.177} = \frac{1.392(6 \text{ sixteenths})}{0.177} = 47.2 \text{ kips}
\]

Since \( r_a \) must be less than or equal to the available strength,

\[
P_a \leq \frac{r_a / \Omega}{0.177} = \frac{0.928(6 \text{ sixteenths})}{0.177} = 31.5 \text{ kips}
\]

Note: The strength of the weld group predicted by the elastic method, as shown here, is significantly less than the predicted by the instantaneous center of rotation method in Example II.A-26a.
Example II.A-28  All-Bolted Single-Angle Connection (beam-to-girder web)

Given:

Design an all-bolted single-angle connection (case I in Table 10-10) between a W18×35 beam and a W21×62 girder-web, to support the following beam end reactions:

\[ R_D = 6.5 \text{ kips} \]
\[ R_L = 20 \text{ kips} \]

Top flange coped 2 in. deep by 4 in. long, \( L_{ve} = 1\frac{1}{2} \text{ in.} \), \( L_{ch} = 1\frac{1}{2} \text{ in.} \) (assumed to be 1¼ in. for calculation purposes to account for possible underrun in beam length),

Use ¾-in. diameter A325-N bolts in standard holes.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Specification</th>
<th>( F_y )</th>
<th>( F_u )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×35</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td>Manual Table 2-3</td>
</tr>
<tr>
<td>Girder W21×62</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
<td></td>
</tr>
<tr>
<td>Angle</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
<td></td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>( t_w )</th>
<th>( d )</th>
<th>( t_f )</th>
<th>( e )</th>
<th>( h_0 )</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam W18×35</td>
<td>0.300 in.</td>
<td>17.7 in.</td>
<td>0.425 in.</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Cope</td>
<td>4 in.</td>
<td>2 in.</td>
<td>4½-in.</td>
<td>15.7 in.</td>
<td>Manual Table 1-1</td>
<td></td>
</tr>
<tr>
<td>Girder W21×62</td>
<td>0.400 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Solution:</strong></td>
<td><strong>Design the bolts and angle</strong></td>
</tr>
<tr>
<td>Since half-web dimension of W18×35 is less than ¼ in., the tabular values may conservatively be used. Bolt shear is more critical than bolt bearing in this example; thus, ( r_v = 1.59 \text{ kips} ).</td>
<td>Since half-web dimension of W18×35 is less than ¼ in., the tabular values may conservatively be used. Bolt shear is more critical than bolt bearing in this example; thus, ( r_v / \Omega = 10.6 \text{ kips} ).</td>
</tr>
<tr>
<td>[ C_{\text{min}} = \frac{R_u}{r_v} = \frac{40 \text{ kips}}{15.9 \text{ kips/bolt}} = 2.52 ]</td>
<td>[ C_{\text{min}} = \frac{R_u}{r_v / \Omega} = \frac{26.5 \text{ kips}}{10.6 \text{ kips/bolt}} = 2.52 ]</td>
</tr>
<tr>
<td>Try a four-bolt connection with a ¾-in. thick angle.</td>
<td>Try a four-bolt connection with a ¾-in. thick angle.</td>
</tr>
<tr>
<td>[ C = 3.07 &gt; 2.52 \quad \text{O.K.} ]</td>
<td>[ C = 3.07 &gt; 2.52 \quad \text{O.K.} ]</td>
</tr>
<tr>
<td>The 3-in. leg will be shop bolted to the girder web and the 4-in. leg will be field bolted to the beam web.</td>
<td>The 3-in. leg will be shop bolted to the girder web and the 4-in. leg will be field bolted to the beam web.</td>
</tr>
</tbody>
</table>

**Check shear yielding of the angle**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi (0.6 F_y A_g) )</td>
<td>( R_u / \Omega = 0.6 F_y A_g / \Omega )</td>
</tr>
<tr>
<td>( = 1.0(0.6)(36 \text{ ksi})(11\frac{1}{2} \text{ in.})^{(\frac{3}{8}-\text{in.})} )</td>
<td>( = 0.6(36 \text{ ksi})(11\frac{1}{2} \text{ in.})^{(\frac{3}{8}-\text{in.})} / 1.50 )</td>
</tr>
<tr>
<td>( = 93.2 \text{ kips} &gt; 40 \text{ kips} \quad \text{O.K.} )</td>
<td>( = 62.1 \text{ kips} &gt; 26.5 \text{ kips} \quad \text{O.K.} )</td>
</tr>
</tbody>
</table>

**Check shear rupture of the angle**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi (0.6 F_u A_h) )</td>
<td>( R_u / \Omega = (0.6 F_u A_h) / \Omega )</td>
</tr>
<tr>
<td>( = 0.75 {0.6(58 \text{ ksi})[(11\frac{1}{2} \text{ in.})^{(\frac{3}{8}-\text{in.})} - 4(\frac{3}{8}-\text{in.})(\frac{3}{8}-\text{in.})]} )</td>
<td>( = 0.6(58 \text{ ksi})[(11\frac{1}{2} \text{ in.})^{(\frac{3}{8}-\text{in.})} - 4(\frac{3}{8}-\text{in.})(\frac{3}{8}-\text{in.})] / 2.00 )</td>
</tr>
<tr>
<td>( = 78.3 \text{ kips} &gt; 40 \text{ kips} \quad \text{O.K.} )</td>
<td>( = 52.2 \text{ kips} &gt; 26.5 \text{ kips} \quad \text{O.K.} )</td>
</tr>
</tbody>
</table>

**Check block shear rupture of the angle**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>From the tables, with ( n = 4, L_{ev} = 1\frac{1}{2} \text{ in.} ) ( L_{eh} = 1\frac{3}{4} \text{ in.} ), ( \phi R_u = \phi F_y A_g U_{bx} + \min \left( \phi 0.6 F_y A_g, \phi F_u A_{nv} \right) )</td>
<td>From the tables, with ( n = 4, L_{ev} = 1\frac{1}{2} \text{ in.} ) ( L_{eh} = 1\frac{3}{4} \text{ in.} ), ( R_u / \Omega = F_y A_g U_{bx} / \Omega + \min \left( 0.6 F_y A_{gw}, F_u A_{mv} / \Omega \right) )</td>
</tr>
<tr>
<td>Tension rupture component ( \phi F_y A_g = 35.3 \text{ kips/in.}(\frac{3}{8}-\text{in.}) )</td>
<td>Tension rupture component ( F_y A_g / \Omega = 23.6 \text{ kips/in.}(\frac{3}{8}-\text{in.}) )</td>
</tr>
</tbody>
</table>

Manual Table 7-1

Section J4.2

Eqn J4-3

Manual Table 10-10

Section J4.2

Eqn J4-4

Manual Table 9-3a
Shear yielding component
\[ 0.6 F_y A_y = 170 \text{kips/in.}(\% \text{in.}) \]
Shear rupture component
\[ 0.6 F_y A_y = 194 \text{kips/in.}(\% \text{in.}) \]
Tension stress is uniform, therefore
\[ U_{bs} = 1.0 \]
\[ \phi R_u = (170 \text{kips/in.} + 35.3 \text{kips/in.})(\% \text{in.}) \]
\[ = 77.0 \text{kips} > 40 \text{kips} \quad \text{o.k.} \]

Check flexure of the support-leg of the angle
The required strength is
\[ M_u = R_u e = (40 \text{kips})(1\% \text{-in.} + 0.300 \text{in.}/2) \]
\[ = 76.0 \text{in.-kips} \]
For flexural yielding
\[ \phi = 0.90 \]
\[ \phi M_u = \phi F_y Z_x \]
\[ = 0.90(36 \text{ksi}) \left[ \frac{(\% \text{in.})(11\% \text{ in.})^2}{4} \right] \]
\[ = 402 \text{in.-kips} > 76.0 \text{in.-kips} \quad \text{o.k.} \]

For flexural rupture,
\[ Z_{net} \approx \frac{t}{4} \left( d - \frac{s^2 n (n^2 - 1)}{d} \left[ d + \frac{\gamma_0}{2} \right] \right) \]
\[ \approx \frac{\gamma_0}{4} \left[ (11\% \text{ in.})^2 - \frac{(3 \text{ in.})^2 (4^2 - 1)(\% \text{ in.} + \frac{\gamma_0}{2} \text{ in.})}{11\% \text{ in.}} \right] = 8.55 \text{ in.}^3 \]

\[ \phi M_u = \phi F_y Z_{net} = 0.75(58 \text{ksi})(8.55 \text{ in.}^3) \]
\[ = 372 \text{ in.-kips} > 76.0 \text{ in.-kips} \quad \text{o.k.} \]
Check beam web for bolt bearing and block shear rupture.

\[ n = 4, L_{ev} = 1\frac{1}{2} \text{ in.}, \text{ and } L_{vb} = 1\frac{1}{2} \text{ in.} \]

(Assumed to be 1¼ in. for calculation purposes to provide for possible underrun in beam length),

\[ \phi R_n = (257 \text{ kips/in.})(0.300 \text{ in.}) \]

\[ = 77.1 \text{ kips} \geq 40 \text{ kips} \quad \text{O.K.} \]

Check beam web for bolt bearing and block shear rupture.

\[ n = 4, L_{ev} = 1\frac{1}{2} \text{ in.}, \text{ and } L_{vb} = 1\frac{1}{2} \text{ in.} \]

(Assumed to be 1¼ in. for calculation purposes to provide for possible underrun in beam length),

\[ R_n / \Omega = (171 \text{ kips/in.})(0.300 \text{ in.}) \]

\[ = 51.3 \text{ kips} \geq 26.5 \text{ kips} \quad \text{O.K.} \]
Example II.A-29  Bolted/Welded Single-Angle Connection (beam-to-column flange).

Given:

Design a single-angle connection between a W16×50 beam and a W14×90 column flange to support the following beam end reactions:

\[ R_D = 9 \text{kips} \]
\[ R_L = 27 \text{kips} \]

Use ¾ in. diameter ASTM A325-N bolts to connect the supported beam to the single angle. Use 70 ksi electrode welds to connect the single angle to the column flange.

Material Properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>Material</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×50</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>W14×90</td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>Angle</td>
<td>ASTM A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>( t_w )</th>
<th>( d )</th>
<th>( t_f )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W16×50</td>
<td>0.380 in.</td>
<td>16.3 in.</td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>0.710</td>
<td>Manual</td>
</tr>
</tbody>
</table>

Solution:

\[ R_u = 1.2 (9 \text{kips}) + 1.6 (27 \text{kips}) = 54 \text{kips} \]
\[ R_u = 9 \text{kips} + 27 \text{kips} = 36 \text{kips} \]

**Design single angle, bolts, and welds**

Since the half-web dimension of a W16×50 is less than ¼ in., the tabulated values may conservatively be used.

Try a four bolt single angle (L4×3×¾).

\[ \phi R_u = 63.6 \text{kips} > 54 \text{kips} \quad \text{O.K.} \]

\[ R_u / \Omega = 42.4 \text{kips} > 36 \text{kips} \quad \text{O.K.} \]
Also with a $\frac{3}{8}$-in. fillet weld size

$$\phi R_u = 56.6 \text{ kips} > 54 \text{ kips} \quad \text{o.k.}$$

Use four-bolt single angle L4×3×$\frac{3}{8}$. The 3-in. leg will be shop welded to the column flange and the 4 in. leg will be field bolted to the beam web.

*Check supported beam web*

The bearing strength of the beam web per bolt is

$$s = 3 \text{ in., } \frac{3}{4} \text{ in. diameter bolts, standard holes}$$

$$R_u = \phi r \tau n$$

$$= (87.8 \text{ kips/in.})(0.380 \text{ in.})(4 \text{ bolts})$$

$$= 133 \text{ kips} > 54 \text{ kips} \quad \text{o.k.}$$

*Check support*

Minimum support thickness for the $\frac{3}{8}$-in. welds is 0.286 in.

$$t_w = 0.710 \text{ in.} > 0.286 \text{ in.} \quad \text{o.k.}$$

Also with a $\frac{3}{8}$-in. fillet weld size

$$R_u/\Omega = 37.7 \text{ kips} > 36 \text{ kips} \quad \text{o.k.}$$

Use four-bolt single angle L4×3×$\frac{3}{8}$. The 3-in. leg will be shop welded to the column flange and the 4 in. leg will be field bolted to the beam web.

*Check supported beam web*

The bearing strength of the beam web per bolt is

$$s = 3 \text{ in., } \frac{3}{4} \text{ in. diameter bolts, standard holes}$$

$$R_u/\Omega = \frac{r_f \tau n}{\Omega}$$

$$= (58.5 \text{ kips/in.})(0.380 \text{ in.})(4 \text{ bolts})$$

$$= 88.9 \text{ kips} > 36 \text{ kips} \quad \text{o.k.}$$

*Check support*

Minimum support thickness for the $\frac{3}{8}$-in. welds is 0.286 in.

$$t_w = 0.710 \text{ in.} > 0.286 \text{ in.} \quad \text{o.k.}$$
Example II.A-30  All-Bolted Tee Connection (beam-to-column flange)

Given:

Design an all-bolted tee connection between a W16×50 beam and a W14×90 column flange to support the following beam end reactions:

\[ R_D = 9 \text{ kips} \]
\[ R_L = 27 \text{ kips} \]

Use ¾ in. diameter ASTM A325-N bolts in standard holes. Try a WT5×22.5 with a four-bolt connection.

Material Properties:

<table>
<thead>
<tr>
<th>Section</th>
<th>Width × Depth</th>
<th>Material</th>
<th>$F_y$</th>
<th>$F_u$</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×50</td>
<td></td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>W14×90</td>
<td></td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
<tr>
<td>WT5×22.5</td>
<td></td>
<td>ASTM A992</td>
<td>50 ksi</td>
<td>65 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Width × Depth</th>
<th>Width</th>
<th>Depth</th>
<th>Thickness</th>
<th>Flange</th>
<th>Tabbing</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W16×50</td>
<td>0.380 in.</td>
<td>16.3 in.</td>
<td>0.630 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column</td>
<td>W14×90</td>
<td>0.710 in.</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Tee</td>
<td>WT5×22.5</td>
<td>5.050 in.</td>
<td></td>
<td>0.350 in.</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(see W10×45 Manual Table 1-1)
Solution:

| $R_u$ | 1.2 (9 kips) + 1.6 (27 kips) = 54 kips | $R_u$ = 9 kips + 27 kips = 36 kips |

Check limitation on tee stem thickness

See Rotational Ductility discussion at the beginning of the Manual Part 9

$$t_{s\text{ max}} = \frac{d_b}{2} + \frac{\gamma_{th}}{2} \text{ in.} = \frac{\gamma_{th}}{2} \text{ in.} = 0.438 \text{ in.} > 0.350 \text{ in.} \quad \text{o.k.}$$

Check limitation on bolt diameter for bolts through tee flange

Assuming a 5½ in. gage,

$$b = \text{flexible width in connection element}$$

$$b = \frac{g - 2k_i}{2} = 5\frac{1}{2} \text{ in.} - 2(\frac{\gamma_{th}}{2} \text{ in.}) = 1.94 \text{ in.}$$

$$d_{b\text{ min}} = 0.163f_v\sqrt{\frac{F_y}{b\frac{b^2}{L} + 2}} \leq 0.69\sqrt{t_s}$$

$$= 0.163(0.620 \text{ in.})\sqrt{\frac{50 \text{ ksi}}{1.94 \text{ in.} \left(\frac{1.94 \text{ in.}}{2}\right)^2 + 2}} \leq 0.69\sqrt{0.350 \text{ in.}}$$

$$= 0.731 \text{ in.} \leq 0.408 \text{ in.}$$

$$= 0.408 \text{ in.}$$

Since $d_b = \frac{\gamma_{th}}{2} \text{ in.} > d_{b\text{ min}} = 0.408 \text{ in.} \quad \text{o.k.}$

Check bolt group through beam web for shear and bearing

$a = d - L_{ch} = 5.05 \text{ in.} - 1\frac{1}{4} \text{ in.} = 3.80 \text{ in.}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Since bolt shear is more critical than bolt bearing in this example, $\phi r_n = 15.9 \text{ kips}$, Thus, $\phi R_u = n\phi r_n = (4 \text{ bolts})(15.9 \text{ kips}) = 63.6 \text{ kips} &gt; 54 \text{ kips} \quad \text{o.k.}$</td>
<td>Since bolt shear is more critical than bolt bearing in this example, $r_n/\Omega = 10.6 \text{ kips}$, Thus, $R_u/\Omega = nr_n/\Omega = (4 \text{ bolts})(10.6 \text{ kips}) = 42.4 \text{ kips} &gt; 36 \text{ kips} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>
Check shear yielding of the tee stem

\[ \phi R_n = \phi 0.6F_y A_g \]
\[ \phi = 1.00 \]
\[ = 1.00 \left[ 0.6 \left( 50 \text{ ksi} \right) \left( 1\frac{3}{4} \text{ in.} \right) \left( 0.350 \text{ in.} \right) \right] \]
\[ = 121 \text{ kips} > 54 \text{ kips} \quad \text{o.k.} \]

Check shear rupture of the tee stem

\[ \phi R_n = \phi 0.6F_u A_u \]

Where \( \phi = 0.75 \)
\[ = 0.75 \left( 0.6 \right) \left( 65 \text{ ksi} \right) \left[ 11\frac{3}{4} \text{ in.} - 4 \left( 0.875 \text{ in.} \right) \right] \left( 0.350 \text{ in.} \right) \]
\[ = 81.9 \text{ kips} > 54 \text{ kips} \quad \text{o.k.} \]

Check block shear rupture of the tee stem

\[ L_{eb} = L_{ev} = 1\frac{3}{4} \text{ in.}, \]

Thus,
\[ \phi R_n = \phi F_u A_u U_{bs} + \min \left( \phi 0.6F_y A_g, \phi F_u A_{uv} \right) \]

Tension rupture component

\[ \phi F_u A_u = 39.6 \text{ kips/in.} \left( 0.350 \text{ in.} \right) \]

Shear yielding component

\[ \phi 0.6F_y A_g = 231 \text{ kips/in.} \left( 0.350 \text{ in.} \right) \]

Shear rupture component

\[ \phi 0.6F_u A_v = 210 \text{ kips/in.} \left( 0.350 \text{ in.} \right) \]

\[ \phi R_n = (210 \text{ kips/in.} + 39.6 \text{ kips/in.}) \left( 0.350 \text{ in.} \right) \]
\[ = 87.4 \text{ kips} > 54 \text{ kips} \quad \text{o.k.} \]
Check bolt group through support for shear and bearing combined with tension due to eccentricity

The following calculation follows the Case II approach in the Section “Eccentricity Normal to the Plane of the Faying Surface” in the introduction of Part 7 of the Manual

Calculate tensile force per bolt $r_{at}$.

$$r_{at} = \frac{R_e e}{n'd_m}$$

$$r_{at} = \frac{54 \text{ kips} \cdot (5.05 \text{ in.} - 1\frac{3}{8} \text{ in.})}{4 \text{ bolts} \cdot 6 \text{ in.}}$$

$$= 8.55 \text{ kips/bolts}$$

Check design strength of bolts for tension-shear interaction

When threads are not excluded from the shear planes of ASTM A325 bolts

$$r_{av} = \frac{54 \text{ kips}}{8 \text{ bolts}}$$

$$= 6.75 \text{ kips/bolt} < 15.9 \text{ kips/bolt} \quad \text{o.k.}$$

$$F_{nt} = 90 \text{ ksi}, F_{nv} = 48 \text{ ksi}$$

$$F_{nt} ' = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{nv}} f_v \leq F_{nt}$$

$$\phi = 0.75$$

$$= 1.3(90 \text{ ksi}) - \left[ \frac{90 \text{ ksi}}{(0.75)(48 \text{ ksi})} \right]$$

$$= 78.8 \text{ ksi} < 90 \text{ ksi}$$

$$\phi R_n = F_{nt} ' A_b$$

$$= 0.75(78.8 \text{ ksi})(0.442 \text{ in}^2)$$

$$= 26.1 \text{ kips/bolt} > 8.55 \text{ kips/bolt} \quad \text{o.k.}$$

Calculate allowable strength of bolts for tension-shear interaction

When threads are not excluded from the shear planes of ASTM A325 bolts

$$r_{av} = \frac{36 \text{ kips}}{8 \text{ bolts}}$$

$$= 4.50 \text{ kips/bolt} < 10.6 \text{ kips/bolt} \quad \text{o.k.}$$

$$F_{nt} = 90 \text{ ksi}, F_{nv} = 48 \text{ ksi}$$

$$F_{nt} ' = 1.3F_{nt} - \frac{\Omega F_{nt}}{F_{nv}} f_v \leq F_{nt}$$

$$\Omega = 2.00$$

$$= 1.3(90 \text{ ksi}) - \left[ \frac{2.00(90 \text{ ksi})}{(48 \text{ ksi})} \right]$$

$$= 78.8 \text{ ksi} < 90 \text{ ksi}$$

$$R_n/\Omega = F_{nt} ' A_b/\Omega$$

$$= (78.8 \text{ ksi})(0.442 \text{ in}^2)/2.00$$

$$= 17.4 \text{ kips/bolt} > 5.7 \text{ kips/bolt} \quad \text{o.k.}$$
Check bearing strength at bolt holes

With \( L_e = 1\frac{1}{4} \) in. and \( s = 3 \) in., the bearing strength of the tee flange exceeds the single shear strength of the bolts. Therefore, bearing strength is o.k.

Check prying action

\[
b = \frac{(g - t_f)}{2} = \frac{(5\frac{5}{8} \text{ in.} - 0.350 \text{ in.})}{2} = 2.58 \text{ in.}
\]

\[
a = \frac{(b_f - g)}{2} = \frac{(8.02 \text{ in.} - 5\frac{5}{8} \text{ in.})}{2} = 1.26 \text{ in.}
\]

Since \( a = 1.26 \) in. is less than \( 1.25b = 3.23 \) in., use \( a = 1.26 \) in. for calculation purposes

\[
b' = b - \frac{d}{2} = 2.58 \text{ in.} - \left(\frac{\frac{5}{8} \text{ in.}}{2}\right) = 2.21 \text{ in.}
\]

\[
a' = a + \left(\frac{d}{2}\right) = 1.26 \text{ in.} + \left(\frac{\frac{5}{8} \text{ in.}}{2}\right) = 1.64 \text{ in.}
\]

\[
\rho = \frac{b'}{a'} = \frac{2.21 \text{ in.}}{1.64 \text{ in.}} = 1.35
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T = r_{wT} = 8.55 \text{ kips/bolt} )</td>
<td>( T = r_{wT} = 5.70 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( B = \phi r_{nT} = 26.1 \text{ kips/bolt} )</td>
<td>( B = r_{nT}/\Omega = 17.4 \text{ kips/bolt} )</td>
</tr>
<tr>
<td>( \beta = \frac{1}{\rho} \left( \frac{B}{T} - 1 \right) )</td>
<td>( \beta = \frac{1}{\rho} \left( \frac{B}{T} - 1 \right) )</td>
</tr>
<tr>
<td>( \frac{1}{1.35} \left( \frac{26.1 \text{ kips/bolts}}{8.55 \text{ kips/bolts}} - 1 \right) = 1.52 )</td>
<td>( \frac{1}{1.35} \left( \frac{17.4 \text{ kips/bolts}}{5.70 \text{ kips/bolts}} - 1 \right) = 1.52 )</td>
</tr>
</tbody>
</table>

Since \( \beta \geq 1 \), set \( \alpha' = 1.0 \)

\[
p = \frac{11\frac{1}{2}}{4} \text{ in.} = 2.88 \text{ in./bolt}
\]

\[
\delta = 1 - \frac{d'}{p} = 1 - \frac{1\frac{3}{16}}{2.88 \text{ in.}} = 0.718
\]
Similarly, checks of the tee flange for shear yielding, shear rupture, and block shear rupture will show that the tee flange is o.k.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual</th>
</tr>
</thead>
</table>
| \[
t_{\text{req}} = \frac{4.44 \nu_{\text{ut}} b'}{\sqrt{p F_u (1 + \delta \alpha')}}
\]
| \[
t_{\text{req}} = \frac{6.66 \nu_{\text{ut}} b'}{\sqrt{p F_u (1 + \delta \alpha')}}
\] | \[
= \frac{4.44(8.55 \text{ kips/bolt})(2.21 \text{ in.})}{\sqrt{(2.88 \text{ in./bolt})(65 \text{ ksi})[1 + (0.718)(1.0)]}}
\]
| \[
= 0.511 \text{ in.} < 0.620 \text{ in.} \quad \text{o.k.}
\] | \[
\]
| \[
= \frac{6.66(5.70 \text{ kips/bolt})(2.21 \text{ in.})}{\sqrt{(2.88 \text{ in./bolt})(65 \text{ ksi})[1 + (0.718)(1.0)]}}
\]
| \[
= 0.511 \text{ in.} < 0.620 \text{ in.} \quad \text{o.k.}
\] | \[
\]
Example II.A-31   Bolted/Welded Tee Connection (beam-to-column flange)

Given:

Design a tee connection bolted to a W16×50 supported beam and welded to a W14×90 supporting column flange, to support the following beam end reactions:

\[ R_D = 6 \text{ kips} \]
\[ R_L = 18 \text{ kips} \]

Use ¾ in. diameter ASTM A325-N bolts in standard holes and E70 electrode welds. Try a WT5×22.5 with four-bolts.

Material Properties:

- **W16×50** - ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

- **W14×90** - ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

- **WT5×22.5** - ASTM A992
  - \( F_y = 50 \text{ ksi} \)
  - \( F_u = 65 \text{ ksi} \)

Geometric Properties:

- **Beam** W16×50
  - \( t_w = 0.380 \text{ in.} \)
  - \( d = 16.3 \text{ in.} \)
  - \( t_f = 0.630 \text{ in.} \)

- **Column** W14×90
  - \( t_f = 0.710 \text{ in.} \)
  - \( b_y = 8.02 \text{ in.} \)
  - \( t_f = 0.620 \text{ in.} \)

- **Tee** WT5×22.5
  - \( d = 5.05 \text{ in.} \)
  - \( b_y = 8.02 \text{ in.} \)
  - \( t_f = 0.620 \text{ in.} \)

(see W10×45 Manual Table 1-1)

\[ k_1 = \frac{3}{16} \text{ in.} \]
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2 \times 6 \text{ kips} + 1.6 \times 18 \text{ kips} = 36 \text{ kips}$</td>
<td>$R_u = 6 \text{ kips} + 18 \text{ kips} = 24 \text{ kips}$</td>
</tr>
</tbody>
</table>

Check limitation on tee stem thickness

See Rotational Ductility discussion at the beginning of the Manual Part 9

$$t_{s,\text{max}} = \frac{d_b}{2} + \frac{\gamma_6}{2} \text{ in.} = \frac{\gamma_6}{2} \text{ in.} = 0.438 \text{ in.} > 0.350 \text{ in.} \quad \text{o.k.}$$

Design the welds connecting the tee flange to the column flange

$b = \text{flexible width in connection element}$

$$b = \frac{b_t - 2k_i}{2} = \frac{8.02 \text{ in.} - 2(0.56 \text{ in.})}{2} = 3.20 \text{ in.}$$

$$w_{\text{min}} = 0.0158 \frac{F_{t_t} b}{b} \left( \frac{b^2}{L^2} + 2 \right) \leq \gamma t_s$$

$$= 0.0158 \left[ \frac{(50 \text{ ksi})(0.620 \text{ in.})^2}{(3.20 \text{ in.})^2} \left( \frac{(3.20 \text{ in.})^2}{(11\frac{1}{2} \text{ in.})^2} + 2 \right) \leq (\gamma)(0.350 \text{ in.}) \right]$$

$$= 0.197 \text{ in.} \leq 0.212 \text{ in.}$$

Design welds for direct load only with no eccentricity

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Try $\frac{3}{8}$-in. fillet welds.</td>
<td>Try $\frac{3}{8}$-in. fillet welds.</td>
</tr>
<tr>
<td>$\phi R_u = 1.392 Dl$</td>
<td>$R_u/\Omega = 0.928 Dl$</td>
</tr>
<tr>
<td>$= 1.392 \times (4 \text{ sixteenths})(2 \text{ sides})(11\frac{1}{2} \text{ in.})$</td>
<td>$= 0.928 \times (4 \text{ sixteenths})(2 \text{ sides})(11\frac{1}{2} \text{ in.})$</td>
</tr>
<tr>
<td>$= 128 \text{ kips} &gt; 36 \text{ kips}$</td>
<td>$= 85.4 \text{ kips} &gt; 24 \text{ kips}$ \quad \text{o.k.}</td>
</tr>
<tr>
<td>Use $\frac{3}{8}$-in. fillet welds.</td>
<td>Use $\frac{3}{8}$-in. fillet welds.</td>
</tr>
</tbody>
</table>

Check the stem side of the connection

Since the connection is flexible at the support, the tee stem and bolts must be designed for eccentric shear, where the eccentricity, $e_b$, is

$$a = d - L_{ob} = 5.05 \text{ in.} - 1\frac{1}{4} \text{ in.} = 3.80 \text{ in.}$$

$$e_b = a = 3.80 \text{ in.}$$
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thus the tee stem and bolts must be designed for $R_u = 36$ kips and $R_ew = 137$ in.-kips.</td>
<td>Thus the tee stem and bolts must be designed for $R_u = 24$ kips and $R_ew = 91.2$ in.-kips.</td>
</tr>
<tr>
<td><strong>Check bolt group through beam web for shear and bearing</strong></td>
<td><strong>Check bolt group through beam web for shear and bearing</strong></td>
</tr>
<tr>
<td>For $\theta = 0^\circ$, with $s = 3$ in., $e_x = e_y = 3.80$ in., and $n = 4$ bolts,</td>
<td>For $\theta = 0^\circ$, with $s = 3$ in., $e_x = e_y = 3.80$ in., and $n = 4$ bolts,</td>
</tr>
<tr>
<td>$C = 2.45$</td>
<td>$C = 2.45$</td>
</tr>
<tr>
<td>and, since bolt shear is more critical than bolt bearing in this example,</td>
<td>and, since bolt shear is more critical than bolt bearing in this example,</td>
</tr>
<tr>
<td>$\phi R_u = C \phi r_u = 2.45 (15.9$ kips/bolt)</td>
<td>$R_u/\Omega = C r_u/\Omega = 2.45 (10.6$ kips/bolt)</td>
</tr>
<tr>
<td>$= 39.0$ kips $&gt; 36.0$ kips o.k.</td>
<td>$= 26.0$ kips $&gt; 24.0$ kips o.k.</td>
</tr>
<tr>
<td><strong>Check flexure on the tee stem</strong></td>
<td><strong>Check flexure on the tee stem</strong></td>
</tr>
<tr>
<td>For flexural yielding</td>
<td>For flexural yielding</td>
</tr>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi M_u = \phi F_u Z_x$</td>
<td>$M_u/\Omega = F_u Z_x/\Omega$</td>
</tr>
<tr>
<td>$= (0.9)(50 \text{ ksi}) \left( \frac{(0.350 \text{ in.})(11\frac{1}{2} \text{ in.})^2}{4} \right)$</td>
<td>$= (50 \text{ ksi}) \left( \frac{(0.350 \text{ in.})(11\frac{1}{2} \text{ in.})^2}{4} \right)/1.67$</td>
</tr>
<tr>
<td>$= 520$ in.-kips $&gt; 137$ in.-kips o.k.</td>
<td>$= 346$ in-kips $&gt; 91.2$ in.-kips o.k.</td>
</tr>
</tbody>
</table>

For flexural rupture,

$$Z_{net} \approx \frac{1}{4} \left[ d^2 - s^2 n \left( n^2 - 1 \right) \left( d_x + \frac{\gamma_x}{d_y} \text{ in.} \right) \right]$$

$$\approx \frac{0.350 \text{ in.}}{4} \left[ (11\frac{1}{2} \text{ in.})^2 - \left( \frac{3 \text{ in.}}{4} \right) \left( 4^2 - 1 \right) \left( \frac{\gamma_x}{d_y} \text{ in.} + \frac{\gamma_y}{d_y} \text{ in.} \right) \right] = 7.98 \text{ in.}^3$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi M_u = \phi F_u Z_{net}$</td>
<td>$M_u/\Omega = F_u Z_{net}/\Omega$</td>
</tr>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$= (0.75)(65 \text{ ksi})(7.98 \text{ in.}^3)$</td>
<td>$= (65 \text{ ksi})(7.98 \text{ in.}^3)/2.00$</td>
</tr>
<tr>
<td>$= 389$ in.-kips $&gt; 137$ in.-kips o.k.</td>
<td>$= 259$ in.-kips $&gt; 91.2$ in.-kips o.k.</td>
</tr>
</tbody>
</table>
Check shear yielding of the tee stem

\[ \phi R_a = \phi 0.6 F_y A_g \]

\[ \phi = 1.00 \]

\[ = 1.00(0.6)(50 \text{ ksi})(11\frac{3}{4} \text{ in.})(0.350 \text{ in.}) \]

\[ = 121 \text{ kips} > 36 \text{ kips} \quad \text{o.k.} \]

Check shear rupture of the tee stem

\[ \phi = 0.75 \]

\[ \phi R_a = 0.6 F_u A_w \]

\[ \phi R_a = 0.75(0.6)(65 \text{ ksi}) \]

\[ \times \left[11\frac{3}{4} - 4(0.875 \text{ in.})\right](0.350 \text{ in.}) \]

\[ = 81.9 \text{ kips} > 36 \text{ kips} \quad \text{o.k.} \]

Check block shear rupture of the tee stem

\[ L_{eh} = L_{ev} = 1\frac{1}{4} \text{ in.} \]

Thus.

\[ \phi R_a = \phi F_u A_w U_{bs} + \min\left(\phi 0.6 F_y A_{gy}, \phi F_u A_{nv}\right) \]

Tension rupture component

\[ \phi F_u A_w = 39.6 \text{ kips/in. (0.350 in.)} \]

Shear yielding component

\[ \phi 0.6 F_y A_{gy} = 231 \text{ kips/in. (0.350 in.)} \]

Shear rupture component

\[ \phi F_u A_{nv} = 210 \text{ kips/in. (0.350 in.)} \]

\[ \phi R_a = (210 \text{ kips/in.} + 39.6 \text{ kips/in.} )(0.350 \text{ in.}) \]

\[ = 87.4 \text{ kips} > 36 \text{ kips} \quad \text{o.k.} \]

Check shear yielding of the tee stem

\[ R_n / \Omega = 0.60 F_y A_g / \Omega \]

\[ \Omega = 1.50 \]

\[ = 0.6(50 \text{ ksi})(11\frac{3}{4} \text{ in.})(0.350 \text{ in.})/1.50 \]

\[ = 80.5 \text{ kips} > 24 \text{ kips} \quad \text{o.k.} \]

Check shear rupture of the tee stem

\[ \Omega = 2.00 \]

\[ R_n / \Omega = 0.6 F_u A_w / \Omega \]

\[ = 0.6(65 \text{ ksi}) \]

\[ \times \frac{1}{2.00} \]

\[ \times \left[11\frac{3}{4} - 4(0.875 \text{ in.})\right](0.350 \text{ in.}) \]

\[ = 54.6 \text{ kips} > 24 \text{ kips} \quad \text{o.k.} \]

Check block shear rupture of the tee stem

\[ L_{eh} = L_{ev} = 1\frac{1}{4} \text{ in.} \]

Thus.

\[ \frac{R_n}{\Omega} = \frac{F_u A_w U_{bs}}{\Omega} + \min\left(\frac{0.6 F_y A_{gy}}{\Omega}, \frac{F_u A_{nv}}{\Omega}\right) \]

Tension rupture component

\[ F_u A_w / \Omega = 26.4 \text{ kips/in. (0.350 in.)} \]

Shear yielding component

\[ 0.6 F_y A_{gy} / \Omega = 154 \text{ kips/in. (0.350 in.)} \]

Shear rupture component

\[ 0.6 F_u A_{nv} / \Omega = 140 \text{ kips/in. (0.350 in.)} \]

\[ \frac{R_n}{\Omega} = (140 \text{ kips/in.} + 26.4 \text{ kips/in.})(0.350 \text{ in.}) \]

\[ = 58.2 \text{ kips} > 24 \text{ kips} \quad \text{o.k.} \]
Check beam web for bolt bearing, block shear rupture, shear yielding and shear rupture

From the table, single shear, four rows of \( \frac{3}{4} \)-in. diameter bolts and an uncoped beam with \( F_y = 50 \text{ ksi} \) and \( F_u = 65 \text{ ksi} \),

\[
\phi R_n = (702 \text{ kips/in.})(0.380 \text{ in.})/2 = 133 \text{ kips} > 36 \text{ kips} \quad \text{o.k.}
\]

Check the supporting column flange

From the table, for column flange material with \( F_y = 50 \text{ ksi} \), \( n = 4 \), \( L = 11\frac{1}{2} \), Welds B, Weld size = \( \frac{3}{4} \)-in. the minimum support thickness is 0.190 in.

\[
t_f = 0.710 \text{ in.} > 0.190 \text{ in.} \quad \text{o.k.}
\]

Manual Table 10-1

Check beam web for bolt bearing, block shear rupture, shear yielding and shear rupture

From the table, single shear, four rows of \( \frac{3}{4} \)-in. diameter bolts and an uncoped beam with \( F_y = 50 \text{ ksi} \) and \( F_u = 65 \text{ ksi} \),

\[
R_{u/\Omega} = (468 \text{ kips/in.})(0.380 \text{ in.})/2 = 88.9 \text{ kips} > 24 \text{ kips} \quad \text{o.k.}
\]

Check the supporting column flange

From the table, for column flange material with \( F_y = 50 \text{ ksi} \), \( n = 4 \), \( L = 11\frac{1}{2} \), Welds B, Weld size = \( \frac{3}{4} \)-in. the minimum support thickness is 0.190 in.

\[
t_f = 0.710 \text{ in.} > 0.190 \text{ in.} \quad \text{o.k.}
\]

Manual Table 10-2
Chapter IIB
Fully-Restrained (FR) Moment Connections

The design of fully restrained (FR) moment connections is covered in Part 11 of the AISC Steel Construction Manual.
Example II.B-1  Bolted Flange-Plate FR Moment Connection
(beam-to-column flange)

Given:

Design a bolted flange-plated FR moment connection between a W18×50 beam and a W14×99 column flange to transfer the following forces:

\[ R_D = 7 \text{ kips} \quad M_D = 42 \text{ kip-ft} \]
\[ R_L = 21 \text{ kips} \quad M_L = 126 \text{ kip-ft} \]

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes and E70 electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>Plate</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>W18×50</td>
<td>W14×99</td>
<td>ASTM A36</td>
<td>50</td>
<td>65</td>
<td>Manual</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Beam</th>
<th>Column</th>
<th>$d$ (in.)</th>
<th>$b_f$ (in.)</th>
<th>$t_f$ (in.)</th>
<th>$t_w$ (in.)</th>
<th>$S_x$ (ksi)</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>W18×50</td>
<td>W14×99</td>
<td>18.0</td>
<td>7.50</td>
<td>0.570</td>
<td>0.355</td>
<td>88.9</td>
<td>Manual</td>
</tr>
<tr>
<td></td>
<td></td>
<td>14.2</td>
<td>14.6</td>
<td>0.780</td>
<td></td>
<td></td>
<td>Table 1-1</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(7 \text{kips}) + 1.6(21 \text{kips})$</td>
<td>$R_u = 7 \text{kips} + 21 \text{kips}$</td>
</tr>
<tr>
<td>= 24 kips</td>
<td>= 28 kips</td>
</tr>
<tr>
<td>$M_u = 1.2(42 \text{kip-ft}) + 1.6(126 \text{kip-ft})$</td>
<td>$M_u = 42 \text{kip-ft} + 126 \text{kip-ft}$</td>
</tr>
<tr>
<td>= 252 kip-ft</td>
<td>= 168 kip-ft</td>
</tr>
</tbody>
</table>

Check the beam available flexural strength

Assume two rows of bolts in standard holes.

$$A_{yb} = b_y t_y = (7.50 \text{ in.})(0.570 \text{ in.}) = 4.28 \text{ in.}^2$$

$$A_{yb} = A_y - 2(d_y + \frac{\ell}{2} \text{ in.})t_y = 4.28 \text{ in.}^2 - 2(0.570 \text{ in.} + \frac{\ell}{2} \text{ in.})(0.570 \text{ in.}) = 3.14 \text{ in.}^2$$

$$\frac{F_y}{F_u} = \frac{50 \text{ ksi}}{65 \text{ ksi}} = 0.769 < 0.8 \text{, therefore } Y_t = 1.0.$$

$$F_u A_{yb} = (65 \text{ ksi})(3.14 \text{ in.}^2) = 204 \text{ kips}$$

$$Y_t F_y A_{yb} = (1.0)(50 \text{ ksi})(4.28 \text{ in.}^2) = 214 \text{ kips} > 204 \text{ kips}$$

Therefore the nominal flexural strength, $M_n$, at the location of the holes in the tension flange is as follows:

$$M_n = \frac{F_u A_{yb} S_y}{A_{yb}} = \frac{(65 \text{ ksi})(3.14 \text{ in.}^2)(88.9 \text{ in.}^3)}{4.28 \text{ in.}^2} = 4240 \text{ kip-in. or 353 kip-ft}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$\phi_b M_n = 0.90(353 \text{ kip-ft}) = 318 \text{ kip-ft}$</td>
<td>$M_n / \Omega_b = \frac{353 \text{ kip-ft}}{1.67} = 211 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

$$318 \text{ kip-ft} > 252 \text{ kip-ft} \quad \text{o.k.}$$

$$211 \text{ kip-ft} > 168 \text{ kip-ft} \quad \text{o.k.}$$

Design single-plate web connection

Try a PL $\frac{3}{8} \times 4 \times 0\text{'}-9$, with three $\frac{3}{8}$-in. diameter ASTM A325-N bolts and $\frac{1}{4}$-in. fillet welds.
### LRFD

<table>
<thead>
<tr>
<th><strong>Design shear strength of the bolts</strong></th>
<th><strong>Allowable shear strength of bolts</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single shear;</td>
<td>Single shear;</td>
</tr>
<tr>
<td>$\phi r_n = 21.6$ kips/bolt</td>
<td>$r_n/\Omega = 14.4$ kips/bolt</td>
</tr>
<tr>
<td>$= 42$ kips/(21.6 kips/bolt) = 1.94$</td>
<td>$= 28$ kips/(14.4 kips/bolt) = 1.94$</td>
</tr>
<tr>
<td><strong>Bearing strength of bolts</strong></td>
<td><strong>Bearing strength of bolts</strong></td>
</tr>
<tr>
<td>Bolt spacing = 3 in.</td>
<td>Bolt spacing = 3 in.</td>
</tr>
<tr>
<td>$\phi r_n = (91.4$ kips/in./bolt)(½ in.)</td>
<td>$r_n/\Omega = (60.9$ kips/in./bolt)(½ in.)</td>
</tr>
<tr>
<td>$= 34.3$ kips/bolt</td>
<td>$= 22.8$ kips/bolt</td>
</tr>
<tr>
<td>$= 42$ kips/(34.3 kips/bolt) = 1.23$</td>
<td>$= 28$ kips/(22.8 kips/bolt) = 1.23$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Plate shear yielding</strong></th>
<th><strong>Plate shear yielding</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi r_n = 0.60 \phi F_y A_g$</td>
<td>$r_n/\Omega = 0.60 F_y A_g/\Omega =$</td>
</tr>
<tr>
<td>$= 0.60(1.00)(36$ ksi)(9 in.)(½ in.)</td>
<td>$= 0.60(36$ ksi)(9 in.)(½ in.)/(1.50)</td>
</tr>
<tr>
<td>$= 72.9$ kips &gt; 42 kips o.k.</td>
<td>$= 48.6$ kips &gt; 28 kips o.k.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Plate shear rupture</strong></th>
<th><strong>Plate shear rupture</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Where $\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi r_n = 0.60 \phi F_u A_{nv}$</td>
<td>$r_n/\Omega = 0.60 F_u A_{nv}/\Omega =$</td>
</tr>
<tr>
<td>(3 bolts)(½ in. + ¼ in. + ¹⁄₁₆ in.) = 3 in.</td>
<td>(3 bolts)(½ in. + ¼ in. + ¹⁄₁₆ in.) = 3 in.</td>
</tr>
<tr>
<td>$A_{nv} = (9$ in. – 3 in.)(½ in.) = 2.25 in$²</td>
<td>$A_{nv} = (9$ in. – 3 in.)(½ in.) = 2.25 in$²</td>
</tr>
<tr>
<td>$= 0.60(0.75)(58$ ksi)(2.25 in$²$)</td>
<td>$= 0.60(58$ ksi)(2.25 in$²$)/(2.00)</td>
</tr>
<tr>
<td>$= 58.7$ kips &gt; 42 kips o.k.</td>
<td>$= 39.2$ kips &gt; 28 kips o.k.</td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th><strong>Design shear strength of the bolts</strong></th>
<th><strong>Allowable shear strength of bolts</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>Single shear;</td>
<td>Single shear;</td>
</tr>
<tr>
<td>$r_n/\Omega = 14.4$ kips/bolt</td>
<td>$r_n/\Omega = 21.8$ kips/in(½ in)</td>
</tr>
</tbody>
</table>

### Block shear rupture strength of the plate

$L_{eh} = 1\frac{1}{4}$ in.; $L_{en} = 1\frac{1}{2}$ in.; $U_{en} = 1.0$; $n = 3$
Shear yielding component

\[ \phi_{0.60} F_y A_{gv} = 121 \text{kips/in}(\% \text{ in}) \]

Shear rupture component

\[ \phi_{0.60} F_y A_{gv} = 131 \text{kips/in}(\% \text{ in}) \]

\[ \phi R_n = (121 \text{kips/in} + 32.6 \text{kips/in})(\% \text{ in}) \]

\[ = 57.6 \text{kips} > 42 \text{kips} \quad \text{o.k.} \]

Weld Strength

\[ \phi R_s = 1.392 D l (2) \]

\[ = 1.392(4 \text{ sixteenths})(9 \text{ in.})(2) \]

\[ = 100 \text{kips} > 42 \text{kips} \quad \text{o.k.} \]

Shear yielding component

\[ 0.60 F_y A_{gv} / \Omega = 81.0 \text{kips/in}(\% \text{ in}) \]

Shear rupture component

\[ 0.60 F_y A_{gv} / \Omega = 87.0 \text{kips/in}(\% \text{ in}) \]

\[ R_n / \Omega = (81.0 \text{kips/in} + 21.6 \text{kips/in})(\% \text{ in}) \]

\[ = 38.5 \text{kips} > 24 \text{kips} \quad \text{o.k.} \]

Weld Strength

\[ R_n / \Omega = 0.928 D l (2) \]

\[ = 0.928(4 \text{ sixteenths})(9 \text{ in.})(2) \]

\[ = 66.8 \text{kips} > 28 \text{kips} \quad \text{o.k.} \]

Connecting Elements Rupture Strength at Welds

Shear rupture strength of base metal

\[ t_{min} = \frac{0.6 F_{ext} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right) \left( 3.09 D \right)}{0.6 F_u} = 3.09 D \]

Column flange; \( t_f = 0.780 \text{ in.} \)

\[ t_{min} = \frac{3.09 D}{F_u} = \frac{(3.09)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.190 \text{ in.} \]

Plate; \( t = \frac{3}{8} \text{ in.} \)

\[ t_{min} = \frac{3.09(2) D}{F_u} = \frac{(6.19)(4 \text{ sixteenths})}{58 \text{ ksi}} = 0.427 \text{ in.} > \frac{3}{8} \text{ in.} \quad \text{proration required.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_s = \frac{% \text{ in.}}{0.427 \text{ in.}} (50.1 \text{kips}) = 44 \text{kips} ]</td>
<td>[ R_n / \Omega = \frac{% \text{ in.}}{0.427 \text{ in.}} (33.4 \text{kips}) = 29.3 \text{kips} ]</td>
</tr>
</tbody>
</table>

\[ 44 \text{kips} > 42 \text{kips} \quad \text{o.k.} \]

\[ 29.3 \text{kips} > 28 \text{kips} \quad \text{o.k.} \]
Design tension flange plate and connection

Design of bolts

### LRFD

\[
P_{sf} = \frac{M_u}{d} = \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}} = 168 \text{ kips}
\]

Try a PL \(\frac{3}{4} \times 7\)

Determine critical bolt strength

For shear, \(\phi_u = 21.6 \text{ kips/bolt}\)

\[
\phi_u = (102 \text{ kips/bolt}) t_f
\]

\[
= (102 \text{ kips/bolt})(0.570 \text{ in.})
\]

\[
= 58.1 \text{ kips/bolt}
\]

For bearing on flange;

\[
\phi_u = (91.4 \text{ kips/bolt}) t_f
\]

\[
= (91.4 \text{ kips/bolt})(0.570 \text{ in.})
\]

\[
= 52.1 \text{ kips/bolt}
\]

Shear controls, therefore the number of bolts required is as follows:

\[
n_{\text{min}} = \frac{P_{sf}}{\phi_u} = \frac{168 \text{ kips}}{21.6 \text{ kips/bolt}} = 7.78 \text{ bolts}
\]

Use eight bolts.

### ASD

\[
P_{sf} = \frac{M_u}{d} = \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.}} = 112 \text{ kips}
\]

Try a PL \(\frac{3}{4} \times 7\)

Determine critical bolt strength

For shear, \(r_n / \Omega = 14.4 \text{ kips/bolt}\)

\[
r_n / \Omega = (68.3 \text{ kips/bolt}) t_f
\]

\[
= (68.3 \text{ kips/bolt})(0.570 \text{ in.})
\]

\[
= 38.9 \text{ kips/bolt}
\]

For bearing on flange;

\[
r_n / \Omega = (60.9 \text{ kips/bolt}) t_f
\]

\[
= (60.9 \text{ kips/bolt})(0.570 \text{ in.})
\]

\[
= 34.7 \text{ kips/bolt}
\]

Shear controls, therefore the number of bolts required is as follows:

\[
n_{\text{min}} = \frac{P_{sf}}{r_n / \Omega} = \frac{112 \text{ kips}}{14.4 \text{ kips/bolt}} = 7.78 \text{ bolts}
\]

Use eight bolts.

---

Check flange plate tension yielding

\[
P_a = F_y A_y = (36 \text{ ksi})(\frac{7}{4} \text{ in.}) = 189 \text{ kips}
\]  
Eqn. D2-1

### LRFD

\[
P_{sf} = \frac{M_u}{d + t_p} = \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.} + \frac{3}{4} \text{ in.}} = 161 \text{ kips}
\]

\[
\phi = 0.90
\]

### ASD

\[
P_{sf} = \frac{M_u}{d + t_p} = \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{18.0 \text{ in.} + \frac{3}{4} \text{ in.}} = 108 \text{ kips}
\]

\[
\Omega = 1.67
\]
Check flange plate tension rupture

\[ A_u \leq 0.85 \ A_g = 0.85(7 \text{ in.})(0.75 \text{ in.}) = 4.46 \text{ in}^2 \]

\[ A_u = [8 - 2\left(\frac{d_g}{2} + \frac{d}{2}\right)]t_p = [\left(7 \text{ in.}\right) - 2\left(\frac{3}{8} \text{ in.} + \frac{1}{2} \text{ in.}\right)](\frac{3}{8} \text{ in.}) = 3.75 \text{ in}^2 \]

\[ A_g = 3.75 \text{ in}^2 \]

\[ P_u = F_u A_g = (58 \text{ ksi})(3.75 \text{ in}^2) = 217 \text{ kips} \]

Eqn. J4-1

Eqn. D2-2

Check flange plate block shear rupture

There are two cases for which block shear rupture must be checked. The first case involves the tearout of the two blocks outside the two rows of bolt holes in the flange plate; for this case \( L_{eh} = 1\frac{1}{2} \text{ in.} \) and \( L_{ev} = 1\frac{1}{2} \text{ in.} \). The second case involves the tearout of the block between the two rows of the holes in the flange plate. Manual Tables 9-3a, 9-3b, and 9-3c may be adapted for this calculation by considering the 4 in. width to be comprised of two, 2-in. wide blocks where \( L_{eh} = 2 \text{ in.} \) and \( L_{ev} = 1\frac{1}{2} \text{ in.} \). Thus, the former case is the more critical.
Determine the required size of the fillet weld to supporting column flange

The applied tension load is perpendicular to the weld, therefore \( \theta = 90^\circ \) and \( 1.0 + 0.5 \sin^3 \theta = 1.5 \).

### LRFD

\[
D_{\text{min}} = \frac{P_{nf}}{2(1.5)(1.392)l}
\]

\[
= \frac{161 \text{ kips}}{2(1.5)(1.392)(7 \text{ in.})} = 5.51 \text{ sixteenths}
\]

Use \( \frac{3}{8} \)-in. fillet welds, 6 > 5.51 **o.k.**

### ASD

\[
D_{\text{min}} = \frac{P_{nf}}{2(1.5)(0.928)l}
\]

\[
= \frac{108 \text{ kips}}{2(1.5)(0.928)(7 \text{ in.})} = 5.54 \text{ sixteenths}
\]

Use \( \frac{3}{8} \)-in. fillet welds, 6 > 5.54 **o.k.**

### Connecting Elements Rupture Strength at Welds

#### Tension rupture strength of base metal

\[
t_{\text{min}} = \frac{0.6F_{EXX} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right)}{F_{u}} = 1.86D
\]

Column flange; \( t_f = 0.780 \text{ in.} \)

\[
t_{\text{min}} = \frac{1.86D}{F_{u}} = \frac{(1.86)(6 \text{ sixteenths})}{65 \text{ ksi}} = 0.171 \text{ in.}
\]

Flange plate; \( t_f = 0.75 \text{ in.} \)

\[
t_{\text{min}} = \frac{3.71D}{F_{u}} = \frac{(1.86)(2)(6 \text{ sixteenths})}{58 \text{ ksi}} = 0.384 \text{ in.}
\]

### Design compression flange plate and connection

Try PL \( \frac{3}{4} \times 7 \)

Assume \( K = 0.65 \) and \( l = 2.0 \text{ in.} \) (1½ in. edge distance and ½ in. setback).

\[
\frac{KL}{r} = \frac{0.65(2.0 \text{ in.})}{\left( \frac{\frac{3}{4} \text{ in.}}{\sqrt{12}} \right)} = 6.00 < 25
\]

Therefore, \( F_v = F_y \)  

\[
A = (7 \text{ in.})(\frac{3}{4} \text{ in.}) = 5.25 \text{ in.}^2
\]
The compression flange plate will be identical to the tension flange plate; a \( \frac{3}{4} \) in. \( \times \) 7 in. plate with eight bolts in two rows of four bolts on a 4 in. gage and \( \frac{7}{8} \) in. fillet welds to the supporting column flange.

Note: Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

The column must be checked for stiffening requirements. For further information, see AISC Design Guide No. 13 *Wide-Flange Column Stiffening at Moment Connections – Wind and Seismic Applications* (Carter, 1999).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \phi P_y = \phi F_y A_y = 0.90(36 \text{ ksi })(5.25 \text{ in.}^2) )</td>
<td>( P_y / \Omega = \frac{F_y A_y}{\Omega} = \frac{(36 \text{ ksi })(5.25 \text{ in.}^2)}{1.67} )</td>
</tr>
<tr>
<td>= 170 kips &gt; 161 kips <strong>O.K.</strong></td>
<td>= 113 kips &gt; 108 kips <strong>O.K.</strong></td>
</tr>
</tbody>
</table>
Example II.B-2 Welded Flange-Plated FR Moment Connection (beam-to-column flange)

Given:

Design a welded flange-plated FR moment connection between a W18×50 beam and a W14×99 column flange to transfer the following forces:

\[ R_D = 7 \text{ kips} \quad M_D = 42 \text{ kip-ft} \]
\[ R_L = 21 \text{ kips} \quad M_L = 126 \text{ kip-ft} \]

Use \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts in standard holes and E70 electrodes.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W18×50</th>
<th>Column W14×99</th>
<th>Plate</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Material</td>
<td>ASTM A992</td>
<td>ASTM A992</td>
<td>ASTM A36</td>
<td></td>
</tr>
<tr>
<td>( F_y )</td>
<td>50 ksi</td>
<td>50 ksi</td>
<td>36 ksi</td>
<td>Table 2-3</td>
</tr>
<tr>
<td>( F_u )</td>
<td>65 ksi</td>
<td>65 ksi</td>
<td>58 ksi</td>
<td>Table 2-4</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W18×50</th>
<th>Column W14×99</th>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d ) in.</td>
<td>18.0 in.</td>
<td>14.2 in.</td>
<td></td>
</tr>
<tr>
<td>( b_f ) in.</td>
<td>7.50 in.</td>
<td>14.6 in.</td>
<td></td>
</tr>
<tr>
<td>( t_f ) in.</td>
<td>0.570 in.</td>
<td>0.780 in.</td>
<td></td>
</tr>
<tr>
<td>( t_w ) in.</td>
<td>0.355 in.</td>
<td>0.780 in.</td>
<td></td>
</tr>
<tr>
<td>( Z_x )</td>
<td>101 in.³</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Handbook Table 2-3
Handbook Table 1-1
## Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(7 \text{ kips}) + 1.6(21 \text{ kips}) = 42 \text{ kips}$</td>
<td>$R_u = 7 \text{ kips} + 21 \text{ kips} = 28 \text{ kips}$</td>
</tr>
<tr>
<td>$M_u = 1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$</td>
<td>$M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft}$</td>
</tr>
<tr>
<td>= 252 kip-ft</td>
<td>= 168 kip-ft</td>
</tr>
</tbody>
</table>

### Check the beam flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a W18×50</td>
<td>For a W18×50</td>
</tr>
<tr>
<td>$\phi R_u M_u = 379 \text{ kip-ft} &gt; 252 \text{ kip-ft}$</td>
<td>$\phi R_u M_u = 252 \text{ kip-ft} &gt; 168 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

### Design the single-plate web connection

Try a PL $\frac{3}{8} \times 4 \times 0' - 9$, with three $\frac{3}{8}$-in. diameter ASTM A325-N bolts and $\frac{1}{4}$ in. fillet welds.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear strength of bolts</td>
<td>Shear strength of bolts</td>
</tr>
<tr>
<td>Single shear;</td>
<td>Single shear;</td>
</tr>
<tr>
<td>$\phi R_u = 21.6 \text{ kips/bolt}$</td>
<td>$r_n/\Omega = 14.4 \text{ kips/bolt}$</td>
</tr>
<tr>
<td>= 42 kips/(21.6 kips/bolt) = 1.94 bolts</td>
<td>= 28 kips/(21.6 kips/bolt) = 1.94 bolts</td>
</tr>
<tr>
<td>Bearing Strength of bolts</td>
<td>Bearing Strength of bolts</td>
</tr>
<tr>
<td>Bolt spacing = 3 in.</td>
<td>Bolt spacing = 3 in.</td>
</tr>
<tr>
<td>$\phi r_n = (91.4 \text{ kips/in./bolt})(\frac{3}{8} \text{ in.})$</td>
<td>$r_n/\Omega = (60.9 \text{ kips/in./bolt})(\frac{3}{8} \text{ in.})$</td>
</tr>
<tr>
<td>= 34.3 kips/bolt</td>
<td>= 22.8 kips/bolt</td>
</tr>
<tr>
<td>= 42 kips/(34.3 kips/bolt) = 1.22 bolts</td>
<td>= 28 kips/(22.8 kips/bolt) = 1.22 bolts</td>
</tr>
<tr>
<td>Plate shear yielding</td>
<td>Plate shear yielding</td>
</tr>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_u = 0.60 \phi F_y A_g$</td>
<td>$r_n/\Omega = 0.60 F_y A_g/\Omega$</td>
</tr>
<tr>
<td>= 0.60(1.00)(36 ksi)(9 in.)/(% in.)</td>
<td>= 0.60(36 ksi)(9 in.)/(% in.)/(1.50)</td>
</tr>
<tr>
<td>= 72.9 kips &gt; 42 kips</td>
<td>= 48.6 kips &gt; 28 kips</td>
</tr>
</tbody>
</table>
Plate shear rupture

\[ \phi = 0.75 \]

\[ \phi R_n = 0.60 \phi F_u A_{nv} \]

\[ A_{nv} = (9 \text{ in.} - 3 \text{ in.})(\% \text{ in.}) = 2.25 \text{ in}^2 \]

\[ R_n = 0.60(0.75)(58 \text{ ksi})(2.25 \text{ in}^2) = 58.7 \text{ kips} > 42 \text{ kips} \quad \text{o.k.} \]

Block shear rupture strength for plate

\[ L_{eb} = 1\frac{1}{4} \text{ in.}; L_{ev} = 1\frac{1}{2} \text{ in.}; U_{bs} = 1.0; n = 3 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_n = \phi F_u A_{nt} U_{bs} + \min \left( \phi 0.6F_y A_{gv} , \phi F_u A_{nv} \right) ]</td>
<td>[ \frac{R_n}{\Omega} = \frac{F_u A_{nt} U_{bs}}{\Omega} + \min \left( \frac{0.6F_y A_{gv}}{\Omega} , \frac{F_u A_{nv}}{\Omega} \right) ]</td>
</tr>
<tr>
<td>Tension component</td>
<td>Tension component</td>
</tr>
<tr>
<td>[ \phi U_{bs} F_u A_{nt} = 32.6 \text{ kips/in}(% \text{ in}) ]</td>
<td>[ U_{bs} F_u A_{nt} / \Omega = 21.8 \text{ kips/in}(% \text{ in}) ]</td>
</tr>
<tr>
<td>Shear yielding</td>
<td>Shear yielding</td>
</tr>
<tr>
<td>[ \phi 0.6F_y A_{gv} = 121 \text{ kips/in}(% \text{ in}) ]</td>
<td>[ 0.6F_y A_{gv} / \Omega t = 81.0 \text{ kips/in}(% \text{ in}) ]</td>
</tr>
<tr>
<td>Shear rupture</td>
<td>Shear rupture</td>
</tr>
<tr>
<td>[ \phi 0.6F_u A_{nv} = 131 \text{ kips/in}(% \text{ in}) ]</td>
<td>[ 0.6F_u A_{nv} / \Omega = 87.0 \text{ kips/in}(% \text{ in}) ]</td>
</tr>
<tr>
<td>[ \phi R_n = (121 \text{ kips/in} + 32.6 \text{ kips/in})(% \text{ in}) ]</td>
<td>[ R_n / \Omega = (81.0 \text{ kips/in} + 21.6 \text{ kips/in})(% \text{ in}) ]</td>
</tr>
<tr>
<td>= 57.6 kips &gt; 42 kips \quad \text{o.k.}</td>
<td>= 38.5 kips &gt; 24 kips \quad \text{o.k.}</td>
</tr>
</tbody>
</table>

Weld Strength

\[ \phi R_n = 1.392Dl(2) \]

\[ = 1.392(4 \text{ sixteenths})(9 \text{ in.})(2) \]

\[ = 100 \text{ kips} > 42 \text{ kips} \quad \text{o.k.} \]

Connecting Elements Rupture Strength at Welds

Shear rupture strength of base metal

\[ t_{min} = \frac{0.6F_{EXX} \left( \sqrt{\frac{D}{2}} \right)}{0.6F_u} = \frac{3.09D}{F_u} \]
Column flange: $t_f = 0.780$ in.

$$t_{\min} = \frac{3.09D}{F_u} = \frac{(3.09)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.190 \text{ in.}$$

Plate; $t = \frac{3}{8}$ in.

$$t_{\min} = \frac{3.09(2)D}{F_u} = \frac{(6.19)(4 \text{ sixteenths})}{58 \text{ ksi}} = 0.427 \text{ in.} > \frac{3}{8} \text{ in.} \quad \text{proration required}$$

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_w = \frac{\frac{3}{8} \text{ in.}}{0.247 \text{ in.}}(50.1 \text{ kips}) = 44 \text{ kips}$</td>
<td>$R_y = \frac{\frac{3}{8} \text{ in.}}{0.427 \text{ in.}}(33.4 \text{ kips}) = 29.3 \text{ kips}$</td>
<td>[44 \text{ kips} &gt; 42 \text{ kips} \quad \text{o.k.}] [29.3 \text{ kips} &gt; 28 \text{ kips} \quad \text{o.k.}]</td>
</tr>
</tbody>
</table>

**Design tension flange plate and connection**

Determine the flange force

\[
P_{sf} = \frac{M_p}{d + t_p} = \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{(18.0 \text{ in.} + \frac{3}{8} \text{ in.})} = 161 \text{ kips}
\]

\[
P_{af} = \frac{M_p}{d + t_p} = \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{(18.0 \text{ in.} + \frac{3}{8} \text{ in.})} = 108 \text{ kips}
\]

The top flange width $b_f = 7\frac{1}{2}$ in. Assume a shelf dimension of $\frac{3}{8}$ in. on both sides of the plate. The plate width, then, is $7\frac{1}{2}$ in. – 2($\frac{3}{8}$ in.) = 6$\frac{1}{4}$ in. Try a 1 in.×6$\frac{1}{4}$ in. flange plate.

Check top flange plate tension yielding

\[
R_y = F/A_t = (36 \text{ ksi})(6 \frac{1}{4} \text{ in.})(1 \text{ in.}) = 225 \text{ kips}
\]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
<td>$\frac{R_y}{\Omega} = \frac{225 \text{ kips}}{1.67} = 135 \text{ kips}$</td>
</tr>
</tbody>
</table>

Determine the force in the welds

\[
P_{sf} = \frac{M_p}{d} = \frac{(252 \text{ kip-ft})(12 \text{ in./ft})}{(18.0 \text{ in.})} = 168 \text{ kips}
\]

\[
P_{af} = \frac{M_p}{d} = \frac{(168 \text{ kip-ft})(12 \text{ in./ft})}{(18.0 \text{ in.})} = 112 \text{ kips}
\]
Determine the required weld size and length for fillet welds to beam flange. Try a $\frac{3}{16}$-in. fillet weld. The minimum length of weld $l_{min}$ is as follows:

For weld compatibility, disregard the increased capacity due to perpendicular loading of the end weld.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{min} = \frac{P_{uf}}{1.392D} = \frac{168 \text{ kips}}{1.392(5)} = 24.1 \text{ in.}$</td>
<td>$l_{min} = \frac{P_{uf}}{0.928D} = \frac{112 \text{ kips}}{0.928(5)} = 24.1 \text{ in.}$</td>
</tr>
</tbody>
</table>

Use 9 in. of weld along each side and $6\frac{1}{4}$ in. of weld along the end of the flange plate.

$\Rightarrow 24.2 \text{ in.} > 24.1 \text{ in.}$ \textbf{o.k.}

Use 9 in. of weld along each side and $6\frac{1}{4}$ in. of weld along the end of the flange plate.

$\Rightarrow 24.2 \text{ in.} > 24.1 \text{ in.}$ \textbf{o.k.}

**Connecting Elements Rupture Strength at Welds**

*Shear rupture strength of base metal*

$$t_{min} = \frac{0.6F_{exc} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right)}{0.6F_{u}} = \frac{3.09D}{F_{u}}$$

Beam flange; $t_f = 0.570$ in.

$$t_{min} = \frac{3.09D}{F_{u}} = \frac{(3.09)(5 \text{ sixteenths})}{65 \text{ ksi}} = 0.238 \text{ in.}$$

Flange plate; $t_p = 1.00$ in.

$$t_{min} = \frac{3.09D}{F_{u}} = \frac{(3.09)(5 \text{ sixteenths})}{58 \text{ ksi}} = 0.266 \text{ in.}$$

*Tension rupture strength of base metal*

$$t_{min} = \frac{0.6F_{exc} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right)}{F_{u}} = \frac{1.86D}{F_{u}}$$

Beam flange; $t_f = 0.570$ in.

$$t_{min} = \frac{1.86D}{F_{u}} = \frac{(1.86)(5 \text{ sixteenths})}{65 \text{ ksi}} = 0.143 \text{ in.}$$
Flange plate; \( t_f = 1.00 \) in.

\[
t_{\text{min}} = \frac{1.86D}{F_u} = \frac{(1.86)(5 \text{ sixteenths})}{58 \text{ ksi}} = 0.160 \text{ in.}
\]

**Determine the required fillet weld size to supporting column flange**

The applied tension load is perpendicular to the weld, therefore \( \theta = 90^\circ \) and \( 1.0 + \sin^{1/2} \theta = 1.5 \).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{\text{min}} = \frac{P_{of}}{2(1.5)(1.392)/l} )</td>
<td>( D_{\text{min}} = \frac{P_{of}}{2(1.5)(0.928)/l} )</td>
</tr>
<tr>
<td>( = \frac{168 \text{ kips}}{2(1.5)(1.392)(6 \text{ \textonehalf in.})} )</td>
<td>( = \frac{112 \text{ kips}}{2(1.5)(0.928)(6 \text{ \textonehalf in.})} )</td>
</tr>
<tr>
<td>( = 6.44 \text{ sixteenths} )</td>
<td>( = 6.44 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

Use \( 7/16\)-in. fillet welds, \( 7 > 6.44 \) o.k. Use \( 7/16\)-in. fillet welds, \( 7 > 6.44 \) o.k.

Note: Tension due to load reversal must also be considered in the design of the fillet weld to the supporting column flange.

**Connecting Elements Rupture Strength at Welds**

**Tension rupture strength of base metal**

\[
t_{\text{min}} = \frac{0.6F_{exx} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right)}{F_u} = \frac{1.86D}{F_u}
\]

Column flange; \( t_f = 0.780 \) in.

\[
t_{\text{min}} = \frac{1.86D}{F_u} = \frac{(1.86)(7 \text{ sixteenths})}{65 \text{ ksi}} = 0.200 \text{ in.}
\]

Flange plate; \( t_p = 1.00 \) in.

\[
t_{\text{min}} = \frac{1.86(2)D}{F_u} = \frac{(3.71)(7 \text{ sixteenths})}{58 \text{ ksi}} = 0.448 \text{ in.}
\]

**Design compression flange plate and connection**

The compression flange plate should have approximately the same area as the tension flange plate (\( A = 6 \text{ \textonehalf in.} \times 1 \text{ in.} = 6.25 \text{ in.}^2 \)). Assume a shelf dimension of \( 7/8 \) in. The plate width, then, is \( 7\frac{1}{2} \text{ in.} + 2(\% \text{ in.}) = 8\frac{3}{4} \text{ in.} \). To approximately balance the flange plate areas, try a \( 3/4 \text{ in.} \times 8\frac{3}{4} \text{ in.} \) compression flange plate.
Assume \( K = 0.65 \) and \( l = 2.0 \) in. (1½ in. edge distance and ½-in. setback).

\[
\frac{KI}{r} = \frac{0.65(2.0 \text{ in.})}{\left(\frac{\frac{1}{4} \text{ in.}}{\sqrt{12}}\right)} = 6.00 < 25
\]

\[
A = \left(\frac{8}{24} \text{ in.}\right)\left(\frac{1}{2} \text{ in.}\right) = 6.56 \text{ in.}^2
\]

Determine the required weld size and length for fillet welds to beam flange

Based upon the weld length required for the tension flange plate, use 3⁄8 in. fillet weld and \( 12\frac{1}{2} \) in. of weld along each side of the beam flange.

Connecting Elements Rupture Strength at Welds

Shear rupture strength of base metal

\[
t_{\text{min}} = \frac{0.6 F_{eq} \left(\frac{\sqrt{2}}{2}\right) \left(\frac{D}{16}\right)}{0.6 F_u} = \frac{3.09D}{F_u}
\]

Beam flange; \( t_f = 0.570 \) in.

\[
t_{\text{min}} = \frac{3.09D}{F_u} = \frac{(3.09)(5 \text{ sixteenths})}{65 \text{ ksi}} = 0.238 \text{ in.}
\]

Bottom flange plate; \( t_p = 0.750 \) in.

\[
t_{\text{min}} = \frac{3.09D}{F_u} = \frac{(3.09)(5 \text{ sixteenths})}{58 \text{ ksi}} = 0.266 \text{ in.}
\]

The column must be checked for stiffening requirements. For further information, see AISC Design Guide No. 13 Wide-Flange Column Stiffening at Moment Connections – Wind and Seismic Applications. (Carter, 1999).
Example II.B-3  Directly-Welded Flange FR Moment Connection (beam-to-column flange).

Given:

Design a directly welded flange FR moment connection between a W18×50 beam and a W14×99 column flange to transfer the following forces:

- $R_D = 7$ kips  \hspace{0.5cm} $M_D = 42$ kip-ft
- $R_L = 21$ kips  \hspace{0.5cm} $M_L = 126$ kip-ft

Use 3/8-in. diameter ASTM A325-N bolts in standard holes and E70 electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>ASTM Grade</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×50</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
<td>Manual</td>
</tr>
<tr>
<td>Column</td>
<td>W14×99</td>
<td>ASTM A992</td>
<td>50</td>
<td>65</td>
<td>Table 2-3</td>
</tr>
<tr>
<td>Plate</td>
<td></td>
<td>ASTM A36</td>
<td>36</td>
<td>58</td>
<td>Table 2-4</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>$d$ (in.)</th>
<th>$b_f$ (in.)</th>
<th>$t_f$ (in.)</th>
<th>$t_w$ (in.)</th>
<th>$Z_c$ (in.)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×50</td>
<td>18.0</td>
<td>7.50</td>
<td>0.570</td>
<td>0.355</td>
<td>101</td>
<td>Manual</td>
</tr>
<tr>
<td>Column</td>
<td>W14×99</td>
<td>14.2</td>
<td>14.6</td>
<td>0.780</td>
<td></td>
<td></td>
<td>Table 1-1</td>
</tr>
</tbody>
</table>
Solution:

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$</td>
<td>$1.2(7 \text{ kips}) + 1.6(21 \text{ kips}) = 42 \text{ kips}$</td>
<td>$R_u = 7 \text{ kips} + 21 \text{ kips} = 28 \text{ kips}$</td>
</tr>
<tr>
<td>$M_u$</td>
<td>$1.2(42 \text{ kip-ft}) + 1.6(126 \text{ kip-ft})$</td>
<td>$M_u = 42 \text{ kip-ft} + 126 \text{ kip-ft}$</td>
</tr>
<tr>
<td></td>
<td>$= 252 \text{ kip-ft}$</td>
<td>$= 168 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Check beam flexural strength

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a $W_{18\times50}$</td>
<td>$\phi_b M_u = 379 \text{ kip-ft} &gt; 252 \text{ kip-ft}$</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>$M_u / \Omega_b = 252 \text{ kip-ft} &gt; 168 \text{ kip-ft}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Design single-plate web connection

Try a PL $\frac{3}{8}\times4\times0'-9$, with three $\frac{3}{8}$-in. diameter ASTM A325-N bolts and $\frac{1}{4}$ in. fillet welds.

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Shear strength of bolts</td>
<td>Single shear;</td>
<td></td>
</tr>
<tr>
<td>$\phi_r u$</td>
<td>$21.6 \text{ kips/bolt}$</td>
<td>$r_u / \Omega = 14.4 \text{ kips/bolt}$</td>
</tr>
<tr>
<td></td>
<td>$= 42 \text{ kips}/(21.6 \text{ kips/bolt}) = 1.94 \text{ bolts}$</td>
<td>$= 28 \text{ kips}/(14.4 \text{ kips/bolt}) = 1.94 \text{ bolts}$</td>
</tr>
</tbody>
</table>

Bearing Strength of bolts

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Bolt spacing</td>
<td>$3 \text{ in.}$</td>
<td>$\Omega = 3 \text{ in.}$</td>
</tr>
<tr>
<td>$\phi_r u$</td>
<td>$(91.4 \text{ kips/in./bolt})(\frac{3}{8} \text{ in.})$</td>
<td>$r_u / \Omega = (60.9 \text{ kips/in./bolt})(\frac{3}{8} \text{ in.})$</td>
</tr>
<tr>
<td></td>
<td>$= 34.3 \text{ kips/bolt}$</td>
<td>$= 22.8 \text{ kips/bolt}$</td>
</tr>
<tr>
<td></td>
<td>$= 42 \text{ kips}/(34.3 \text{ kips/bolt}) = 1.47 \text{ bolts}$</td>
<td>$= 28 \text{ kips}/(22.8 \text{ kips/bolt}) = 1.22 \text{ bolts}$</td>
</tr>
</tbody>
</table>

Plate shear yielding

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>1.00</td>
<td>$\Omega = 1.50$</td>
</tr>
<tr>
<td>$\phi R_u$</td>
<td>$0.60 \phi F_y A_g$</td>
<td>$r_u / \Omega = 0.60 F_u A_u / \Omega = $</td>
</tr>
<tr>
<td></td>
<td>$= 0.60(1.00)(36 \text{ ksi})(9 \text{ in.})(\frac{3}{8} \text{ in.}) = $</td>
<td>$= 0.60(36 \text{ ksi})(9 \text{ in.})(\frac{3}{8} \text{ in.})/(1.50)$</td>
</tr>
<tr>
<td></td>
<td>$= 72.9 \text{ kips} &gt; 42 \text{ kips}$</td>
<td>o.k.</td>
</tr>
<tr>
<td></td>
<td>$\Omega = 1.50$</td>
<td>$= 48.6 \text{ kips} &gt; 28 \text{ kips}$</td>
</tr>
</tbody>
</table>

Plate shear rupture

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi$</td>
<td>0.75</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_u$</td>
<td>$0.60 \phi F_u A_{av}$</td>
<td>$r_u / \Omega = 0.60 F_u A_{av} / \Omega = $</td>
</tr>
<tr>
<td></td>
<td>$(3 \text{ bolts})(\frac{3}{8} \text{ in.} + \frac{1}{6} \text{ in.} + \frac{1}{8} \text{ in.}) = 3 \text{ in.}$</td>
<td>$(3 \text{ bolts})(\frac{3}{8} \text{ in.} + \frac{1}{6} \text{ in.} + \frac{1}{8} \text{ in.}) = 3 \text{ in.}$</td>
</tr>
</tbody>
</table>
### Block shear rupture strength for plate

\[ L_{cb} = 1\frac{1}{4} \text{ in.}; \quad L_{cv} = 1\frac{1}{2} \text{ in.}; \quad U_{hs} = 1.0; \quad n = 3 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_n = \phi F_u A_{nt} U_{hs} + \min \left( \phi 0.6 F_y A_{gy} + \phi F_u A_{nv} \right) ]</td>
<td>[ R_n = \frac{F_u A_{nt} U_{hs}}{\Omega} + \min \left( \frac{0.6 F_y A_{gy}}{\Omega}, \frac{F_u A_{nv}}{\Omega} \right) ]</td>
</tr>
</tbody>
</table>

- **Tension component**
  - \[ \phi F_u A_{nt} = 32.6 \text{ kips/in}(\frac{1}{8} \text{ in}) \]
- **Shear yielding component**
  - \[ \phi 0.6 F_y A_{gy} = 121 \text{ kips/in}(\frac{1}{8} \text{ in}) \]
- **Shear rupture component**
  - \[ \phi 0.6 F_u A_{nv} = 131 \text{ kips/in}(\frac{1}{8} \text{ in}) \]
  - \[ \phi R_n = (121 \text{ kips/in} + 32.6 \text{ kips/in})(\frac{1}{8} \text{ in}) \]
  - \[ = 57.6 \text{ kips > 42 kips} \]

**Weld Strength**

- \[ \phi R_n = 1.392 D(2) \]
  - \[ = 1.392(4 \text{ sixteenths})(9 \text{ in.}) \]
  - \[ = 50.1 \text{ kips > 42 kips} \]

### Connecting Elements Rupture Strength at Welds

**Shear rupture strength of base metal**

\[ t_{\text{min}} = \frac{0.6 F_{y,\text{avg}} \left( \frac{\sqrt{2}}{2} \right) \left( D \right)}{0.6 F_u} = \frac{3.09 D}{F_u} \]

Column flange \( t_f = 0.780 \) in.

\[ t_{\text{min}} = \frac{3.09 D}{F_u} = \frac{(3.09)(4 \text{ sixteenths})}{65 \text{ ksi}} = 0.190 \text{ in.} \]
Plate $t_p = \frac{3}{8} \text{ in.}$

$$t_{\text{min}} = \frac{3.09 (2) D}{F_u} = \frac{(6.19) (4 \text{ sixteenths})}{58 \text{ ksi}} = 0.427 \text{ in.}$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi F_u = \frac{\frac{3}{8} \text{ in.}}{0.427 \text{ in.}} (50.1 \text{ kips}) = 44 \text{ kips}$</td>
<td>$R_u = \frac{\frac{3}{8} \text{ in.}}{0.427 \text{ in.}} (33.4 \text{ kips}) = 29.3 \text{ kips}$</td>
</tr>
<tr>
<td>44 kips &gt; 42 kips</td>
<td>29.3 kips &gt; 28 kips</td>
</tr>
</tbody>
</table>

A complete-joint penetration groove weld will transfer the entire flange force in tension and compression.  

Note: The column must be checked for stiffening requirements. For further information, see AISC Design Guide No. 13 *Wide-Flange Column Stiffening at Moment Connections – Wind and Seismic Applications.* (Carter, 1999).
Example II.B-4  Four-Bolt Unstiffened Extended End-Plate FR Moment Connection (beam-to-column flange).

Given:
Design a four-bolt unstiffened extended end-plate FR moment connection between a W18×50 beam and a W14×99 column-flange to transfer the following forces:

\[ R_D = 7 \text{ kips} \quad M_D = 42 \text{ kip-ft} \]
\[ R_L = 21 \text{ kips} \quad M_L = 126 \text{ kip-ft} \]

Use ASTM A325-N snug-tight bolts in standard holes and E70 electrodes.

a.  Use design procedure 1 (thick end-plate and smaller diameter bolts) from AISC Steel Design Guide 16 *Flush and Extended Multiple-Row Moment End-Plate Connections*.

b.  Use design procedure 2 (thin end-plate and larger diameter bolts) from AISC Steel Design Guide 16 *Flush and Extended Multiple-Row Moment End-Plate Connections*.

Material Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W18×50</th>
<th>Column W14×99</th>
<th>Plate</th>
</tr>
</thead>
<tbody>
<tr>
<td>ASTM</td>
<td>A992</td>
<td>A992</td>
<td>A36</td>
</tr>
<tr>
<td>( F_y )</td>
<td>50 ksi</td>
<td>50 ksi</td>
<td>36 ksi</td>
</tr>
<tr>
<td>( F_u )</td>
<td>65 ksi</td>
<td>65 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th></th>
<th>Beam W18×50</th>
<th>Column W14×99</th>
</tr>
</thead>
<tbody>
<tr>
<td>( d )</td>
<td>18.0 in.</td>
<td>14.2 in.</td>
</tr>
<tr>
<td>( b_f )</td>
<td>7.50 in.</td>
<td>14.6 in.</td>
</tr>
<tr>
<td>( t_f )</td>
<td>0.570 in.</td>
<td>0.780 in.</td>
</tr>
<tr>
<td>( t_u )</td>
<td>0.355 in.</td>
<td>0.355 in.</td>
</tr>
<tr>
<td>( S_e )</td>
<td>88.9 in.(^2)</td>
<td></td>
</tr>
</tbody>
</table>

Manual Table 2-3

Manual Table 1-1
Solution A:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 1.2(7 \text{kips}) + 1.6(21 \text{kips}) = 42 \text{kips}$</td>
<td>$R_u = 7 \text{kips} + 21 \text{kips} = 28 \text{kips}$</td>
</tr>
<tr>
<td>$M_u = 1.2(42 \text{kip-ft}) + 1.6(126 \text{kip-ft})$</td>
<td>$M_u = 42 \text{kip-ft} + 126 \text{kip-ft}$</td>
</tr>
<tr>
<td>= 252 kip-ft</td>
<td>= 168 kip-ft</td>
</tr>
</tbody>
</table>

Check beam flexural strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For a $W_{18 \times 50}$</td>
<td>For a $W_{18 \times 50}$</td>
</tr>
<tr>
<td>$\phi M_u = 379 \text{kip-ft} &gt; 252 \text{kip-ft}$</td>
<td>$M_u/\Omega = 252 \text{kip-ft} &gt; 168 \text{kip-ft}$</td>
</tr>
</tbody>
</table>

Extended end-plate geometric properties:

$b_p = 7 \frac{1}{2} \text{in.}$  $g = 5 \frac{1}{2} \text{in.}$  $p_{fl} = 1 \frac{1}{2} \text{in.}$  $p_{fo} = 1 \frac{1}{2} \text{in.}$  $p_{ext} = 3 \text{in.}$

Calculate secondary dimensions

$h_o = d + p_{fo} = 18.0 \text{in.} + 1 \frac{1}{2} \text{in.} = 19 \frac{1}{2} \text{in.}$

$d_o = h_o - \frac{t_f}{2} = 19 \frac{1}{2} \text{in.} - \frac{0.570 \text{in.}}{2} = 19.22 \text{in.}$

$h_i = d - p_{fl} - t_f = 18.0 \text{in.} - 1 \frac{1}{2} \text{in.} - 0.570 \text{in.} = 15.9 \text{in.}$

$d_i = h_i - \frac{t_f}{2} = 15.93 \text{in.} - \frac{0.570 \text{in.}}{2} = 15.6 \text{in.}$

$\gamma_r = 1.0$ for extended end-plates

Determine the required bolt diameter assuming no prying action

For ASTM A325-N bolts, $F'w = 90 \text{ksi}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$d_{req} = \sqrt{\frac{2M_u}{\pi \Phi F'w \left( \sum d_n \right)}}$</td>
<td>$d_{req} = \sqrt{\frac{2M_u \Omega}{\pi F'w \left( \sum d_n \right)}}$</td>
</tr>
<tr>
<td>= $\sqrt{\frac{2(252 \text{kip-ft})(12 \text{in./ft})}{\pi(0.75)(90 \text{ksi})(19.2 \text{in.} + 15.6 \text{in.})}}$</td>
<td>= $\sqrt{\frac{2(168 \text{kip-ft})(12 \text{in./ft})(2.00)}{\pi(90 \text{ksi})(19.2 \text{in.} + 15.6 \text{in.})}}$</td>
</tr>
<tr>
<td>= 0.905 \text{in.}</td>
<td>= 0.905 \text{in.}</td>
</tr>
</tbody>
</table>

Use 1-in. diameter ASTM A325-N snug-tightened bolts.

Determine the required end-plate thickness
\[ s = \frac{\sqrt{b_y g}}{2} = \frac{\sqrt{\left(\frac{7}{2} \text{ in.}\right)\left(\frac{5}{2} \text{ in.}\right)}}{2} = 3.21 \text{ in.} \]

Verify interior bolt pitch, \( p_{r,i} = 1\frac{3}{4} \text{ in.} \leq s = 3.21 \text{ in.} \quad \text{O.K.} \)

\[ Y = \frac{b_y}{2} \left[ h_i \left( \frac{1}{p_{r,i}} + \frac{1}{s} \right) + h_o \left( \frac{1}{p_{r,o}} \right) - \frac{\gamma}{2} \right] + \frac{2}{g} \left[ h_i \left( p_{r,i} + s \right) \right] \]

\[ = \frac{7\frac{1}{2} \text{ in.}}{2} \left[ \left(15.93 \text{ in.}\right) \left( \frac{1}{1\frac{1}{2} \text{ in.}} + \frac{1}{3.21 \text{ in.}} \right) + \left(19\frac{1}{2} \text{ in.}\right) \left( \frac{1}{1\frac{1}{2} \text{ in.}} \right) - \frac{\gamma}{2} \right] \]

\[ + \frac{2}{5\frac{1}{2} \text{ in.}} \left[ \left(15.93 \text{ in.}\right) \left(1\frac{1}{2} \text{ in.} + 3.21 \text{ in.}\right) \right] \]

\[ = 133 \]

\[ P_i = \frac{\pi d_t^2 F_y}{4} = \frac{\pi \left(1 \text{ in.}\right)^2 \left(90 \text{ ksi}\right)}{4} = 70.7 \text{ kips} \]

\[ M_n = 2P_i \left(\sum d_n\right) = 2 \left(70.7 \text{ kips}\right) \left(19.2 \text{ in.} + 15.6 \text{ in.}\right) = 4930 \text{ kip-in.} \]

<table>
<thead>
<tr>
<th>\text{LRFD}</th>
<th>\text{ASD}</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi M_n = 0.75(4930 \text{ kip-in.}) = 3700 \text{ kip-in.} )</td>
<td>( \frac{M_n}{\Omega} = \frac{4930 \text{ kip-in.}}{2.00} = 2460 \text{ kip-in.} )</td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( \frac{t_p \text{ req'd}}{} = \sqrt{\frac{1.11 \gamma \phi M_w}{\phi_i F_y Y}} )</td>
<td>( \frac{t_p \text{ req'd}}{} = \sqrt{\frac{1.11 \gamma \phi M_w}{\left(\frac{F_y}{\Omega \phi_i}\right) Y}} )</td>
</tr>
<tr>
<td>( = \sqrt{\frac{1.11(1.0)(3700 \text{ kip-in.})}{(0.90)(36 \text{ ksi})(133 \text{ in.})}} )</td>
<td>( = \sqrt{\frac{1.11(1.0)(2460 \text{ kip-in.}) (1.67)}{(36 \text{ ksi})(133 \text{ in.})}} )</td>
</tr>
<tr>
<td>( = 0.977 \text{ in.} )</td>
<td>( = 0.978 \text{ in.} )</td>
</tr>
</tbody>
</table>

Use a 1-in. thick end-plate.

Use a 1-in. thick end-plate.
**Calculate end-plate design strength**

From above, \( \phi M_n = 3700 \text{ kip-in.} \)

\[
\phi M_{pl} = \frac{\phi F_p t_y^2 Y}{\gamma_c} = \frac{0.90(36 \text{ ksi})(1 \text{ in.})^2(133 \text{ in.})}{1.0} = 4310 \text{ kip-in.}
\]

\[\phi M_n = \min \left( \frac{\phi M_{wp}}{\gamma_c} \right) = 3700 \text{ kip-in. or 308 kip-ft} \]

308 kip-ft > 252 kip-ft \textbf{o.k.}

**Calculate end-plate allowable strength**

From above, \( M_n / \Omega = 2464 \text{ kip-in.} \)

\[
M_{pl} / \Omega = \frac{F_p t_y^2 Y}{\Omega_y \gamma_c} = \frac{(36 \text{ ksi})(1 \text{ in.})^2(133 \text{ in.})}{1.67(1.0)} = 2860 \text{ kip-in.}
\]

\[M_n / \Omega = \min \left( \frac{M_{wp}}{\Omega_y}, \frac{M_{pl}}{\Omega_y \gamma_c} \right) = 2460 \text{ kip-in. or 205 kip-ft} \]

205 kip-ft > 168 kip-ft \textbf{o.k.}

**Check bolt shear**

Try the minimum of four bolts at tension flange and two bolts at compression flange.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = n \phi r_n = (2 \text{ bolts})(28.3 \text{ kips/bolt}) )</td>
<td>( R_n / \Omega = n r_n / \Omega = (2 \text{ bolts})(18.8 \text{ kips/bolt}) )</td>
</tr>
<tr>
<td>= 56.6 kips &gt; 42 kips \textbf{o.k.}</td>
<td>= 37.6 kips &gt; 28 kips \textbf{o.k.}</td>
</tr>
</tbody>
</table>

**Determine the required size of the beam web-to end-plate fillet weld**

\[
D_{min} = \frac{\phi F_p t_y}{2(1.39)} = \frac{0.90(50 \text{ ksi})(0.355 \text{ in.})}{2(1.39)} = 5.75 \text{ sixteenths}
\]

Use \( \frac{3}{16} \) in. fillet welds on both sides of the beam web from the inside face of the beam flange to the centerline of the inside bolt holes plus two bolt diameters.

**Determine weld size required for the end reaction**

The end reaction, \( R_u \) or \( R_c \), is resisted by weld between the mid-depth of the beam and the inside face of the compression flange or between the inner row of tension bolts plus two bolt diameters, which ever is smaller. By inspection the former governs for this example.
\[ l = \frac{d}{2}, \quad t_f = \frac{18.0 \text{ in.}}{2} - 0.570 \text{ in.} = 8.43 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{\text{min}} = \frac{R_u}{2(1.39)l} = \frac{42 \text{ kips}}{2(1.39)(8.43 \text{ in.})} )</td>
<td>( D_{\text{min}} = \frac{R_u}{2(0.928)l} = \frac{28 \text{ kips}}{2(0.928)(8.43 \text{ in.})} )</td>
</tr>
<tr>
<td>( = 1.79 \rightarrow 3 \text{ sixteenths (minimum size)} )</td>
<td>( = 1.79 \rightarrow 3 \text{ sixteenths (minimum size)} )</td>
</tr>
</tbody>
</table>

Use \( \frac{1}{8}\)-in. fillet weld on both sides of the beam web below the tension-bolt region. Use \( \frac{1}{16}\)-in. fillet weld on both sides of the beam web below the tension-bolt region.

Table J2.4

Connecting Elements Rupture Strength at Welds

Shear rupture strength of base metal

\[ t_{\text{min}} = \frac{0.6F_{\text{ex}} \sqrt{2}}{2} \left( \frac{D}{16} \right) = \frac{3.09D}{F_u} \]

Beam web \( t_w = 0.355 \text{ in.} \)

\[ t_{\text{min}} = \frac{3.09(2)D}{F_u} = \frac{(6.19)(3 \text{ sixteenths})}{65 \text{ ksi}} = 0.285 \text{ in.} \]

End plate \( t_p = 1 \text{ in.} \)

\[ t_{\text{min}} = \frac{3.09D}{F_u} = \frac{(3.09)(3 \text{ sixteenths})}{58 \text{ ksi}} = 0.160 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = 1.392Dl \left( \frac{0.355 \text{ in.}}{0.285 \text{ in.}} \right) )</td>
<td>( R_u = 0.928Dl \left( \frac{0.355 \text{ in.}}{0.476 \text{ in.}} \right) )</td>
</tr>
<tr>
<td>( = 1.392(3 \text{ sixteenths})(2)(8.43 \text{ in.}) \left( \frac{0.355 \text{ in.}}{0.285 \text{ in.}} \right) )</td>
<td>( = 0.928(3 \text{ sixteenths})(2)(8.43 \text{ in.}) \left( \frac{0.355 \text{ in.}}{0.285 \text{ in.}} \right) )</td>
</tr>
</tbody>
</table>
| \( = 87.7 \text{ kips} > 42 \text{ kips} \) | \( = 58.5 \text{ kips} > 28 \text{ kips} \)

Determine required fillet weld size for the beam flange to end-plate connection

\[ l = 2(b_f + t_f) - t_w = 2(7.50 \text{ in.} + 0.570 \text{ in.}) - 0.355 \text{ in.} = 15.8 \text{ in.} \]
\[ P_{of} = \sum d_n = \frac{2(252 \text{ kip-ft})(12 \text{ in./ft})}{19.2 \text{ in.} + 15.6 \text{ in.}} = 173 \text{ kips} \]

\[ D_{min} = \frac{P_{of}}{1.5(1.39)l} = \frac{173 \text{ kips}}{1.5(1.39)(15.8 \text{ in.})} = 5.27 \rightarrow 6 \text{ sixteenths (minimum size)} \]

**LRFD**

\[ P_{of} = \sum d_n = \frac{2(168 \text{ kip-ft})(12 \text{ in./ft})}{19.2 \text{ in.} + 15.6 \text{ in.}} = 116 \text{ kips} \]

\[ D_{min} = \frac{P_{of}}{1.5(0.928)l} = \frac{116 \text{ kips}}{1.5(0.928)(15.8 \text{ in.})} = 5.27 \rightarrow 6 \text{ sixteenths (minimum size)} \]

**ASD**

Note that the 1.5 factor is from Specification J2.4.

Use 7/16-in. fillet welds at beam tension flange. Welds at compression flange may be 1/4-in. fillet welds (minimum size per Specification Table J2.4).

**Connecting Elements Rupture Strength at Welds**

**Tension rupture strength of base metal**

\[ R_u = F_u A_t \]

\[ t_{min} = \frac{0.6F_{10u}}{F_u} \left( \frac{\sqrt{2}}{2} \right) \left( \frac{D}{16} \right) = \frac{1.86D}{F_u} \]

Beam flange; \( t_f = 0.570 \text{ in.} \)

\[ t_{min} = \frac{1.86(2)D}{F_u} = \frac{(3.71)(6 \text{ sixteenths})}{65 \text{ksi}} = 0.343 \text{ in.} \]

End plate; \( t_p = 1.00 \text{ in.} \)

\[ t_{min} = \frac{1.86D}{F_u} = \frac{(1.86)(6 \text{ sixteenths})}{58 \text{ksi}} = 0.192 \text{ in.} \]
Solution B:

Only those portions of the design that vary from the solution “A” calculations are presented here.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Determine required end-plate thickness</strong></td>
<td><strong>Determine required end-plate thickness</strong></td>
</tr>
<tr>
<td>( \phi = 0.90 )</td>
<td>( \Omega = 1.67 )</td>
</tr>
<tr>
<td>( t_{req} = \frac{\gamma \cdot M}{\phi \cdot F \cdot Y} )</td>
<td>( t_{req} = \sqrt{\frac{\gamma \cdot M \cdot \Omega}{F \cdot Y}} )</td>
</tr>
<tr>
<td>( = \frac{1.0 \cdot (252 \text{ kip-ft})(12 \text{ in./ft})}{0.90 \cdot (36 \text{ ksi})(133 \text{ in.})} )</td>
<td>( = \sqrt{\frac{1.0 \cdot (168 \text{ kip-ft})(12 \text{ in./ft})(1.67)}{(36 \text{ ksi})(133 \text{ in.})}} )</td>
</tr>
<tr>
<td>= 0.84 in.</td>
<td>= 0.84 in.</td>
</tr>
<tr>
<td>Use ( t_p = \frac{3}{8} ) in.</td>
<td>Use ( t_p = \frac{3}{8} ) in.</td>
</tr>
</tbody>
</table>

Select a trial bolt diameter and calculate the maximum prying forces

Try 1-in. diameter bolts.

\[ w' = \frac{b_p}{2} - (d_b + \frac{d_b}{8} \text{ in.}) = \frac{7.50 \text{ in.}}{2} - (1\frac{3}{8} \text{ in.}) = 2.69 \text{ in.} \]

\[ a_t = 3.682 \left( \frac{t_p}{d_b} \right)^3 - 0.085 = 3.682 \left( \frac{\frac{3}{8} \text{ in.}}{1 \text{ in.}} \right)^3 - 0.085 = 2.38 \]

\[ F_t' = \frac{t_p^2 F_{sp} \left[ 0.85 \left( \frac{b_p}{2} \right) + 0.80 w' \right] + \frac{\pi d_b^2 F_{st}}{8}}{4p_{f,i}} \]

\[ = \frac{(\frac{3}{8})^3 (36) \left[ 0.85 \left( \frac{7\frac{3}{8}}{2} \right) + 0.80 (2.69) \right]}{4(1\frac{3}{8})} \]

\[ = \frac{\pi (1)^3 (90)}{8} + \frac{4(1\frac{3}{8})}{4(1\frac{3}{8})} \]

\[ = 30.4 \text{ kips} \]
\[ Q_{\text{max}} = \frac{w't_p^2}{4a_i} \sqrt{F_{sp}^2 - 3 \left( \frac{F_{o}'}{w't_p} \right)^2} \]

\[ = \frac{(2.69)(\%)^2}{4(2.38)} \sqrt{36^2 - 3 \left( \frac{30.4}{2.69(\%)} \right)^2} \]

\[ = 6.10 \text{ kips} \]

\[ a_o = \min\left[ a_i, p_{ext} - p_{f.o} \right] = \min[2.38 \text{ in.}, 1\frac{1}{2} \text{ in.}] = 1\frac{1}{2} \text{ in.} \]

\[ F_o' = \frac{1}{F_i} \left( \frac{p_i}{p_o} \right) = (30.4 \text{ kips})\left( \frac{1\frac{1}{2} \text{ in.}}{1\frac{1}{2} \text{ in.}} \right) \]

\[ = 30.4 \text{ kips} \]

\[ Q_{\text{max}} = \frac{w't_p^2}{4a_o} \sqrt{F_{sp}^2 - 3 \left( \frac{F_{o}'}{w't_p} \right)^2} \]

\[ = \frac{(2.69)(\%)^2}{4(1\frac{1}{2})} \sqrt{36^2 - 3 \left( \frac{30.4}{2.69(\%)} \right)^2} \]

\[ = 9.68 \text{ kips} \]

Calculate the connection available strength for the limit state of bolt rupture with prying action

\[ P_i = \frac{\pi d^2 F_{ut}}{4} = \frac{\pi (1 \text{ in.})^2 (90 \text{ ksi})}{4} = 70.7 \text{ kips} \]

Unmodified Bolt Pretension, \( T_{bo} = 51 \text{ kips} \)

Modify bolt pretension for the snug-tight condition.

\[ T_s = \frac{T_{bo}}{4} = \frac{51 \text{ kips}}{4} = 12.8 \text{ kips} \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| $\phi M_q = \begin{align*}
\phi & \left[ 2\left(P_i - Q_{\text{max}}\right)d_o + 2\left(P_i - Q_{\text{max}}\right)d_i \right] \\
& \left[ 2\left(P_i - Q_{\text{max}}\right)d_o + 2\left(T_i\right)d_i \right] \\
& \left[ 2\left(P_i - Q_{\text{max}}\right)d_i + 2\left(T_i\right)d_o \right] \\
& \left[ 2\left(T_i\right)(d_o + d_i) \right] \\
\end{align*}$ | $\begin{align*}
M_q & = \frac{1}{\Omega} \left[ (P_i - Q_{\text{max}})d_o + 2(P_i - Q_{\text{max}})d_i \right] \\
& \left[ 2(P_i - Q_{\text{max}})d_o + 2(T_i)d_i \right] \\
& \left[ 2(P_i - Q_{\text{max}})d_i + 2(T_i)d_o \right] \\
& \left[ 2(T_i)(d_o + d_i) \right] \\
\end{align*}$ |

$= \max \begin{align*}
0.75 \begin{bmatrix} 2(70.7 - 9.68)(19.2) \\
+2(70.7 - 6.10)(15.6) \end{bmatrix} \\
0.75 \begin{bmatrix} 2(70.7 - 9.68)(19.2) \\
+2(12.8)(15.6) \end{bmatrix} \\
0.75 \begin{bmatrix} 2(70.7 - 6.10)(15.6) \\
+2(12.8)(19.2) \end{bmatrix} \\
0.75 \begin{bmatrix} 2(12.8)(19.215 + 15.6) \end{bmatrix} \\
\end{align*} = 3275 \text{ kip-in.}$

$= \max \begin{align*}
2183 \text{ kip-in.} \\
1373 \text{ kip-in.} \\
1257 \text{ kip-in.} \\
446 \text{ kip-in.} \\
\end{align*} = 2138 \text{ kip-in.}$

$= \max \begin{align*}
3275 \text{ kip-in.} \\
2059 \text{ kip-in.} \\
1885 \text{ kip-in.} \\
669 \text{ kip-in.} \\
\end{align*} = 3275 \text{ kip-in.}$

$M_q / \Omega = \frac{3275}{1.5} = 2183 \text{ kip-in.} > 182 \text{ kip-ft} > 168 \text{ kip-ft}$

For **Example IIB-4**, design procedure 1 produced a design with a 1-in. thick end-plate and 1-in. diameter bolts. Design procedure 2 produced a design with a 7\% in. thick end-plate and 1-in. diameter bolts. Either design is acceptable. Design procedure 1 did not produce a smaller bolt diameter for this example, although in general it should result in a thicker plate and smaller diameter bolt than design procedure 2. It will be noted that the bolt stress is lower in design procedure 1 than in design procedure 2.
Chapter IIC
Bracing and Truss Connections

The design of bracing and truss connections is covered in Part 13 of the AISC Steel Construction Manual.
Example II.C-1  Truss Support Connection

Given:

Determine:

a. the connection requirements between gusset and column,

b. the required gusset size and the weld requirements for member $U_0L_1$ at the gusset.

$R_D = 18.5$ kips
$R_L = 55.5$ kips

Use 3/8-in. diameter ASTM A325-N bolts in standard holes and E70 electrodes.
Material Properties:
Top Chord \(WT8\times38.5\) ASTM A992 \(F_y = 50\ ksi\) \(F_u = 65\ ksi\) \(\text{Manual}\) \(\text{Tables 2-3 and 2-4}\)
Column \(W12\times50\) ASTM A992 \(F_y = 50\ ksi\) \(F_u = 65\ ksi\)
Brace \(2L4\times3\frac{1}{2}\times\frac{3}{8}\) ASTM A36 \(F_y = 36\ ksi\) \(F_u = 58\ ksi\)
Gusset Plate \(\text{ASTM A36}\) \(F_y = 36\ ksi\) \(F_u = 58\ ksi\)
Clip Angles \(2L4\times4\) ASTM A36 \(F_y = 36\ ksi\) \(F_u = 58\ ksi\)

Geometric Properties:
Top Chord \(WT8\times38.5\) \(t_w = 0.455\ in.\) \(\text{Manual}\) \(\text{Table 1-8}\)
Brace \(2L4\times3\frac{1}{2}\times\frac{3}{8}\) \(t = 0.375\ in.\)

Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace axial load</td>
<td>Brace axial load</td>
</tr>
<tr>
<td>(R_u = 174\ kips)</td>
<td>(R_u = 116\ kips)</td>
</tr>
<tr>
<td>Truss end reaction</td>
<td>Truss end reaction</td>
</tr>
<tr>
<td>(R_u = 1.2(18.5) + 1.6(55.5) = 111\ kips)</td>
<td>(R_u = 18.5 + 55.5 = 74\ kips)</td>
</tr>
<tr>
<td>Top chord axial load</td>
<td>Top chord axial load</td>
</tr>
<tr>
<td>(R_u = 140\ kips)</td>
<td>(R_u = 93\ kips)</td>
</tr>
</tbody>
</table>

Design the weld connecting the diagonal to the gusset

Note: Specification Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.

For \(\frac{3}{8}\)-in. angles, \(D_{min} = 3\), try \(\frac{3}{8}\)-in. fillet welds, \(D = 4\).

Determine minimum gusset thickness based on weld size

Note: This check is required only when the stress in the plate is unknown. Since, in this case, the stress is known and gusset block shear rupture will be checked, this is not required. It is included here to illustrate the method.

For two \(\frac{3}{8}\)-in. fillet welds,
\[ I_{min} = \frac{6.19D}{F_y} = \frac{6.19(4)}{58 \text{ ksi}} = 0.427\ in. \]
Use a ½ in. gusset plate. With the diagonal to gusset welds determined, a gusset plate layout as shown in Figure (c) can be made.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Design bolts connecting clip angles to column (shear and tension)</strong></td>
<td><strong>Design bolts connecting clip angles to column (shear and tension)</strong></td>
</tr>
<tr>
<td>The number of ½-in. diameter ASTM A325-N bolts required for shear only is as follows:</td>
<td></td>
</tr>
<tr>
<td>$n_{\text{min}} = \frac{R_u}{\phi R_u} = \frac{111 \text{ kips}}{21.6 \text{ kips/bolt}} = 5.14$</td>
<td>The number of ½-in. diameter ASTM A325-N bolts required for shear only is as follows:</td>
</tr>
<tr>
<td>$n_{\text{min}} = \frac{R_u}{\phi R_u} = \frac{74 \text{ kips}}{14.4 \text{ kips/bolt}} = 5.14$</td>
<td></td>
</tr>
<tr>
<td>Manual Table 7-1</td>
<td></td>
</tr>
</tbody>
</table>

Try a clip angle thickness of ½ in. For a trial calculation, the number of bolts was increased to 10 in pairs at 3-in. spacing; this is done to “square off” the connection as shown.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>With 10 bolts, $f_u = \frac{R_u}{nA_k} = \frac{111 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)} = 18.5 \text{ ksi}$</td>
<td>With 10 bolts, $f_u = \frac{R_u}{nA_k} = \frac{74 \text{ kips}}{(10 \text{ bolts})(0.601 \text{ in.}^2)} = 12.3 \text{ ksi}$</td>
</tr>
<tr>
<td>The eccentric moment at the faying surface is as follows: $M_u = R_u e = (111 \text{ kips})(6.1 \text{ in.}) = 677 \text{ kip-in.}$</td>
<td>The eccentric moment at the faying surface is as follows: $M_u = R_u e = (74 \text{ kips})(6.1 \text{ in.}) = 451 \text{ kip-in.}$</td>
</tr>
</tbody>
</table>

For the bolt group, the Case II approach of Manual Part 7 can be used. Thus, the maximum tensile force per bolt, $T$, is given by:

$n^* =$ number of bolts above the neutral axis $= 4$ bolts

$d_m =$ moment arm between resultant tensile force and resultant compressive force $= 9$ in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_u = \frac{M_u}{n^*d_m} = \frac{677 \text{ kip-in.}}{(4 \text{ bolts})(9 \text{ in.})} = 18.8 \text{ kips/bolt}$</td>
<td>$T_u = \frac{M_u}{n^*d_m} = \frac{451 \text{ kip-in.}}{(4 \text{ bolts})(9 \text{ in.})} = 12.5 \text{ kips/bolt}$</td>
</tr>
</tbody>
</table>

Check design tensile strength of bolts

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$F_u ' = 1.3 \frac{F_{\text{un}}}{\phi F_{\text{un}}} f_u \leq F_{\text{un}}$</td>
<td>$F_u ' = 1.3 \frac{F_{\text{un}}}{\phi F_{\text{un}}} f_u \leq F_{\text{un}}$</td>
</tr>
<tr>
<td>$= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(48 \text{ ksi})}(18.5 \text{ ksi})$</td>
<td>$= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{48 \text{ ksi}}(12.3 \text{ ksi})$</td>
</tr>
<tr>
<td>$= 70.8 \text{ ksi} &lt; 90 \text{ ksi}$ o.k.</td>
<td>$= 70.1 \text{ ksi} &lt; 90 \text{ ksi}$ o.k.</td>
</tr>
</tbody>
</table>

Eqn. J3-3a (LRFD) and J3-3b (ASD) Table J3.2
\[
B = \phi F_u A_e = 0.75 \left( 70.8 \text{ ksi} \right) \left( 0.601 \text{ in.}^2 \right)
\]
\[= 31.9 \text{ kips} > 18.8 \text{ kips} \quad \text{o.k.}\]

\[
B = \frac{F_u}{\Omega} A_b = \frac{70.1 \text{ ksi}}{2.00} \left( 0.601 \text{ in.}^2 \right)
\]
\[= 21.1 \text{ kips} > 12.5 \text{ kips} \quad \text{o.k.}\]

Check the clip angles

Check prying action.

\[p = 3 \text{ in.}\]

\[b = 2 \text{ in.} - \frac{\frac{3}{4} \text{ in.}}{2} = 1.69 \text{ in.}\]

Note: 1\(\frac{1}{4}\) in. entering and tightening clearance accommodated, \text{o.k.}

\[a = 4 \text{ in.} - 2 \text{ in.} \leq 1.25b \text{ (for calculation purposes)}\]

\[= 2 \text{ in.} < 1.25 \left( 1.69 \text{ in.} \right) = 2.11 \text{ in.} \quad \text{o.k.}\]

\[b' = b - \frac{d}{2} = 1.69 \text{ in.} - \frac{\left( \frac{3}{4} \text{ in.} \right)}{2} = 1.25 \text{ in.}\]

\[a' = a + \frac{d}{2} = 2 \text{ in.} + \frac{\left( \frac{3}{4} \text{ in.} \right)}{2} = 2.44 \text{ in.}\]

\[\rho = \frac{b'}{a'} = \frac{1.25 \text{ in.}}{2.44 \text{ in.}} = 0.512\]

\[\delta = 1 - \frac{d'}{p} = 1 - \frac{\frac{3}{4} \text{ in.}}{3 \text{ in.}} = 0.688\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(t_c = \sqrt{\frac{4.44 Bb'}{pF_u}} = \sqrt{\frac{4.44 \left( 31.9 \text{ kips} \right) \left( 1.25 \text{ in.} \right)^2}{\left( 3 \text{ in.} \right) \left( 58 \text{ ksi} \right)}})</td>
<td>(t_c = \sqrt{\frac{6.66 Bb'}{pF_u}} = \sqrt{\frac{6.66 \left( 21.1 \text{ kips} \right) \left( 1.25 \text{ in.} \right)^2}{\left( 3 \text{ in.} \right) \left( 58 \text{ ksi} \right)}})</td>
</tr>
<tr>
<td>= 1.01 in.</td>
<td>= 1.01 in.</td>
</tr>
</tbody>
</table>

\[\alpha' = \frac{1}{\delta (1 + \rho)} \left[ \left( \frac{t_c}{t} \right)^2 - 1 \right]\]

\[= \frac{1}{0.688 \left( 1 + 0.512 \right)} \left[ \left( \frac{1.01 \text{ in.}}{0.625 \text{ in.}} \right)^2 - 1 \right]\]

\[= 1.55\]
Since $\alpha^+ > 1$, use $\alpha^+ = 1.0$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$T_a = B \left( \frac{f}{f_c} \right)^2 (1 + \delta \alpha^+)$</td>
<td>$T_a = B \left( \frac{f}{f_c} \right)^2 (1 + \delta \alpha^+)$</td>
</tr>
<tr>
<td>$(31.9 \text{ kips} \left( \frac{0.625 \text{ in.}}{1.01 \text{ in.}} \right)^2 \left[ 1 + 0.688(1) \right]$</td>
<td>$(21.1 \text{ kips} \left( \frac{0.625 \text{ in.}}{1.01 \text{ in.}} \right)^2 \left[ 1 + 0.688(1) \right]$</td>
</tr>
<tr>
<td>$= 20.6 \text{ kips} &gt; 18.8 \text{ kips}$</td>
<td>$= 13.6 \text{ kips} &gt; 12.5 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Check shear yielding of the clip angles**

$\phi R_a = \phi \left( 0.6 F_y \right) A_g$

$= 1.0(0.6)(36 \text{ ksi})[2(15.0 \text{ in.})\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right)]$

$= 405 \text{ kips} > 111 \text{ kips}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_a / \Omega = \left( \frac{0.6 F_y}{\Omega} \right) A_g$</td>
<td>$R_a / \Omega = \left( \frac{0.6 F_y}{\Omega} \right) A_g$</td>
</tr>
<tr>
<td>$= (0.6)(36 \text{ ksi)}[2(15.0 \text{ in.})\left( \frac{% \text{ in.}}{% \text{ in.}} \right)]$</td>
<td>$= (0.6)(36 \text{ ksi)}[2(15.0 \text{ in.})\left( \frac{% \text{ in.}}{% \text{ in.}} \right)]$</td>
</tr>
<tr>
<td>$= 270 \text{ kips} &gt; 74 \text{ kips}$</td>
<td>$= 270 \text{ kips} &gt; 74 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Check shear rupture of the angles**

$A_g = 2\left[ 15.0 \text{ in.} - 5(1 \text{ in.}) \right]\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right) = 12.5 \text{ in.}^2$

$\phi R_n = \phi \left( 0.6 F_y \right) A_g$

$= 0.75(0.6)(58 \text{ ksi})(12.5 \text{ in.}^2)$

$= 326 \text{ kips} > 111 \text{ kips}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_n / \Omega = \left( \frac{0.6 F_y}{\Omega} \right) A_g$</td>
<td>$R_n / \Omega = \left( \frac{0.6 F_y}{\Omega} \right) A_g$</td>
</tr>
<tr>
<td>$= (0.6)(58 \text{ ksi})(12.5 \text{ in.}^2)$</td>
<td>$= (0.6)(58 \text{ ksi})(12.5 \text{ in.}^2)$</td>
</tr>
<tr>
<td>$= 218 \text{ kips} &gt; 74 \text{ kips}$</td>
<td>$= 218 \text{ kips} &gt; 74 \text{ kips}$</td>
</tr>
</tbody>
</table>

**Check block shear rupture of the clip angles**

Assume uniform tension stress, use $U_{bs} = 1.0$.

Gross area subject to shear, $A_{sv} = 2\left[ 15.0 \text{ in.} - 1\frac{3}{4} \text{ in.} \right]\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right) = 16.9 \text{ in.}^2$

Net area subject to shear, $A_{nv} = 16.9 \text{ in.}^2 - 2\left[ 4\frac{3}{4} \text{ in.} \right]\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right) = 11.3 \text{ in.}^2$

Net area subject to tension, $A_n = 2\left[ 2 \text{ in.} \right]\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right) - 0.5\left( 1 \text{ in.} \right)\left( \frac{\% \text{ in.}}{\% \text{ in.}} \right) = 1.88 \text{ in.}^2$
\[
\phi R_u = \phi \left[ F_u A_u + \min \left\{ 0.6 F_u A_u, 0.6 F_u A_{uy} \right\} \right] \\
= 0.75 \left\{ (58 \text{ ksi})(1.88 \text{ in.}^2) + \min \left\{ 0.6(36 \text{ ksi})(16.9 \text{ in.}^2), 0.6(58 \text{ ksi})(11.3 \text{ in.}^2) \right\} \right\} \\
= 356 \text{ kips} > 111 \text{ kips} \hspace{1cm} \text{o.k.}
\]
\[
R_u / \Omega = \frac{F_u A_u + \min\left\{ 0.6 F_u A_u, 0.6 F_u A_{uy} \right\}}{\Omega} \\
= \frac{(58 \text{ ksi})(1.88 \text{ in.}^2) + \min\left\{ 0.6(36 \text{ ksi})(16.9 \text{ in.}^2), 0.6(58 \text{ ksi})(11.3 \text{ in.}^2) \right\}}{2.00} \\
= 237 \text{ kips} > 74 \text{ kips} \hspace{1cm} \text{o.k.}
\]

Use 2L4x4x\%.  

Check bearing and tearout  

The clear edge distance, \( L_c \), for the top bolts is  
\[ L_c = L_e - \frac{d}{2} \]  
where \( L_e \) is the distance to the center of the hole. Thus,  
\[ L_c = 1\% \text{ in.} - \frac{1}{2}(\% \text{ in.}) = 1.03 \text{ in.} \]  

The bearing/tearout capacity of the top bolt is as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \phi R_u = \phi \left[ F_u A_u + \min \left{ 0.6 F_u A_u, 0.6 F_u A_{uy} \right} \right] ]</td>
<td>[ R_u / \Omega = \frac{F_u A_u + \min\left{ 0.6 F_u A_u, 0.6 F_u A_{uy} \right}}{\Omega} ]</td>
</tr>
</tbody>
</table>
| \[ = 0.75 \left\{ (58 \text{ ksi})(1.88 \text{ in.}^2) + \min \left\{ 0.6(36 \text{ ksi})(16.9 \text{ in.}^2), 0.6(58 \text{ ksi})(11.3 \text{ in.}^2) \right\} \right\} \] | \[
= \frac{1.2(1.03 \text{ in.})(\% \text{ in.})(58 \text{ ksi})}{2.00} \leq \frac{2.4(\% \text{ in.})(\% \text{ in.})(58 \text{ ksi})}{2.00} \\
= 22.4 \text{ kips} < 38.1 \text{ kips} \] |
| = 356 \text{ kips} > 111 \text{ kips} o.k. | = 237 \text{ kips} > 74 \text{ kips} o.k. |
The bearing/tearout capacity of each of the remaining bolts is \( L_c = 3 \text{ in.} - 1(\frac{3}{8} \text{ in.}) = 2.06 \text{ in.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = \phi_1.2L_tF_n &lt; 2.4dtF_u )</td>
<td></td>
</tr>
<tr>
<td>( = 0.75(1.2)(2.06 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) )</td>
<td></td>
</tr>
<tr>
<td>( \leq 0.75(2.4)(\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi}) )</td>
<td></td>
</tr>
<tr>
<td>( = 67.2 \text{ kips} &lt; 57.1 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( = 57.1 \text{ kips/bolt} )</td>
<td></td>
</tr>
<tr>
<td>( \phi R_n = (33.6 \text{ kips})(2 \text{ bolts}) + (57.1 \text{ kips})(8 \text{ bolts}) )</td>
<td></td>
</tr>
<tr>
<td>( = 524 \text{ kips} &gt; 111 \text{ kips} ) o.k.</td>
<td></td>
</tr>
<tr>
<td>( \frac{r_o}{\Omega} = \frac{1.2L_tF_n}{\Omega} \leq \frac{2.4dtF_u}{\Omega} )</td>
<td></td>
</tr>
<tr>
<td>( = \frac{1.2(2.06 \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi})}{2.00} )</td>
<td></td>
</tr>
<tr>
<td>( \leq \frac{2.4(\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(58 \text{ ksi})}{2.00} )</td>
<td></td>
</tr>
<tr>
<td>( = 44.8 \text{ kips} &lt; 38.0 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( = 38.1 \text{ kips/bolt} )</td>
<td></td>
</tr>
</tbody>
</table>

Therefore the bearing/tearout capacity of the bolt group is

\( \phi R_n = (33.6 \text{ kips})(2 \text{ bolts}) + (57.1 \text{ kips})(8 \text{ bolts}) \)
\( = 44.8 \text{ kips} < 38.0 \text{ kips} \)
\( = 38.1 \text{ kips/bolt} \)

Therefore the bearing/tearout capacity of the bolt group is

\( R_n / \Omega = (22.4 \text{ kips})(2 \text{ bolts}) + (38.1 \text{ kips})(8 \text{ bolts}) \)
\( = 350 \text{ kips} > 74 \text{ kips} \) o.k.

**Design clip angle-to-gusset connection**

The minimum weld size is \( \frac{3}{8} \text{ in.} \) with top chord slope being \( \frac{1}{2} \) on 12, the horizontal welds are as shown due to the square cut end. Use the average length. Then,

\( l = 15.0 \text{ in.} \)

\( kl = \frac{3 \frac{3}{8} \text{ in.} + 2 \frac{3}{8} \text{ in.}}{2} = 3.06 \text{ in.} \)

\( k = \frac{kl}{l} = \frac{3.06 \text{ in.}}{15.0 \text{ in.}} = 0.204 \)

With \( \theta = 0^\circ \), by interpolation \( x = 0.030 \) and

\( a = 0.030 \text{ in.} \)

\( al + xl = 10.1 \text{ in.} \)

\( a = \frac{10.1 \text{ in.} - 0.030(15.0 \text{ in.})}{15.0 \text{ in.}} = 0.643 \)

By interpolation, \( C = 1.50 \)
\[
\begin{array}{c|c|c|c|c}
\text{LRFD} & \text{ASD} & \\
\hline
D_{\text{req}} &= \frac{R_u}{2(\phi CC/l)} & \frac{\Omega R_u}{2(\phi \Omega)} \\
&= \frac{111 \text{kips}}{2(0.75)(1.50)(1.0)(15.0 \text{ in.})} & \frac{(2.0)(74 \text{kips})}{2(1.50)(1.0)(15.0 \text{ in.})} \\
&= 3.29 \rightarrow 4 \text{ sixteenths} & \rightarrow 3.29 \rightarrow 4 \text{ sixteenths} \\
\text{Use } 1/8\text{-in. fillet welds.} & \text{Use } 1/4\text{-in. fillet welds.} & \\
\end{array}
\]

**Design chord extension plate**

The chord stem extension plate thickness should match or slightly exceed that of the tee stem; use 1/2-in. plate.

**Check tension yielding on the Whitmore section**

\[L_u = 4 \text{ in.} + 2(8 \text{ in.}) \tan 30^\circ = 13.2 \text{ in.}\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\phi R_u = \phi F_y A_g = 0.9(36 \text{ ksi})(13.2 \text{ in.})(\frac{3}{8} \text{ in.})] &amp; [\frac{R_u}{\Omega} = \frac{F_y A_g}{\Omega} = \frac{(36 \text{ ksi})(13.2 \text{ in.})(\frac{3}{8} \text{ in.})}{1.67}]</td>
<td></td>
</tr>
<tr>
<td>= 214 kips &gt; 174 kips \text{o.k.} &amp; = 142 kips &gt; 116 kips \text{o.k.}</td>
<td></td>
</tr>
</tbody>
</table>

**Check block shear rupture of the clip angles**

Assume uniform tension stress, use \(U_{ts} = 1.0\)  

Section J4.3

Gross area subject to shear, \(A_{gy} = 2(8.0 \text{ in.})(\frac{\sqrt{3}}{2} \text{ in.}) = 8.0 \text{ in.}^2\)

Net area subject to shear, \(A_w = A_{gy} = 8.0 \text{ in.}^2\)

Net area subject to tension, \(A_n = (4 \text{ in.})(\frac{\sqrt{3}}{2} \text{ in.}) = 2.0 \text{ in.}^2\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[\phi R_u = \phi F_u A_m U_{bs} + \min \left( \phi F_y A_g, \phi F_u A_{mv} \right)] &amp;</td>
<td></td>
</tr>
</tbody>
</table>
| \[= 0.75(58 \text{ ksi})(2.0 \text{ in.}^2)\] & Eqn. J4-5  
| \[+ \min \left\{ \frac{0.75(0.6)(36 \text{ ksi})}{1.50(1.0)(15.0 \text{ in.})}, \frac{0.75(0.6)(58 \text{ ksi})}{1.50(1.0)(15.0 \text{ in.})} \right\}\] & \[\frac{R_u}{\Omega} = \frac{F_u A_m U_{bs}}{\Omega} + \min \left( \frac{0.6 F_y A_g}{\Omega}, \frac{F_u A_{mv}}{\Omega} \right)\] |
| = 217 kips > 174 kips \text{o.k.} & = 144 kips > 116 kips \text{o.k.} |
The gusset width must be such that the groove weld connecting it to the stem of the tee can transfer the tee axial force between the gusset and the top chord (note that the slight slope of the top chord has been ignored). The required length is

\[
L_{req} = \frac{R_u}{\phi(0.6P_t)t}
\]

\[
= \frac{140 \text{ kips}}{0.75(0.6)(58 \text{ ksi})(0.455 \text{ in.})} = 11.8 \text{ in.}
\]

Use \(L = 15\) in.

\[
L_{req} = \frac{\Omega R_y}{(0.6P_t)t}
\]

\[
= \frac{(2.00)(93 \text{ kips})}{(0.6)(58 \text{ ksi})(0.455 \text{ in.})} = 11.8 \text{ in.}
\]

Use \(L = 15\) in.

The gusset depth depends upon the connection angles. From a scaled layout, the gusset must extend 1'-0" below the tee stem.

Use PL\(\frac{1}{2}\times12\times1\'-3\).
Example II.C-2  Bracing Connection

Given:

Design the diagonal bracing connection between the W12×87 brace and the W18×106 beam and the W14×605 column.

<table>
<thead>
<tr>
<th></th>
<th>$T_u$</th>
<th>$T_a$</th>
<th>$R_u$</th>
<th>$R_a$</th>
<th>$P_u$</th>
<th>$P_a$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace Axial Load</td>
<td>675 kips</td>
<td>450 kips</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Beam End Reaction</td>
<td>15 kips</td>
<td>10 kips</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Column Axial Load</td>
<td>422 kips</td>
<td>281 kips</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Use 3⁄8-in. diameter ASTM A325-N bolts in standard holes and E70 electrodes.
Material Properties:

- Brace W12×87 ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi  Manual  Tables 2-3 and 2-4
- Beam W18×106 ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi
- Column W14×605 ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi
- Gusset Plate ASTM A36  $F_y = 36$ ksi  $F_u = 58$ ksi

Geometric Properties:

- Brace W12×87 $A=25.6$ in.$^2$  $d=12.5$ in.  $t_w=0.515$ in.  $b_f=12.1$ in.  $t_f=0.810$ in.  Manual  Table 1-1
- Beam W18×106  $d=18.7$ in.  $t_w=0.590$ in.  $b_f=11.2$ in.  $t_f=0.940$ in.  $k=1.34$
- Column W14×605  $d=20.9$ in.  $t_w=2.60$ in.  $b_f=17.4$ in.  $t_f=4.16$ in.
Solution:

**Brace-to-gusset connection**

Distribute brace force in proportion to web and flange areas.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{sf} = \frac{P_{sy} b_f l_f}{A} = \frac{(675 \text{ kips})(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2} )</td>
<td>( P_{sf} = \frac{P_{sy} b_f l_f}{A} = \frac{(450 \text{ kips})(12.1 \text{ in.})(0.810 \text{ in.})}{25.6 \text{ in.}^2} )</td>
</tr>
<tr>
<td>= 259 kips</td>
<td>= 172 kips</td>
</tr>
</tbody>
</table>

\( P_{aw} = P_a - 2P_{sf} = 675 \text{ kips} - 2(259 \text{ kips}) \)

\( = 157 \text{ kips} \)

\( P_{af} = P_a - 2P_{sf} = 450 \text{ kips} - 2(172 \text{ kips}) \)

\( = 106 \text{ kips} \)

**Design brace-flange-to-gusset connection.**

Determine number of \( \frac{3}{8} \)-in. diameter ASTM A325-N bolts required on the brace side for single shear.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{min} = \frac{P_{sf}}{\phi F_a} = \frac{259 \text{ kips}}{21.6 \text{ kips/bolt}} = 11.9 \rightarrow 12 \text{ bolts} )</td>
<td>( n_{min} = \frac{P_{sf}}{r_u/\Omega} = \frac{172 \text{ kips}}{14.4 \text{ kips/bolt}} = 11.9 \rightarrow 12 \text{ bolts} )</td>
</tr>
</tbody>
</table>

On the gusset side, since these bolts are in double shear, half as many bolts will be required. Try six rows of two bolts each through the flange, six bolts through the gusset, and 2L4x4x\( \frac{3}{8} \) angles (\( A = 10.9 \text{ in.}^2 \quad \overline{x} = 1.27 \text{ in.} \)).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Check tension yielding of the angles ( \phi R_u = \phi F_e A_e = 0.90(36 \text{ ksi})(10.9 \text{ in.}^2) )</td>
<td>Check tension yielding of the angles ( R_u / \Omega = \frac{F_e A_e}{1.67} = \frac{(36 \text{ ksi})(10.9 \text{ in.}^2)}{1.67} )</td>
</tr>
<tr>
<td>= 353 kips &gt; 259 kips o.k.</td>
<td>= 235 kips &gt; 172 kips o.k.</td>
</tr>
</tbody>
</table>

Table D3.1: 

Check tension rupture of the angles

\( U = 1 - \overline{x} = 1 - \frac{1.27 \text{ in.}}{15 \text{ in.}} = 0.92 \)

\( A_e = UA_u = 0.92[10.9 \text{ in.}^2 - 2(\frac{3}{8} \text{ in.})(1 \text{ in.})] = 8.65 \text{ in.}^2 \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi F_e A_e = 0.75(58 \text{ ksi})(8.65 \text{ in.}^2) )</td>
<td>( R_u / \Omega = \frac{F_e A_e}{\Omega} = \frac{(58 \text{ ksi})(8.65 \text{ in.}^2)}{2.00} )</td>
</tr>
<tr>
<td>= 376 kips &gt; 259 kips o.k.</td>
<td>= 251 kips &gt; 172 kips o.k.</td>
</tr>
</tbody>
</table>
Check block shear rupture of the angles.

Use \( n = 6, L_{w} = 1\frac{1}{2} \text{ in.}, \) and \( L_{eb} = 1\frac{1}{2} \text{ in.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = \phi F_u A_n U_{b_t} + \min \left( \phi 0.6 F_s A_{sv}, \phi F_u A_{nv} \right) )</td>
<td>( R_n = \frac{F_u A_n U_{b_t}}{\Omega} + \min \left( \frac{0.6 F_s A_{sv}}{\Omega}, \frac{F_u A_{nv}}{\Omega} \right) )</td>
</tr>
</tbody>
</table>

Tension Rupture Component

\( \phi F_u A_n = (43.5 \text{ kips/in.})(\frac{3}{8} \text{ in.})(2) \)

Shear Yielding Component

\( \phi 0.6 F_s A_{sv} = (267 \text{ kips/in.})(\frac{3}{8} \text{ in.})(2) \)

Shear Rupture Component

\( \phi F_u A_{nv} = (267 \text{ kips})(\frac{3}{8} \text{ in.})(2) \)

\( \phi R_n = (43.5 \text{ kips} + 267 \text{ kips})(\frac{3}{8} \text{ in.})(2) = 465 \text{ kips} > 259 \text{ kips} \quad \text{o.k.} \)

Similarly, the block shear rupture strength of the brace flange is o.k.

**Design brace-web-to-gusset connection**

Determine number of %\text{-}in. diameter ASTM A325-N bolts required on the brace side (double shear) for shear.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( n_{min} = \frac{P_{tu}}{\phi R_n} = \frac{157 \text{ kips}}{43.3 \text{ kips/bolt}} = 3.63 \rightarrow 4 \text{ bolts} )</td>
<td>( n_{min} = \frac{P_{tu}}{r_s/\Omega} = \frac{106 \text{ kips}}{28.9 \text{ kips/bolt}} = 3.67 \rightarrow 4 \text{ bolts} )</td>
</tr>
</tbody>
</table>

On the gusset side, the same number of bolts are required. Try two rows of two bolts and two PL\(\frac{3}{8}\times9\).

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check tension yielding of the plates</strong></td>
<td><strong>Check tension yielding of the plates</strong></td>
</tr>
<tr>
<td>( \phi R_s = \phi F_s A_s = 0.90(36 \text{ ksi})(2)(\frac{3}{8} \text{ in.})(9 \text{ in.}) )</td>
<td>( R_s/\Omega = F_s A_s / \Omega = \frac{(36 \text{ ksi})(2)(\frac{3}{8} \text{ in.})(9 \text{ in.})}{1.67} )</td>
</tr>
</tbody>
</table>

\( = 219 \text{ kips} > 157 \text{ kips} \quad \text{o.k.} \)

\( = 146 \text{ kips} > 106 \text{ kips} \quad \text{o.k.} \)
Check tension rupture of the plates

Take \( A_e \) as the lesser of \( A_n \) and \( 0.85A_g \),

\[
A_e = \min\left( A_n, 0.85A_g \right) = \min\left( \left( \frac{\mu}{\text{in.}} \right) \left( \frac{2(9 \text{ in.}) - 4(1 \text{ in.})}{2} \right), 0.85(2) \left( \frac{\mu}{\text{in.}} \right) \left( 9 \text{ in.} \right) \right) = 5.25 \text{ in.}^2
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e = \phi F_u A_e = 0.75(58 \text{ ksi})(5.25 \text{ in.}^2) )</td>
<td>( R_n / \Omega = F_u A_n / \Omega = \frac{58 \text{ ksi}(5.25 \text{ in.}^2)}{2.00} )</td>
</tr>
<tr>
<td>= 228 kips &gt; 157 kips o.k.</td>
<td>=152 kips &gt; 106 kips o.k.</td>
</tr>
</tbody>
</table>

Check block shear rupture of the plates (outer blocks).

Use \( n = 2 \), \( L_{ev} = 1 \frac{1}{2} \text{ in.} \), and \( L_{ob} = 1 \frac{1}{2} \text{ in.} \). The calculations here are done in the same manner as those for the angles. Thus,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = \phi F_u A_n U_{bs} + \min\left( \phi 0.6F_y A_{gy}, \phi F_u A_{ny} \right) )</td>
<td>( R_n / \Omega = F_u A_n / \Omega + \min\left( \frac{0.6F_y A_{gy}}{\Omega}, \frac{F_u A_{ny}}{\Omega} \right) )</td>
</tr>
<tr>
<td>Tension Rupture Component</td>
<td>Tension Rupture Component</td>
</tr>
<tr>
<td>( \phi F_y A_{gy} = (43.5 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
<td>( F_u A_n / \Omega = (29.0 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
</tr>
<tr>
<td>Shear Yielding Component</td>
<td>Shear Yielding Component</td>
</tr>
<tr>
<td>( \phi 0.6F_y A_{gy} = (72.9 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
<td>( 0.6F_y A_{gy} / \Omega = (48.6 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
</tr>
<tr>
<td>Shear Rupture Component</td>
<td>Shear Rupture Component</td>
</tr>
<tr>
<td>( \phi 0.6F_y A_{sv} = (78.3 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
<td>( 0.6F_y A_{sv} / \Omega = (52.2 \text{ kips/in.})(\frac{\mu}{\text{in.}})(2) )</td>
</tr>
<tr>
<td>( \phi R_n = (43.5 \text{ kips} + 72.9 \text{ kips})(\frac{\mu}{\text{in.}})(2) )</td>
<td>( R_n / \Omega = (29.0 \text{ kips} + 48.6 \text{ kips})(\frac{\mu}{\text{in.}})(2) )</td>
</tr>
<tr>
<td>= 175 kips &gt; 157 kips o.k.</td>
<td>= 116 kips &gt; 106 kips o.k.</td>
</tr>
</tbody>
</table>

Similarly, the block shear rupture strength of the interior blocks of the brace-web plates and the brace web are o.k.

Check block shear rupture of the brace web.

Use \( n = 2 \), \( L_{ev} = 1 \frac{1}{2} \text{ in.} \), and \( L_{ob} = 3 \text{ in.} \).
Check tension yielding of the brace

\[ \phi R_n = \phi F_y A_y = 0.90(50 \text{ ksi})(25.6 \text{ in.}^2) \]
\[ = 1150 \text{ kips} > 675 \text{ kips} \quad \text{o.k.} \]

Check tension rupture of the brace

Take \( A_y = A_n = 25.6 \text{ in.}^2 - [4(0.810 \text{ in.}) + 2(0.515 \text{ in.})] = 21.3 \text{ in.}^2 \)

Design the gusset

Check block shear rupture for the force transmitted through web. Use \( n = 2, L_{ev} = 1\frac{1}{2} \text{ in.} \) and \( L_{wb} = 3 \text{ in.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_n = \phi F_y A_y + \min \left( \phi 0.6 F_y A_{gy}, \phi F_y A_{nv} \right) )</td>
<td>( R_n = \frac{F_y A_y U_{bs}}{\Omega} + \min \left( \frac{0.6 F_y A_{gy}}{\Omega}, \frac{F_y A_{nv}}{\Omega} \right) )</td>
</tr>
</tbody>
</table>

Tension Rupture Component

\( \phi F_y A_{wy} = (122 \text{ kips/in.})(0.515 \text{ in.})(2) \)
\[ \frac{R_n}{\Omega} = \frac{F_y A_y}{\Omega} = \frac{(81.3 \text{ kips/in.})(0.515 \text{ in.})(2)}{1.67} \]
\[ = 766 \text{ kips} > 450 \text{ kips} \quad \text{o.k.} \]

Shear Yielding Component

\( \phi 0.6 F_y A_{gy} = (101 \text{ kips/in.})(0.515 \text{ in.})(2) \)
\[ \frac{R_n}{\Omega} = \frac{0.6 F_y A_{gy}}{\Omega} = \frac{(67.5 \text{ kips/in.})(0.515 \text{ in.})(2)}{1.67} \]
\[ = 692 \text{ kips} > 450 \text{ kips} \quad \text{o.k.} \]

Shear Rupture Component

\( \phi 0.6 F_y A_{nv} = (87.8 \text{ kips/in.})(0.515 \text{ in.})(2) \)
\[ \frac{R_n}{\Omega} = \frac{0.6 F_y A_{nv}}{\Omega} = \frac{(58.5 \text{ kips/in.})(0.515 \text{ in.})(2)}{1.67} \]
\[ = 692 \text{ kips} > 450 \text{ kips} \quad \text{o.k.} \]

Design the gusset

From edge distance, spacing, and thickness requirements of the angles and web plates, try PL 3/4.

Check block shear rupture for the force transmitted through web. Use \( n = 2, L_{ev} = 1\frac{1}{2} \text{ in.} \) and \( L_{wb} = 3 \text{ in.} \)
<table>
<thead>
<tr>
<th>Component</th>
<th>Equation</th>
<th>Result</th>
</tr>
</thead>
<tbody>
<tr>
<td>Tension Rupture Component</td>
<td>( \phi F_u A_u = (109 \text{ kips/in.})(% \text{ in.}) )</td>
<td>273 kips &gt; 157 kips</td>
</tr>
<tr>
<td>Shear Yielding Component</td>
<td>( 0.6 F_{yv} A_{yv} = (72.9 \text{ kips/in.})(% \text{ in.}) )</td>
<td>402 kips</td>
</tr>
<tr>
<td>Tension Rupture Component</td>
<td>( F_u A_u / \Omega = (72.5 \text{ kips/in.})(% \text{ in.}) )</td>
<td>539 kips</td>
</tr>
<tr>
<td>Shear Yielding Component</td>
<td>( 0.6 F_{yv} A_{yv} / \Omega = (48.6 \text{ kips/in.})(% \text{ in.}) )</td>
<td>268 kips</td>
</tr>
<tr>
<td>Shear Rupture Component</td>
<td>( \phi R_n = (109 \text{ kips} + 72.9 \text{ kips})(% \text{ in.}) )</td>
<td>431 kips</td>
</tr>
<tr>
<td></td>
<td>( R_n / \Omega = (72.5 \text{ kips} + 48.6 \text{ kips})(% \text{ in.}) )</td>
<td>287 kips</td>
</tr>
<tr>
<td>Shear Rupture Component</td>
<td>( \phi R_n = (109 \text{ kips})(% \text{ in.}) )</td>
<td>539 kips</td>
</tr>
<tr>
<td></td>
<td>( R_n / \Omega = (72.5 \text{ kips})(% \text{ in.}) )</td>
<td>360 kips</td>
</tr>
<tr>
<td></td>
<td>( \phi R_n = 539 \text{ kips} + \min(402 \text{ kips}, 431 \text{ kips}) )</td>
<td>941 kips &gt; 675 kips</td>
</tr>
<tr>
<td>Shear Rupture Component</td>
<td>( R_n / \Omega = 360 \text{ kips} + \min(268 \text{ kips}, 287 \text{kips}) )</td>
<td>628 kips &gt; 450 kips</td>
</tr>
</tbody>
</table>

**Check block shear rupture for total brace force**

With \( A_{yv} = 24.8 \text{ in.}^2 \), \( A_{yv} = 16.5 \text{ in.}^2 \), and \( A_{nt} = 12.4 \text{ in.}^2 \). Thus,

**LRFD**

\[
\phi R_n = \phi F_u A_u U_{bs} + \min\left(\phi 0.6 F_{yv} A_{yv}, \phi F_u A_{yv}\right)
\]

**Tension Rupture Component**

\[
\phi F_u A_u = 0.75(58 \text{ ksi})(12.4 \text{ in.}^2) = 539 \text kips
\]

**Shear Yielding Component**

\[
\phi 0.6 F_{yv} A_{yv} = 0.75(0.6)(36 \text{ ksi})(24.8 \text{ in.}^2) = 402 \text kips
\]

**Shear Rupture Component**

\[
\phi 0.6 F_{yv} A_{yv} = 0.75(0.6)(58 \text{ ksi})(16.5 \text{ in.}^2) = 431 \text kips
\]

\[
\phi R_n = 539 \text kips + \min(402 \text kips, 431 \text kips)
\]

\[
= 941 \text kips > 675 \text kips \quad \text{o.k.}
\]

**ASD**

\[
R_n / \Omega = F_u A_u U_{bs} + \min\left(\frac{0.6 F_{yv} A_{yv}}{\Omega}, \frac{F_u A_{yv}}{\Omega}\right)
\]

**Tension Rupture Component**

\[
\frac{F_u A_u}{\Omega} = \frac{58 \text{ ksi} (12.4 \text{ in.}^2)}{2.00} = 360 \text kips
\]

**Shear Yielding Component**

\[
\frac{0.6 F_{yv} A_{yv}}{\Omega} = \frac{(0.6)(36 \text{ ksi})(24.8 \text{ in.}^2)}{2.00} = 268 \text kips
\]

**Shear Rupture Component**

\[
\frac{0.6 F_{yv} A_{yv}}{\Omega} = \frac{(0.6)(58 \text{ ksi})(16.5 \text{ in.}^2)}{2.00} = 287 \text kips
\]

\[
R_n / \Omega = 360 \text kips + \min(268 \text kips, 287 \text kips)
\]

\[
= 628 \text kips > 450 \text kips \quad \text{o.k.}
\]
Check tension yielding on the Whitmore section of the gusset.

The Whitmore section, as illustrated with dashed lines in Figure (b), is 34.8 in. long; 30.9 in. occurs in the gusset and 3.90 in. occurs in the beam web. Thus,

\[
\phi R_s = \phi F_y A_w
\]

\[
= 0.90 \left[ \frac{36 \text{ ksi}(30.9 \text{ in.})}{(\frac{3}{4} \text{ in.})} + \frac{50 \text{ ksi}(3.90 \text{ in.})(0.590 \text{ in.})}{(\frac{3}{4} \text{ in.})} \right] = 854 \text{ kips} > 675 \text{ kips}
\]

o.k.

Note: The beam web thickness is used, conservatively ignoring the larger thickness in the beam-flange and the flange-to-web fillet area.

Check bearing strength of the angles, brace flange and gusset

The bearing strength per bolt is given by Specification Section J3.10 as:

\[ r_u = 1.2L_c tF_y \leq 2.4dtF_y \]

Because of the edge distance requirement, the angles, brace flange, and gusset, must be considered simultaneously. The angles have edge bolts and interior bolts. For an edge bolt,

\[ L_c = 1\% \text{ in.} - (\frac{1}{4})(\frac{1}{4}\%) \text{ in.} = 1.03 \text{ in.} \]

\[
\phi r_s = \phi 1.2L_c tF_y < 2.4dtF_y
\]

\[
= 0.75(1.2)(1.03 \text{ in.})(\frac{3}{4} \text{ in.})(58 \text{ ksi}) \leq 2.4(0.75\%)(\frac{3}{4} \text{ in.})(\frac{3}{4} \text{ in.})(58 \text{ ksi})
\]

= 40.4 kips < 68.6 kips

= 40.4 kips/bolt

For an interior bolt,

\[ L_c = 3 \text{ in.} - (1)(\frac{1}{4}\% \text{ in.}) = 2.06 \text{ in.} \]

\[
r_s / \Omega = \frac{1.2L_c tF_y}{\Omega} \leq \frac{2.4dtF_y}{\Omega}
\]

\[
= 1.2(1.03 \text{ in.})(\frac{3}{4} \text{ in.})(58 \text{ ksi}) \leq 2.4(\frac{3}{4} \text{ in.})(\frac{3}{4} \text{ in.})(58 \text{ ksi})
\]

= 26.9 kips < 45.7 kips

= 26.9 kips/bolt
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_u = \phi 1.2 L_c t F_u &lt; 2.4 d t F_u )</td>
<td>( \frac{r_n}{\Omega} = \frac{1.2 L_c t F}{\Omega} \leq \frac{2.4 d t F_u}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(1.2)(2.06 in.)(( \frac{3}{8} ) in.)(58 ksi)</td>
<td>= 0.75( \frac{2}{3} )(2.4)(( \frac{3}{16} ) in.)(( \frac{3}{8} ) in.)(58 ksi)</td>
</tr>
<tr>
<td>( \leq 0.75( \frac{2}{3} )(2.4)(( \frac{3}{16} ) in.)(( \frac{3}{8} ) in.)(58 ksi) )</td>
<td>( \leq 2.4( \frac{3}{16} ) in.)(( \frac{3}{8} ) in.)(58 ksi)</td>
</tr>
<tr>
<td>= 81.0 kips &lt; 68.6 kips</td>
<td>= 54 kips &lt; 45.7 kips</td>
</tr>
<tr>
<td>= 68.6 kips/bolt</td>
<td>= 45.7 kips/bolt</td>
</tr>
</tbody>
</table>

Note: The above strengths are per angle.

The gusset plate bolt strengths are the same as the angle bolt strengths since the gusset plate is the same thickness and material, and has the same edge distance. Thus,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For an edge bolt, ( \phi_u = 40.4 ) kips/bolt</td>
<td>For an edge bolt, ( \frac{r_n}{\Omega} = 26.9 ) kips/bolt</td>
</tr>
<tr>
<td>For an interior bolt, ( \phi_u = 68.6 ) kips/bolt</td>
<td>For an interior bolt, ( \frac{r_n}{\Omega} = 45.7 ) kips/bolt</td>
</tr>
</tbody>
</table>

For the brace, edge bolt \( L_c = 1.03 \) in. and interior bolt \( L_c = 2.06 \) in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace edge bolt,</td>
<td>Brace edge bolt,</td>
</tr>
<tr>
<td>( \phi_u = \phi 1.2 L_c t F_u &lt; 2.4 d t F_u )</td>
<td>( \frac{r_n}{\Omega} = \frac{1.2 L_c t F}{\Omega} \leq \frac{2.4 d t F_u}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(1.2)(1.03 in.)(0.810 in.)(65 ksi)</td>
<td>= 0.75(1.2)(1.03 in.)(0.810 in.)(65 ksi)</td>
</tr>
<tr>
<td>( \leq 0.75( \frac{2}{3} )(2.4)(( \frac{3}{16} ) in.)(0.810 in.)(65 ksi) )</td>
<td>( \leq 2.4( \frac{3}{16} ) in.)(0.810 in.)(65 ksi)</td>
</tr>
<tr>
<td>= 48.9 kips &lt; 83.3 kips</td>
<td>= 32.6 kips &lt; 55.5 kips</td>
</tr>
<tr>
<td>= 48.9 kips/bolt</td>
<td>= 32.6 kips/bolt</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace interior bolt,</td>
<td>Brace interior bolt,</td>
</tr>
<tr>
<td>( \phi_u = \phi 1.2 L_c t F_u &lt; 2.4 d t F_u )</td>
<td>( \frac{r_n}{\Omega} = \frac{1.2 L_c t F}{\Omega} \leq \frac{2.4 d t F_u}{\Omega} )</td>
</tr>
<tr>
<td>= 0.75(1.2)(2.06 in.)(0.810 in.)(65 ksi)</td>
<td>= 0.75(1.2)(2.06 in.)(0.810 in.)(65 ksi)</td>
</tr>
<tr>
<td>( \leq 0.75( \frac{2}{3} )(2.4)(( \frac{3}{16} ) in.)(0.810 in.)(65 ksi) )</td>
<td>( \leq 2.4( \frac{3}{16} ) in.)(0.810 in.)(65 ksi)</td>
</tr>
<tr>
<td>= 97.8 kips &lt; 83.2 kips</td>
<td>= 65.2 kips &lt; 55.5 kips</td>
</tr>
<tr>
<td>= 83.2 kips/bolt</td>
<td>= 55.5 kips/bolt</td>
</tr>
</tbody>
</table>
The bearing strength of the flange connection can now be calculated. Summarizing the various strengths:

<table>
<thead>
<tr>
<th>Member</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (2)</td>
<td>80.8 kips</td>
<td>137 kips</td>
</tr>
<tr>
<td>Gusset</td>
<td>40.4 kips</td>
<td>68.6 kips</td>
</tr>
<tr>
<td>Brace</td>
<td>48.9 kips</td>
<td>83.2 kips</td>
</tr>
</tbody>
</table>

From the above table,

\( \phi R_u = (1 \text{ bolt})(40.4 \text{ kips/bolt}) \)
\( + (5 \text{ bolts})(68.6 \text{ kips/bolt}) \)
\( = 383 \text{ kips} > 259 \text{ kips} \quad \text{O.K.} \)

Note: The gusset edge bolt bearing strength is less than the bolt double shear strength; the bolt shear strength must be re-checked. Thus, the revised bolt shear strength is

<table>
<thead>
<tr>
<th>Member</th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Angle (2)</td>
<td>53.8 kips</td>
<td>91.4 kips</td>
</tr>
<tr>
<td>Gusset</td>
<td>26.9 kips</td>
<td>45.7 kips</td>
</tr>
<tr>
<td>Brace</td>
<td>32.6 kips</td>
<td>55.5 kips</td>
</tr>
</tbody>
</table>

From the above table,

\( R_{n}/\Omega = (1 \text{ bolt})(26.9 \text{ kips/bolt}) \)
\( + (5 \text{ bolts})(45.7 \text{ kips/bolt}) \)
\( = 255 \text{ kips} > 172 \text{ kips} \quad \text{O.K.} \)

Note: When the brace force is compression gusset buckling would have to be checked; refer to the comments at the end of this example.

**Distribution of the brace force to beam and column**

From the members and frame geometry

\( e_b = \frac{d_{beam}}{2} = \frac{18.7 \text{ in.}}{2} = 9.35 \text{ in.} \)
\( e_c = \frac{d_{column}}{2} = \frac{20.9 \text{ in.}}{2} = 10.5 \text{ in.} \)

\( \tan \theta = \frac{12}{9/\sqrt{6}} = 1.25 \)

and \( e_b \tan \theta - e_c = (9.35 \text{ in.})(1.25) - (10.5 \text{ in.}) = 1.19 \text{ in.} \)

Try gusset PL \( \frac{3}{8} \times 42 \text{ in.} \) horizontally \( \times 33 \text{ in.} \) vertically (several intermediate gusset dimensions were inadequate). With connection centroids at the midpoint of the gusset edges

\( \alpha = \frac{42 \text{ in.}}{2} + \frac{1}{2} \text{ in.} = 21\frac{1}{2} \text{ in.} \)

where \( \frac{1}{2} \text{ in.} \) is allowed for the setback between the gusset and the column, and
IIC - 21

\[ \beta = \frac{33 \text{ in.}}{2} = 16.5 \text{ in.} \]

Choosing \( \beta = \bar{\beta} \), the \( \alpha \) required for the uniform forces is

\[ \alpha = eb \tan \theta - ec + \beta \tan \theta = 1.19 + (16.5 \text{ in.})(1.25) = 21.8 \text{ in.} \]

The resulting eccentricity is \( \alpha - \bar{\alpha} \), where

\[ \alpha - \bar{\alpha} = 21.8 \text{ in.} - 21.5 \text{ in.} = 0.3 \text{ in.} \]

Since slight eccentricity is negligible. Use \( \alpha = 21.8 \text{ in.} \) and \( \beta = 16.5 \text{ in.} \).

**Calculate gusset interface forces**

\[ r = \sqrt{(\alpha + e_c)^2 + (\beta + e_e)^2} = \sqrt{(21.8 \text{ in.} + 10.5)^2 + (16.5 \text{ in.} + 9.35 \text{ in.})^2} = 41.4 \text{ in.} \]

On the gusset-to-column connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{uc} )</td>
<td>( H_{uc} )</td>
</tr>
<tr>
<td>( \frac{e_{ uc} P_{ uc}}{r} = \frac{(10 \frac{1}{2} \text{ in.})(675 \text{ kips})}{41.4 \text{ in.}} = 171 \text{ kips} )</td>
<td>( \frac{e_{ uc} P_{ uc}}{r} = \frac{(10 \frac{1}{2} \text{ in.})(450 \text{ kips})}{41.4 \text{ in.}} = 114 \text{ kips} )</td>
</tr>
<tr>
<td>( V_{uc} )</td>
<td>( V_{uc} )</td>
</tr>
<tr>
<td>( \frac{\beta P_{ uc}}{r} = \frac{(16 \frac{1}{2} \text{ in.})(675 \text{ kips})}{41.4 \text{ in.}} = 269 \text{ kips} )</td>
<td>( \frac{\beta P_{ uc}}{r} = \frac{(16 \frac{1}{2} \text{ in.})(450 \text{ kips})}{41.4 \text{ in.}} = 179 \text{ kips} )</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{ub} )</td>
<td>( H_{ub} )</td>
</tr>
<tr>
<td>( \frac{\alpha P_{ ub}}{r} = \frac{(21.8 \text{ in.})(675 \text{ kips})}{41.4 \text{ in.}} = 355 \text{ kips} )</td>
<td>( \frac{\alpha P_{ ub}}{r} = \frac{(21.8 \text{ in.})(450 \text{ kips})}{41.4 \text{ in.}} = 237 \text{ kips} )</td>
</tr>
<tr>
<td>( V_{ub} )</td>
<td>( V_{ub} )</td>
</tr>
<tr>
<td>( \frac{e_{ ub} P_{ ub}}{r} = \frac{(9.35 \text{ in.})(675 \text{ kips})}{41.4 \text{ in.}} = 153 \text{ kips} )</td>
<td>( \frac{e_{ ub} P_{ ub}}{r} = \frac{(9.35 \text{ in.})(450 \text{ kips})}{41.4 \text{ in.}} = 102 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Design gusset-to-column connection**

The forces involved are \( V_{uc} = 269 \text{ kips} \) (LRFD ASD) shear and \( H_{uc} = 171 \text{ kips} \) (ASD) tension.

Try 2L4×4×\( \sqrt{3} \times 2'\)-6 welded to the gusset and bolted with 10 rows of \( \frac{7}{8} \) in. diameter A325-N bolts in standard holes to the column flange.

**Calculate the required tensile strength per bolt**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T_a = \frac{H_{uc}}{n} = \frac{171 \text{ kips}}{20 \text{ bolts}} = 8.55 \text{ kips/bolt} )</td>
<td>( T_a = \frac{H_{uc}}{n} = \frac{114 \text{ kips}}{20 \text{ bolts}} = 5.70 \text{ kips/bolt} )</td>
</tr>
</tbody>
</table>
Check design strength of bolts for tension-shear interaction.

\[ r_{av} = \frac{V_{aw}}{n} = \frac{269 \text{ kips}}{20 \text{ bolts}} = 13.5 \text{ kips/bolt} \]

13.5 kips/bolt < 21.6 kips/bolt \textbf{o.k.}

\[ f_{av} = \frac{r_{av}}{A_b} = \frac{13.5 \text{ kips}}{0.6013 \text{ in.}^2} = 22.5 \text{ ksi} \]

\[ F_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{av}} f_{av} \leq F_{nt} \]

\[ = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(48 \text{ ksi})}(22.5 \text{ ksi}) \]

\[ = 60.8 \text{ ksi} < 90 \text{ ksi} \]

Use \( F_{nt} = 60.8 \text{ ksi} \)

\[ B_u = \phi F_{nt} A_b = 0.75(60.8 \text{ ksi})(0.6013 \text{ in.}^2) \]

\[ = 27.4 \text{ kips/bolt} > 8.55 \text{ kips/bolt} \textbf{o.k.} \]

Check allowable strength of bolts for tension-shear interaction.

\[ r_{av} = \frac{V_{aw}}{n} = \frac{179 \text{ kips}}{20 \text{ bolts}} = 8.95 \text{ kips/bolt} \]

8.95 kips/bolt < 14.4 kips/bolt \textbf{o.k.}

\[ f_{av} = \frac{r_{av}}{A_b} = \frac{8.95 \text{ kips}}{0.6013 \text{ in.}^2} = 14.9 \text{ ksi} \]

\[ F_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{av}} f_{av} \leq F_{nt} \]

\[ = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}(2.00)}{48 \text{ ksi}}(14.9 \text{ ksi}) \]

\[ = 61.1 \text{ ksi} < 90 \text{ ksi} \]

Use \( F_{nt} = 61.1 \text{ ksi} \)

\[ B_u = \frac{F_{nt}}{A_b} = \frac{(61.1 \text{ ksi})(0.6013 \text{ in.}^2)}{2.00} \]

\[ = 18.4 \text{ kips/bolt} > 5.70 \text{ kips/bolt} \textbf{o.k.} \]

Check bearing strength at bolt holes

The bearing strength per bolt is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_u = \phi 1.2L_t F_{tu} &lt; 2.4dF_u )</td>
<td></td>
</tr>
<tr>
<td>( = 0.75(1.2)(1.03 \text{ in.})(\frac{1}{2} \text{ in.})(58 \text{ ksi}) )</td>
<td></td>
</tr>
<tr>
<td>( \leq 0.75(2.4)(\frac{1}{6} \text{ in.})(\frac{1}{6} \text{ in.})(58 \text{ ksi}) )</td>
<td></td>
</tr>
<tr>
<td>( = 26.9 \text{ kips} &lt; 45.7 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( = 26.9 \text{ kips/bolt} )</td>
<td></td>
</tr>
</tbody>
</table>

Since this edge bolt value exceeds the single-shear strength of the bolts 21.6 kips, and the actual shear per bolt of 13.5 kips, bearing strength is \textbf{o.k.}

Check allowable strength of bolts for tension-shear interaction.

\[ r_{av} = \frac{V_{aw}}{n} = \frac{179 \text{ kips}}{20 \text{ bolts}} = 8.95 \text{ kips/bolt} \]

8.95 kips/bolt < 14.4 kips/bolt \textbf{o.k.}

\[ f_{av} = \frac{r_{av}}{A_b} = \frac{8.95 \text{ kips}}{0.6013 \text{ in.}^2} = 14.9 \text{ ksi} \]

\[ F_{nt} = 1.3F_{nt} - \frac{F_{nt}}{\phi F_{av}} f_{av} \leq F_{nt} \]

\[ = 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}(2.00)}{48 \text{ ksi}}(14.9 \text{ ksi}) \]

\[ = 61.1 \text{ ksi} < 90 \text{ ksi} \]

Use \( F_{nt} = 61.1 \text{ ksi} \)

\[ B_u = \frac{F_{nt}}{A_b} = \frac{(61.1 \text{ ksi})(0.6013 \text{ in.}^2)}{2.00} \]

\[ = 18.4 \text{ kips/bolt} > 5.70 \text{ kips/bolt} \textbf{o.k.} \]

Since this edge bolt value exceeds the single-shear strength of the bolts 14.4 kips, and the actual shear per bolt of 8.95 kips, bearing strength is \textbf{o.k.}
Check prying action

\[ b = g - \frac{t}{2} = 2\frac{1}{2} \text{ in.} - \frac{\frac{1}{2}}{2} \text{ in.} = 2\frac{1}{4} \text{ in.} \]

Note: 1¼ in. entering and tightening clearance is accommodated, o.k.

\[ a = 4 \text{ in.} - g = 4 \text{ in.} - 2\frac{1}{2} \text{ in.} = 1\frac{1}{2} \text{ in.} \]

Since \( a = 1\frac{1}{2} \text{ in.} \) is less than \( 1.25b = 2.81 \text{ in.} \), use \( a = 1\frac{1}{2} \text{ in.} \).

\[ b' = b - \frac{d}{2} = 2\frac{1}{2} \text{ in.} - \frac{\frac{1}{2}}{2} \text{ in.} = 1.81 \text{ in.} \]

\[ a' = a + \frac{d}{2} = 1\frac{1}{2} \text{ in.} + \frac{\frac{1}{2}}{2} \text{ in.} = 1.94 \text{ in.} \]

\[ \rho = \frac{b'}{a'} = \frac{1.81}{1.94} \text{ in.} = 0.935 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) = \frac{1}{0.935} \left( \frac{27.4 \text{ kips/bolt}}{8.55 \text{ kips/bolt}} - 1 \right) ]</td>
<td>[ \beta = \frac{1}{\rho} \left( \frac{B_u}{T_u} - 1 \right) = \frac{1}{0.935} \left( \frac{18.4 \text{ kips/bolt}}{5.70 \text{ kips/bolt}} - 1 \right) ]</td>
</tr>
<tr>
<td>= 2.36</td>
<td>= 2.38</td>
</tr>
</tbody>
</table>

Since \( \beta > 1 \), set \( \alpha' = 1.0 \)

\[ \delta = 1 - \frac{d'}{\rho} = 1 - \frac{1\frac{1}{2}}{3} \text{ in.} = 0.688 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_{req} = \sqrt{\frac{4.44T_u b'}{pF_u (1 + \delta \alpha')}} ]</td>
<td>[ t_{req} = \sqrt{\frac{6.66T_u b'}{pF_u (1 + \delta \alpha')}} ]</td>
</tr>
<tr>
<td>[ = \sqrt{\frac{4.44(8.55 \text{ kips/bolt})(1.81 \text{ in.})}{(3 \text{ in.})(58 \text{ ksi})[1 + 0.688(1.0)]}} ]</td>
<td>[ = \sqrt{\frac{6.66(5.70 \text{ kips/bolt})(1.81 \text{ in.})}{(3 \text{ in.})(58 \text{ ksi})[1 + 0.688(1.0)]}} ]</td>
</tr>
<tr>
<td>= 0.484 in.</td>
<td>= 0.484 in.</td>
</tr>
</tbody>
</table>

Since \( t = \frac{1}{2} \text{ in.} > 0.484 \text{ in.} \), angles are o.k. Since \( t = \frac{1}{2} \text{ in.} > 0.484 \text{ in.} \), angles are o.k.
Design welds

Try fillet welds around the perimeter (three sides) of both angles.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_{ae} = \sqrt{H_{ae}^2 + V_{ae}^2} )</td>
<td>( P_{ae} = \sqrt{H_{ae}^2 + V_{ae}^2} )</td>
</tr>
<tr>
<td>( = \sqrt{(171 \text{ kips})^2 + (269 \text{ kips})^2} = 319 \text{ kips} )</td>
<td>( = \sqrt{(114 \text{ kips})^2 + (179 \text{ kips})^2} = 212 \text{ kips} )</td>
</tr>
<tr>
<td>( \theta = \tan^{-1}\left(\frac{H_{ae}}{V_{ae}}\right) = \tan^{-1}\left(\frac{171 \text{ kips}}{269 \text{ kips}}\right) = 32.4^\circ )</td>
<td>( \theta = \tan^{-1}\left(\frac{H_{ae}}{V_{ae}}\right) = \tan^{-1}\left(\frac{114 \text{ kips}}{179 \text{ kips}}\right) = 32.5^\circ )</td>
</tr>
</tbody>
</table>

From Manual Table 8-8 with \( \theta = 30^\circ \),

\( l = 30 \text{ in.}, k_l = 3\frac{1}{2} \text{ in.}, \) therefore \( k = 0.117 \)

By interpolation

\( x = 0.011, x_l = 0.011(30 \text{ in.}) = 0.33 \text{ in.} \)

\( a_l = 4 \text{ in.} - x_l = 4 \text{ in.} - 0.33 \text{ in.} = 3.67 \text{ in.} \)

\( a = 0.122 \)

By interpolation

\( C = 2.60 \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( D_{req} = \frac{P_{ae}}{\phi \Omega CC_l} )</td>
<td>( D_{req} = \frac{P_{ae} \Omega}{CC_l} )</td>
</tr>
<tr>
<td>( = \frac{319 \text{ kips}}{0.75(2.60)(1.0)(2 \text{ welds})(30 \text{ in.})} = 2.73 \rightarrow 3 \text{ sixteenths} )</td>
<td>( = \frac{(212 \text{ kips})(2.00)}{(2.60)(1.0)(2 \text{ welds})(30 \text{ in.})} = 2.72 \rightarrow 3 \text{ sixteenths} )</td>
</tr>
</tbody>
</table>

From Specification Table J2.4, minimum weld size is \( \frac{1}{4} \text{ in.} \). Use \( \frac{1}{4} \)-in. fillet welds.

**Check gusset thickness against weld size required for strength**

For two fillet welds, \( t_{min} = \frac{6.19D}{F_y} = \frac{6.19(2.73 \text{ sixteenths})}{58 \text{ ksi}} = 0.291 \text{ in.} < \frac{1}{4} \text{ in.} \) **O.K.**
Check strength of angles

Check shear yielding (due to $V_{uc}$ or $V_{ac}$)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi \left(0.60 F_y A_g\right)$</td>
<td>$R_u / \Omega = \frac{0.60 F_y A_g}{\Omega}$</td>
</tr>
<tr>
<td>$= 1.00 \left[0.60 \times (36 \text{ ksi}) \times (2) \times (30 \text{ in.}) \times (\frac{3}{2} \text{ in.})\right]$</td>
<td>$= \frac{0.60 \times (36 \text{ ksi}) \times (2) \times (30 \text{ in.})}{1.50}$</td>
</tr>
<tr>
<td>$= 648 \text{ kips} &gt; 269 \text{ kips}$</td>
<td>$= 432 \text{ kips} &gt; 179 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

Similarly, shear yielding of the angles due to $H_{uc}$ is not critical.

Check shear rupture

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi \left(0.60 F_y A_{sv}\right)$</td>
<td>$R_u / \Omega = \frac{0.60 F_y A_{sv}}{\Omega}$</td>
</tr>
<tr>
<td>$= 0.75 \times (0.60) \times (58 \text{ ksi}) \times (\frac{3}{2} \text{ in.}) \times \left[(2) \times (30 \text{ in.}) - 20 \times (1 \text{ in.})\right]$</td>
<td>$= \frac{0.60 \times (58 \text{ ksi}) \times (\frac{3}{2} \text{ in.}) \times [(2) \times (30 \text{ in.}) - 20 \times (1 \text{ in.})]}{2.00}$</td>
</tr>
<tr>
<td>$= 522 \text{ kips} &gt; 269 \text{ kips}$</td>
<td>$= 348 \text{ kips} &gt; 179 \text{ kips}$ o.k.</td>
</tr>
</tbody>
</table>

Block shear rupture

Use $n = 10$, $L_{cv} = 1\frac{1}{2}$ in. and $L_{cv} = 1\frac{1}{2}$ in. Thus,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_u = \phi F_y A_{m} U_{bs} + \min \left(\phi 0.6 F_y A_{ay}, \phi F_y A_{nv}\right)$</td>
<td>$R_u / \Omega = \frac{F_y A_{m} U_{bs} + \min \left(0.6 F_y A_{ay}, F_y A_{nv}\right)}{\Omega}$</td>
</tr>
</tbody>
</table>

Tension Rupture Component

$\phi F_y A_{m} = (43.5 \text{ kips/in.}) \times (\frac{3}{2} \text{ in.}) \times (2)$

Shear Yielding Component

$0.6 F_y A_{ay} / \Omega = (29.0 \text{ kips/in.}) \times (\frac{3}{2} \text{ in.}) \times (2)$

Shear Rupture Component

$0.6 F_y A_{ay} = (496 \text{ kips/in.}) \times (\frac{3}{2} \text{ in.}) \times (2)$

$\phi R_u = (43.5 \text{ kips} + 496 \text{ kips}) \times (\frac{3}{2} \text{ in.}) \times (2)$

$= 506 \text{ kips} > 269 \text{ kips}$ o.k.

$R_u / \Omega = (29.0 \text{ kips} + 308 \text{ kips}) \times (\frac{3}{2} \text{ in.}) \times (2)$

$= 337 \text{ kips} > 179 \text{ kips}$ o.k.
Check column flange

By inspection, the 4.16 in. thick column flange has adequate flexural strength, stiffness, and bearing strength.

Design the gusset-to-beam connection

The forces involved are

\[
\begin{align*}
H_{ab} &= 355 \text{ kips} & H_{ab} &= 237 \text{ kips} \\
V_{ab} &= 153 \text{ kips} & V_{ab} &= 102 \text{ kips}
\end{align*}
\]

Because one edge of the gusset is welded and the other is bolted, the distribution of force on the welded edge is uneven. To account for this, the required strength of the gusset edge is amplified by a factor of 1.25 to allow for the redistribution of forces on the weld.

The stresses on the gusset plate at the welded edge are as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( f_{us} = \frac{V_{us}}{tl} \leq \phi F_y )</td>
<td>( f_{as} = \frac{V_{as}}{tl} \leq \frac{F_y}{\Omega} )</td>
</tr>
<tr>
<td>( = \frac{153 \text{ kips}}{(\frac{\gamma}{4} \text{ in.})(42 \text{ in.})} \leq 0.90(36 \text{ ksi}) )</td>
<td>( = \frac{102 \text{ kips}}{(\frac{\gamma}{4} \text{ in.})(42 \text{ in.})} \leq \frac{36 \text{ ksi}}{1.67} )</td>
</tr>
<tr>
<td>( = 4.86 \text{ ksi} &lt; 32.4 \text{ ksi} )</td>
<td>( = 3.24 \text{ ksi} &lt; 21.6 \text{ ksi} )</td>
</tr>
</tbody>
</table>

\( f_{us} = \frac{H_{us}}{tl} \leq \phi(0.60F_y) \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( = \frac{355 \text{ kips}}{(\frac{\gamma}{4} \text{ in.})(42 \text{ in.})} \leq 1.00(0.60)(36 \text{ ksi}) )</td>
<td>( = \frac{237 \text{ kips}}{(\frac{\gamma}{4} \text{ in.})(42 \text{ in.})} \leq \frac{0.60(36 \text{ ksi})}{1.50} )</td>
</tr>
<tr>
<td>( = 11.3 \text{ ksi} &lt; 21.6 \text{ ksi} )</td>
<td>( = 7.52 \text{ ksi} &lt; 14.4 \text{ ksi} )</td>
</tr>
</tbody>
</table>

\( \theta = \tan^{-1}\left(\frac{V_{us}}{H_{us}}\right) = \tan^{-1}\left(\frac{153 \text{ kips}}{355 \text{ kips}}\right) = 23.3^\circ \)

\( \mu = 1.0 + 0.5 \sin^{1.5} \theta = 1.0 + 0.5 \sin^{1.5}(23.3^\circ) = 1.12 \)

The weld strength per \( \frac{\gamma}{6} \) in. is as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi r_w = (1.392 \text{ kips/in.})(1.12) = 1.56 \text{ kips/in.} )</td>
<td>( r_w / \Omega = (0.928 \text{ kips/in.})(1.12) = 1.04 \text{ kips/in.} )</td>
</tr>
</tbody>
</table>

The peak weld stress is given by

\[
\begin{align*}
\frac{f_{u, \text{peak}}}{\frac{1}{2} \sqrt{(f_{us} + f_{ub})^2 + f_{w}^2}}
\end{align*}
\]
The average stress is
\[ f_{u, ave} = \frac{\left( f_{u, peak} + f_{u, ave} \right)}{2} \]
\[ = \frac{\left( 4.61 \text{ kips/in.} + 4.61 \text{ kips/in.} \right)}{2} = 4.61 \text{ kips/in.} \]

The average stress is
\[ f_{a, ave} = \frac{\left( f_{a, peak} + f_{a, ave} \right)}{2} \]
\[ = \frac{\left( 3.07 \text{ kips/in.} + 3.07 \text{ kips/in.} \right)}{2} = 3.07 \text{ kips/in.} \]

Since \( f_{ab} = 0 \text{ ksi} \) (there is no moment on the edge), \( f_{u, ave} = f_{u, peak} = 4.61 \text{ kips/in.} \)

The required weld size is
\[ D_{req} = \frac{f_{u, weld}}{\Phi_{n}} \]
\[ = \frac{5.76 \text{ kips/in.}}{1.56 \text{ kips/in.}} = 3.69 \rightarrow 4 \text{ sixteensths} \]

Minimum weld size is \( \frac{3}{16} \text{ in.} \). Use a \( \frac{3}{16} \text{-in.} \) fillet weld.

---

LRFD | ASD
---|---
Check local web yielding of the beam
\[ \Phi R_u = \Phi(N + 2.5k) F_{yw} t_w \]
\[ = 1.0 \left[ 42 \text{ in.} + 2.5(1.34 \text{ in.}) \right] (50 \text{ ksi}) (0.590 \text{ in.}) \]
\[ = 1340 \text{ kips} > 153 \text{ kips} \text{ o.k.} \]

Check local web yielding of the beam
\[ R_u / \Omega = \frac{(N + 2.5k) F_{yw} t_w}{\Omega} \]
\[ = \left[ 42 \text{ in.} + 2.5(1.34 \text{ in.}) \right] (50 \text{ ksi}) (0.590 \text{ in.}) \]
\[ = 892 \text{ kips} > 102 \text{ kips} \text{ o.k.} \]

Eqn. J10-3
**Design beam-to-column connection**

Since the brace may be in tension or compression, the required strength the beam-to-column connection is as follows. The required shear strength is

\[
R_{ub} \pm V_{ub} = 15 \text{kips} + 153 \text{kips} = 168 \text{kips}
\]

and the required axial strength is

\[
A_{ub} \pm (H_u - H_{ub}) = 0 \pm 171 \text{kips} = 171 \text{kips}
\]

**LRFD**

<table>
<thead>
<tr>
<th></th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_{ub} \pm V_{ub} = 10 \text{kips} + 102 \text{kips} = 112 \text{kips})</td>
<td>(R_{ub} \pm V_{ub} = 10 \text{kips} + 102 \text{kips} = 112 \text{kips})</td>
</tr>
</tbody>
</table>

Try 2L8×6×¾×1'-2½ (leg gage = 2¼ in.) welded to the beam web, bolted with five rows of ¾ in. diameter A325-N bolts in standard holes to the column flange.

**Calculate tensile force per bolt**

<table>
<thead>
<tr>
<th></th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(T_u = \frac{A_{ub} \pm (H_u - H_{ub})}{n} = \frac{171 \text{kips}}{10 \text{ bolts}} = 17.1 \text{kips/bolt})</td>
<td>(T_u = \frac{A_{ub} \pm (H_u - H_{ub})}{n} = \frac{114 \text{kips}}{10 \text{ bolts}} = 11.4 \text{kips/bolt})</td>
</tr>
</tbody>
</table>

**Check available strength of bolts for tension-shear interaction**

<table>
<thead>
<tr>
<th></th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_u = \frac{R_{ub} \pm V_{ub}}{n} = \frac{168 \text{kips}}{10 \text{ bolts}} = 16.8 \text{kips/bolt})</td>
<td>(V_u = \frac{R_{ub} \pm V_{ub}}{n} = \frac{112 \text{kips}}{10 \text{ bolts}} = 11.2 \text{kips/bolt})</td>
</tr>
<tr>
<td>(f_{wu} = \frac{V_u}{A_b} = \frac{16.8 \text{kips/bolt}}{0.6013 \text{ in.}^2} = 27.4 \text{ ksi})</td>
<td>(f_{wu} = \frac{V_u}{A_b} = \frac{11.2 \text{kips/bolt}}{0.6013 \text{ in.}^2} = 18.6 \text{ ksi})</td>
</tr>
<tr>
<td>(F_{ul} = 1.3F_{wu} - \frac{F_{st}}{\phi F_{wu}}f_{wu} \leq F_{st})</td>
<td>(F_{ul} = 1.3F_{wu} - \frac{\Omega F_{wu}}{f_{wu}}f_{wu} \leq F_{st})</td>
</tr>
<tr>
<td>(= 1.3(90 \text{ ksi}) - \frac{90 \text{ ksi}}{0.75(48 \text{ ksi})} (27.4 \text{ ksi}) &lt; 90 \text{ ksi})</td>
<td>(= 1.3(90 \text{ ksi}) - \frac{2.00(90 \text{ ksi})}{48 \text{ ksi}} (18.6 \text{ ksi}) &lt; 90 \text{ ksi})</td>
</tr>
<tr>
<td>= 48.5 ksi &lt; 90 ksi o.k.</td>
<td>(= 47.2 \text{ ksi} &lt; 90 \text{ ksi} \text{ o.k.})</td>
</tr>
<tr>
<td>Use (F_{ul} = 48.5 \text{ ksi.})</td>
<td>Use (F_{ul} = 47.2 \text{ ksi.})</td>
</tr>
<tr>
<td>(B_u = \phi F_{ul} A_b = 0.75(48.5 \text{ ksi})(0.6013 \text{ in.}^2))</td>
<td>(B_u = \frac{F_{ul} A_b}{\Omega} = \frac{(48.5 \text{ ksi})(0.6013 \text{ in.}^2)}{2.00})</td>
</tr>
<tr>
<td>(= 21.9 \text{kips/bolt} &gt; 17.1 \text{kips/bolt} \text{ o.k.})</td>
<td>(= 14.2 \text{kips/bolt} &gt; 11.4 \text{kips/bolt} \text{ o.k.})</td>
</tr>
</tbody>
</table>

Table J3.2, Eqs. J3-3a and J3-3b
Check bearing strength

\[ \phi_u = \phi = 1.2L_c t_{u} < 2.4dt_{u} \]

\[ = 0.75(1.2)(1.03 \text{ in.})\left(\frac{3}{8} \text{ in.}\right)(58 \text{ ksi}) \]

\[ \leq 0.75(2.4)\left(\frac{3}{8} \text{ in.}\right)(\frac{3}{8} \text{ in.})(58 \text{ ksi}) \]

= 40.4 kips < 68.6 kips

= 40.4 kips/bolt

Since this edge bolt value exceeds the single-shear strength of the bolts 21.6 kips, bearing does not control. \textbf{o.k.}

Check bearing strength

\[ r_u / \Omega = \frac{1.2L_c t_{F}}{\Omega} \leq \frac{2.4dt_{F}}{\Omega} \]

\[ = \frac{1.2(1.03 \text{ in.})\left(\frac{3}{8} \text{ in.}\right)(58 \text{ ksi})}{2.00} \]

\[ \leq \frac{2.4\left(\frac{3}{8} \text{ in.}\right)(\frac{3}{8} \text{ in.})(58 \text{ ksi})}{2.00} \]

= 26.9 kips < 45.7 kips

= 26.9 kips/bolt

Since this edge bolt value exceeds the single-shear strength of the bolts 14.4 kips, bearing does not control. \textbf{o.k.}

Check prying action

\[ b = g - \frac{t}{2} = 2\frac{3}{8} \text{ in.} - \frac{0.75 \text{ in.}}{2} = 2\frac{3}{8} \text{ in.} \]

Note: 1\frac{3}{8} in. entering and tightening clearance is accommodated, \textbf{o.k.}

\[ a = 6 \text{ in.} - g = 6 \text{ in.} - 2\frac{3}{8} = 3\frac{3}{8} \text{ in.} \]

Since \( a = 3\frac{3}{8} \text{ in.}\) exceeds 1.25\( b = 2.97 \text{ in.}\), use \( a = 2.97 \text{ in.}\) for calculation purposes.

\[ b' = b - \frac{d}{2} = 2\frac{3}{8} \text{ in.} - \frac{\frac{3}{8} \text{ in.}}{2} = 1.94 \text{ in.} \]

\[ a' = a + \frac{d}{2} = 2.97 \text{ in.} + \frac{\frac{3}{8} \text{ in.}}{2} = 3.41 \text{ in.} \]

\[ \rho = \frac{b'}{a'} = \frac{1.94 \text{ in.}}{3.41 \text{ in.}} = 0.569 \]

\[ p = \frac{14\frac{1}{2} \text{ in.}}{5 \text{ bolts}} = 2.90 \text{ in./bolt} \]

\[ \delta = 1 - \frac{d'}{p} = 1 - \frac{\frac{3}{8} \text{ in.}}{2.90 \text{ in.}} = 0.677 \]

\begin{tabular}{|c|c|}
\hline
\textbf{LRFD} & \textbf{ASD} \\
\hline
\textbf{Check prying action} & \\
\hline
\textbf{LRFD} & \textbf{ASD} \\
\hline
\textbf{Check prying action} & \\
\hline
\end{tabular}
Design welds

Try fillet welds around perimeter (three sides) of both angles.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{wc} = \sqrt{(171 \text{ kips})^2 + (168 \text{ kips})^2} = 240 \text{ kips}$</td>
<td>$P_{wc} = \sqrt{(114 \text{ kips})^2 + (112 \text{ kips})^2} = 160 \text{ kips}$</td>
</tr>
<tr>
<td>$\theta = \tan^{-1}\left(\frac{171 \text{ kips}}{168 \text{ kips}}\right) = 45.5^\circ$</td>
<td>$\theta = \tan^{-1}\left(\frac{114 \text{ kips}}{112 \text{ kips}}\right) = 45.5^\circ$</td>
</tr>
</tbody>
</table>

For $\theta = 45^\circ$, $l = 14\frac{1}{2} \text{ in.}$, $kl = 7\frac{1}{2} \text{ in.}$, and $k = 0.517$

By interpolation

- $x = 0.132$,
- $xl = 0.132(14\frac{1}{2} \text{ in.}) = 1.91 \text{ in.}$
- $al = 8 \text{ in.} - xl = 8 \text{ in.} - 1.91 \text{ in.} = 6.09 \text{ in.}$
- $a = 0.420$

By interpolation

$$C = 3.55$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$D_{req} = \frac{P_{wc}}{\phi CCl}$</td>
<td>$D_{req} = \frac{\Omega P_{wc}}{CCl}$</td>
</tr>
<tr>
<td>$= \frac{240 \text{ kips}}{0.75(3.55)(1.0)(2 \text{ welds})(14\frac{1}{2} \text{ in.})}$</td>
<td>$= \frac{2.00(160 \text{ kips})}{(3.55)(1.0)(2 \text{ welds})(14\frac{1}{2} \text{ in.})}$</td>
</tr>
<tr>
<td>$= 3.11 \rightarrow 4 \text{ sixteenths}$</td>
<td>$= 3.11 \rightarrow 4 \text{ sixteenths}$</td>
</tr>
</tbody>
</table>

Minimum weld size is $\frac{1}{4} \text{ in.}$ Use $\frac{1}{4}$-in. fillet welds.

Minimum weld size is $\frac{1}{4} \text{ in.}$ Use $\frac{1}{4}$-in. fillet welds.
Check beam web thickness (against weld size required for strength)

For two fillet welds

\[ t_{\text{min}} = \frac{6.19D}{F_v} = \frac{6.19 \times (3.11 \text{ sixteenths})}{65 \text{ ksi}} = 0.296 \text{ in.} < 0.590 \text{ in.} \quad \text{o.k.} \]

Check the strength of the angles

Shear yielding

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi \left( 0.60 F_y A_{yw} \right) )</td>
<td>( R_u / \Omega = \frac{0.60 F_y A_{yw}}{\Omega} )</td>
</tr>
<tr>
<td>( = 1.0(0.60)(36 \text{ ksi})(2)(14 \frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.}) )</td>
<td>( = 0.60(36 \text{ ksi})(2)(14 \frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.}) )</td>
</tr>
<tr>
<td>( = 470 \text{ kips} &gt; 168 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 313 \text{ kips} &gt; 112 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Similarly, shear yielding of the angles due to \( H_{aw} \) is not critical.

Shear rupture

\[ A_{sv} = (\frac{3}{8} \text{ in.}) \left[ 2(14 \frac{3}{8} \text{ in.}) - 10(1 \text{ in.}) \right] = 14.3 \text{ in.}^2 \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = \phi \left( 0.60 F_y A_{yw} \right) )</td>
<td>( R_u / \Omega = \frac{0.60 F_y A_{yw}}{\Omega} )</td>
</tr>
<tr>
<td>( = 0.75(0.60)(58 \text{ ksi})(14.3 \text{ in.}^2) )</td>
<td>( = \frac{0.60(58 \text{ ksi})(14.3 \text{ in.}^2)}{2.00} )</td>
</tr>
<tr>
<td>( = 372 \text{ kips} &gt; 168 \text{ kips} \quad \text{o.k.} )</td>
<td>( = 248 \text{ kips} &gt; 11 \text{ kips} \quad \text{o.k.} )</td>
</tr>
</tbody>
</table>

Block shear rupture

With \( n = 5, L_{sv} = 1\frac{1}{4} \text{ in.}, L_{eh} = 3\frac{1}{4} \text{ in.} \) Section J4.3

\[ A_{sw} = \left[ (3\frac{3}{8} \text{ in.}) - (\frac{3}{8})(1.0 \text{ in.}) \right](\frac{3}{8} \text{ in.})(2) = 4.12 \text{ in.}^2 \]

\[ A_{sv} = (13\frac{3}{8} \text{ in.})(\frac{3}{8} \text{ in.})(2) = 19.8 \text{ in.}^2 \]

\[ A_{sv} = \left[ (13\frac{3}{8} \text{ in.}) - (4\frac{3}{8})(1 \text{ in.}) \right](\frac{3}{8} \text{ in.})(2) = 13.1 \text{ in.}^2 \]

\[ F_y A_y = (58 \text{ ksi})(4.12 \text{ in.}^2) = 239 \text{ kips} \]

\[ 0.60 F_y A_y = 0.60(36 \text{ ksi})(19.9 \text{ in.}^2) = 430 \text{ kips} \quad \text{controls} \]

\[ 0.60 F_y A_y = 0.6(58 \text{ ksi})(13.1 \text{ in.}^2) = 457 \text{ kips} \]
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Eqn. J4-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = 0.75(239 \text{ kips} + 430 \text{ kips})$</td>
<td>$R_n / \Omega = \frac{(239 \text{ kips} + 430 \text{ kips})}{2.00}$</td>
<td></td>
</tr>
<tr>
<td>$= 502 \text{ kips} &gt; 168 \text{ kips}$</td>
<td>$= 334 \text{ kips} &gt; 112 \text{ kips}$</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

**Check column flange**

By inspection, the 4.16 in. thick column flange has adequate flexural strength, stiffness, and bearing strength.

Note: When the brace is in compression, the buckling strength of the gusset would have to be checked, where

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi R_n = \phi_c F_{cr} A_w$</td>
<td>$R_n / \Omega = F_{cr} A_w / \Omega_c$</td>
</tr>
</tbody>
</table>

In the above equation, $\phi, F_{cr},$ or $F_{cr} / \Omega,$ may be determined from $\frac{Kl}{r}$ with Specification Section E3 or J4.4, where $l_i$ is the perpendicular distance from the Whitmore section to the interior edge of the gusset. Alternatively, the average value of $l = \frac{l_1 + l_2 + l_3}{3}$ may be substituted, where these quantities are illustrated in the figure. Note that, for this example $l_2$ is negative since part of the Whitmore section is in the beam web.

The effective length factor $K$ has been established as 0.5 by full scale tests on bracing connections (Gross, 1990). It assumes that the gusset is supported on both edges. In cases where the gusset is supported on one edge only, such as illustrated in Example 3, Figure (d), the brace can more readily move out-of-plane and a sidesway mode of buckling can occur in the gusset. For this case, $K$ should be taken as 1.2.

**Check gusset buckling**

The area of the Whitmore section is

$$A_w = (30.9 \text{ in.})(\frac{1}{2} \text{ in.}) + (3.90 \text{ in.})(0.590 \text{ in.})\left(\frac{50 \text{ ksi}}{36 \text{ ksi}}\right) = 26.4 \text{ in.}^2$$

In the above equation, the area in the beam web is multiplied by the ratio 50/36 to convert the area to an equivalent area of A36 plate.

The slenderness ratio is

$$\frac{Kl}{r} = \frac{0.5(17.0 \text{ in.})\left(\sqrt{2}\right)}{\frac{1}{2} \text{ in.}} = 39.3$$

From Specification Section E3

$$F_e = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2} = \frac{\pi^2 \left(29000 \text{ ksi}\right)}{39.3^2} = 185$$
Check $4.71 \sqrt{\frac{E}{F_y}} = 4.71 \sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}} = 134$

The buckling stress is given by $F_{cr} = \left[ \frac{F_y}{0.658} \right] = \left[ \frac{36 \text{ ksi}}{0.658} \right] = 55 \text{ ksi}$

The required stress is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$f_{ud} = \frac{675 \text{ kips}}{26.4 \text{ in.}^2} = 25.6 \text{ ksi}$</td>
<td>$f_{ud} = \frac{450 \text{ kips}}{26.4 \text{ in.}^2} = 17.0 \text{ ksi}$</td>
</tr>
<tr>
<td>The available stress is</td>
<td>The available stress is</td>
</tr>
<tr>
<td>$\phi F_{cr} = 0.90(33.2 \text{ ksi})$</td>
<td>$F_{cr} / \Omega = \frac{33.2 \text{ ksi}}{1.67}$</td>
</tr>
</tbody>
</table>
| = 29.9 ksi > 25.6 ksi | = 19.9 ksi > 17.0 ksi | o.k. | o.k.
Example II.C-3  Bracing Connection

Given:

Each of the four designs shown for the diagonal bracing connection between the W14×68 brace, W24×55 beam, and W14×211 column web have been developed using the Uniform Force Method (the General Case and Special Cases 1, 2, and 3) for the load case of 1.2D + 1.3W for LRFD and D + W for ASD.

For the given values of $\alpha$ and $\beta$, determine the interface forces on the gusset-to-column and gusset-to-beam connections for

a. General Case of Figure (a)
b. Special Case 1 of Figure (b)
c. Special Case 2 of Figure (c)
d. Special Case 3 of Figure (d)

<table>
<thead>
<tr>
<th>Brace Axial Load</th>
<th>$P_u = \pm 195$ kips</th>
<th>$P_o = \pm 130$ kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam End Reaction</td>
<td>$R_u = 44$ kips</td>
<td>$R_o = 29$ kips</td>
</tr>
<tr>
<td>Beam Axial Load</td>
<td>$A_u = 26$ kips</td>
<td>$A_o = 17$ kips</td>
</tr>
</tbody>
</table>
Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>ASTM</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
<th>Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace</td>
<td>W14×68</td>
<td>A992</td>
<td>50</td>
<td>65</td>
<td>Manual</td>
</tr>
<tr>
<td>Beam</td>
<td>W24×55</td>
<td>A992</td>
<td>50</td>
<td>65</td>
<td>Tables 2-3 and 2-4</td>
</tr>
<tr>
<td>Column</td>
<td>W14×211</td>
<td>A992</td>
<td>50</td>
<td>65</td>
<td></td>
</tr>
<tr>
<td>Gusset Plate</td>
<td>W14×211</td>
<td>A36</td>
<td>36</td>
<td>58</td>
<td></td>
</tr>
</tbody>
</table>
Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Width</th>
<th>Height</th>
<th>A</th>
<th>d</th>
<th>tw</th>
<th>bf</th>
<th>tf</th>
<th>k</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brace</td>
<td>W14×68</td>
<td>14 in.</td>
<td>68 in.</td>
<td>20.0 in²</td>
<td>14.0 in.</td>
<td>10.0 in.</td>
<td>0.720 in.</td>
<td>Manual</td>
</tr>
<tr>
<td>Beam</td>
<td>W24×55</td>
<td>23.6 in.</td>
<td>55 in.</td>
<td>26.7 kips</td>
<td>7.01 in.</td>
<td>5.05 in.</td>
<td>1.11</td>
<td>Table 1-1</td>
</tr>
<tr>
<td>Column</td>
<td>W14×211</td>
<td>15.7 in.</td>
<td>211 in.</td>
<td>195 kips</td>
<td>15.8 in.</td>
<td>1.56 in.</td>
<td>21.9 in.</td>
<td></td>
</tr>
</tbody>
</table>

Solution A (General Case):

Assume $\beta = \bar{\beta} = 3$ in.

$$\alpha = e_e \tan \theta - e_e + \beta \tan \theta = (11.8 \text{ in.}) \left( \frac{12}{11 \frac{1}{\kappa_e}} \right) - 0 + (3 \text{ in.}) \left( \frac{12}{11 \frac{1}{\kappa_e}} \right) = 16.1 \text{ in.}$$

Since $\alpha \neq \bar{\alpha}$, an eccentricity exists on the gusset-to-beam connection.

Calculate the interface forces

$$r = \sqrt{(\alpha + e_e)^2 + (\beta + e_e)^2} = \sqrt{(16.1 \text{ in.} + 0 \text{ in.})^2 + (3 \text{ in.} + 11.8 \text{ in.})^2} = 21.9 \text{ in.}$$

On the gusset-to-column connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{uc} = \frac{\beta}{r} P_a = \frac{3 \text{ in.}}{21.9 \text{ in.}} \cdot (195 \text{ kips}) = 26.7 \text{ kips}$</td>
<td>$V_{uc} = \frac{\beta}{r} P_a = \frac{3 \text{ in.}}{21.9 \text{ in.}} \cdot (130 \text{ kips}) = 17.8 \text{ kips}$</td>
</tr>
<tr>
<td>$H_{uc} = \frac{e_e}{r} P_a = 0 \text{ kips}$</td>
<td>$H_{uc} = \frac{e_e}{r} P_a = 0 \text{ kips}$</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ub} = \frac{\alpha}{r} P_a = \frac{16.1 \text{ in.}}{21.9 \text{ in.}} \cdot (195 \text{ kips}) = 143 \text{ kips}$</td>
<td>$H_{ub} = \frac{\alpha}{r} P_a = \frac{16.1 \text{ in.}}{21.9 \text{ in.}} \cdot (130 \text{ kips}) = 95.6 \text{ kips}$</td>
</tr>
<tr>
<td>$V_{ub} = \frac{e_e}{r} P_a = \frac{11.8 \text{ in.}}{21.9 \text{ in.}} \cdot (195 \text{ kips}) = 105 \text{ kips}$</td>
<td>$V_{ub} = \frac{e_e}{r} P_a = \frac{11.8 \text{ in.}}{21.9 \text{ in.}} \cdot (130 \text{ kips}) = 70.0 \text{ kips}$</td>
</tr>
<tr>
<td>$M_{ub} = V_{ub} \left( \alpha - \bar{\alpha} \right)$</td>
<td>$M_{ub} = V_{ub} \left( \alpha - \bar{\alpha} \right)$</td>
</tr>
<tr>
<td>$= \frac{(105 \text{ kips})(15 \frac{1}{2} \text{ in.} - 16.1 \text{ in.})}{12 \text{ in./ft}}$</td>
<td>$= \frac{(70.0 \text{ kips})(15 \frac{1}{2} \text{ in.} - 16.1 \text{ in.})}{12 \text{ in./ft}}$</td>
</tr>
<tr>
<td>$= -3.06 \text{ kip-ft}$</td>
<td>$= -2.04 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

In this case, this small moment is negligible.

On the beam-to-column connection, the shear is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ab} + V_{ab} = 44 \text{ kips} + 105 \text{ kips} = 149 \text{ kips}$</td>
<td>$R_{ab} + V_{ab} = 29 \text{ kips} + 70 \text{ kips} = 99 \text{ kips}$</td>
</tr>
</tbody>
</table>
and the axial force is

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
A_{ab} + H_{ac} = 26 \text{ kips} \pm 0 \text{ kips} = 26 \text{ kips} & A_{ab} + H_{ac} = 17 \text{ kips} \pm 0 \text{ kips} = 17 \text{ kips}
\end{array}
\]

For a discussion of the sign use between \(A_{ab}\) and \(H_{ac}\) (\(A_{ab}\) an \(H_{ac}\) for ASD), refer to AISC (1992).

**Solution B (Special Case 1):**

In this case, the centroidal positions of the gusset-edge connections are irrelevant; \(\bar{\alpha}\) and \(\bar{\beta}\) are given to define the geometry of the connection, but are not needed to determine the gusset edge forces.

The angle of the brace from the vertical is

\[
\theta = \tan^{-1}\left(\frac{12}{10\frac{1}{12}}\right) = 49.8^\circ
\]

The horizontal and vertical components of the brace force are

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
H_u = P_u \sin \theta = (195 \text{ kips}) \sin 49.8^\circ = 149 \text{ kips} & H_a = P_a \sin \theta = (130 \text{ kips}) \sin 49.8^\circ = 99.3 \text{ kips} \\
V_u = P_u \cos \theta = (195 \text{ kips}) \cos 49.8^\circ = 126 \text{ kips} & V_a = P_a \cos \theta = (130 \text{ kips}) \cos 49.8^\circ = 83.9 \text{ kips}
\end{array}
\]

On the gusset-to-column connection

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
V_{uc} = V_u = 126 \text{ kips} & V_{ac} = V_a = 83.9 \text{ kips} \\
H_{ac} = 0 \text{ kips} & H_{ac} = 0 \text{ kips}
\end{array}
\]

On the gusset-to-beam connection

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
V_{ab} = 0 \text{ kips} & V_{ab} = 0 \text{ kips} \\
H_{ac} = H_a = 149 \text{ kips} & H_{ab} = H_a = 99.3 \text{ kips}
\end{array}
\]

On the beam-to-column connection

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
R_{ab} = 44 \text{ kips (shear)} & R_{ab} = 29 \text{ kips (shear)} \\
A_{ab} = 26 \text{ kips (axial transfer force)} & A_{ab} = 17 \text{ kips (axial transfer force)}
\end{array}
\]

In addition to the forces on the connection interfaces, the beam is subjected to a moment \(M_{ub}\) or \(M_{ab}\), where

\[
\begin{array}{l|l}
\text{LRFD} & \text{ASD} \\
M_{ub} = H_{ab}e_b = \frac{(149 \text{ kips})(11.8 \text{ in.})}{12 \text{ in./ft}} & M_{ab} = H_{ab}e_b = \frac{(99.3 \text{ kips})(11.8 \text{ in.})}{12 \text{ in./ft}} \\
= 147 \text{ kip-ft} & = 97.6 \text{ kip-ft}
\end{array}
\]
This moment, as well as the beam axial load \( H_{ab} = 149 \text{ kips} \) or \( H_{ab} = 99.3 \text{ kips} \) and the moment and shear in the beam associated with the end reaction \( R_{ab} \) or \( R_{ab} \), must be considered in the design of the beam.

**Solution C (Special Case 2):**

Assume \( \beta = 10/\ell \text{ in.} \).

\[
\alpha = e_a \tan \theta - e_c + \beta \tan \theta = (11.8 \text{ in.}) \left( \frac{12}{11\ell_a} \right) - 0 + (10/\ell \text{ in.}) \left( \frac{12}{11\ell_a} \right) = 24.2 \text{ in.}
\]

Calculate the interface forces for the general case before applying Special Case 2.

\[
r = \sqrt{(\alpha + e_a)^2 + (\beta + e_a)^2} = \sqrt{(24.2 \text{ in.} + 0 \text{ in.})^2 + (10/\ell \text{ in.} + 11.8 \text{ in.})^2} = 32.9 \text{ in.}
\]

On the gusset-to-column connection

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_{uc} )</td>
<td>( \frac{\beta}{r} P_a = 10/\ell \text{ in.} ) (195 kips) = 62.2 kips</td>
<td>( V_{uc} )</td>
</tr>
<tr>
<td>( H_{uc} )</td>
<td>( \frac{e_a}{r} P_a = 0 \text{ kips} )</td>
<td>( H_{uc} )</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{ab} )</td>
<td>( \frac{\alpha}{r} P_a = 24.2 \text{ in.} ) (195 kips) = 143 kips</td>
<td>( H_{ab} )</td>
</tr>
<tr>
<td>( V_{ab} )</td>
<td>( \frac{e_a}{r} P_a = 11.8 \text{ in.} ) (195 kips) = 69.9 kips</td>
<td>( V_{ab} )</td>
</tr>
</tbody>
</table>

On the beam-to-column connection, the shear is

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_{ab} + V_{ab} )</td>
<td>44 kips + 69.9 kips = 114 kips</td>
<td>( R_{ab} + V_{ab} )</td>
</tr>
</tbody>
</table>

and the axial force is

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( A_{ab} + H_{uc} )</td>
<td>26 kips ± 0 kips = 26 kips</td>
<td>( A_{ab} + H_{uc} )</td>
</tr>
</tbody>
</table>

Next, applying Special Case 2 with \( \Delta V_{ab} = V_{ab} = 69.9 \text{ kips} \) (\( \Delta V_{ab} = V_{ab} = 46.6 \text{ kips} \) for ASD), calculate the interface forces.
On the gusset-to-column connection (where $V_{uc}$ is replaced by $V_{uc} + \Delta V_{ub}$) or (where $V_{uc}$ is replaced by $V_{uc} + \Delta V_{ub}$ for ASD)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{uc} = 62.2 \text{ kips} + 69.9 \text{ kips} = 132 \text{ kips}$</td>
<td>$V_{uc} = 41.5 \text{ kips} + 46.6 \text{ kips} = 88.1 \text{ kips}$</td>
</tr>
<tr>
<td>$H_{uc} = 0 \text{ kips} \ (\text{unchanged})$</td>
<td>$H_{uc} = 0 \text{ kips} \ (\text{unchanged})$</td>
</tr>
</tbody>
</table>

On the gusset-to-beam connection (where $V_{ub}$ is replaced by $V_{ub} - \Delta V_{ub}$) or (where $V_{ab}$ is replaced by $V_{ab} - \Delta V_{ab}$)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ub} = 143 \text{ kips} \ (\text{unchanged})$</td>
<td>$H_{ub} = 95.6 \text{ kips} \ (\text{unchanged})$</td>
</tr>
<tr>
<td>$V_{ub} = 69.9 \text{ kips} - 69.9 \text{ kips} = 0 \text{ kips}$</td>
<td>$V_{ab} = 46.6 \text{ kips} - 46.6 \text{ kips} = 0 \text{ kips}$</td>
</tr>
<tr>
<td>$M_{ub} = (\Delta V_{ub})\alpha = \frac{(69.9 \text{ kips})(24.2 \text{ in.})}{12\text{ in./ft}} = 141 \text{ kip-ft}$</td>
<td>$M_{ab} = (\Delta V_{ab})\alpha = \frac{(46.6 \text{ kips})(24.2 \text{ in.})}{12\text{ in./ft}} = 94.0 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

On the beam-to-column connection, the shear is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ub} + \Delta V_{ub} - \Delta V_{ub} = 44 \text{ kips} + 69.9 \text{ kips} - 69.9 \text{ kips}$</td>
<td>$R_{ab} + \Delta V_{ab} - \Delta V_{ab} = 29 \text{ kips} + 46.6 \text{ kips} - 46.6 \text{ kips}$</td>
</tr>
<tr>
<td>$= 44 \text{ kips}$</td>
<td>$= 29 \text{ kips}$</td>
</tr>
</tbody>
</table>

and the axial force is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ub} + H_{uc} = 26 \text{ kips} \pm 0 \text{ kips} = 26 \text{ kips}$</td>
<td>$A_{ab} + H_{uc} = 17 \text{ kips} \pm 0 \text{ kips} = 17 \text{ kips}$</td>
</tr>
</tbody>
</table>
Solution D (Special Case 3):

Set $\beta = \bar{\beta} = 0$ in.

$$\alpha = e_b \tan \theta = (11.8 \text{ in.}) \left( \frac{12}{11.8 \text{ in.}} \right) = 12.8 \text{ in.}$$

Since $\alpha \neq \bar{\alpha}$, an eccentricity exists on the gusset-to-beam connection.

Calculate the interface forces

$$r = \sqrt{\alpha^2 + e_b^2} = \sqrt{(12.8 \text{ in.})^2 + (11.8 \text{ in.})^2} = 17.4 \text{ in.}$$

On the gusset-to-beam connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$H_{ab} = \frac{\alpha}{r} P_a = \frac{12.8 \text{ in.}}{17.4 \text{ in.}} (195 \text{ kips}) = 143 \text{ kips}$</td>
<td>$H_{ab} = \frac{\alpha}{r} P_a = \frac{12.8 \text{ in.}}{17.4 \text{ in.}} (130 \text{ kips}) = 95.6 \text{ kips}$</td>
</tr>
<tr>
<td>$V_{ab} = \frac{e_b}{r} P_a = \frac{11.8 \text{ in.}}{17.4 \text{ in.}} (195 \text{ kips}) = 132 \text{ kips}$</td>
<td>$V_{ab} = \frac{e_b}{r} P_a = \frac{11.8 \text{ in.}}{17.4 \text{ in.}} (130 \text{ kips}) = 88.2 \text{ kips}$</td>
</tr>
<tr>
<td>$M_{ab} = V_{ab} (\alpha - \bar{\alpha})$</td>
<td>$M_{ab} = V_{ab} (\alpha - \bar{\alpha})$</td>
</tr>
<tr>
<td>$\frac{(132 \text{ kips})(12.8 \text{ in.} - 13 \frac{1}{2} \text{ in.})}{12 \text{ in./ft}} = -7.70 \text{ kip-ft}$</td>
<td>$\frac{(88.2 \text{ kips})(12.8 \text{ in.} - 13 \frac{1}{2} \text{ in.})}{12 \text{ in./ft}} = -5.15 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

In this case, this small moment is negligible.

On the beam-to-column connection, the shear is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ab} + V_{ab} = 44 \text{ kips} + 132 \text{ kips} = 176 \text{ kips}$</td>
<td>$R_{ab} + V_{ab} = 29 \text{ kips} + 88.2 \text{ kips} = 117 \text{ kips}$</td>
</tr>
</tbody>
</table>

And the axial force is

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_{ab} + H_{ac} = 26 \text{ kips} \pm 0 \text{ kips} = 26 \text{ kips}$</td>
<td>$A_{ab} + H_{ac} = 17 \text{ kips} \pm 0 \text{ kips} = 17 \text{ kips}$</td>
</tr>
</tbody>
</table>

Note: From the foregoing results, designs by Special Case 3 and the General Case of the Uniform Force Method provide the more economical designs. Additionally, note that designs by Special Case 1 and Special Case 2 result in moments on the beam and/or column that must be considered.
Example II.C-4  Truss Support Connection

Given:

Determine the requirements for the following cases:

a. Joint $L_1$

b. Joint $U_1$

$R_D = 18.5$ kips
$R_L = 55.5$ kips

Use E70 electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>Section</th>
<th>ASTM</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Chord</td>
<td>WT8×38.5</td>
<td>A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Bottom Chord</td>
<td>WT8×28.5</td>
<td>A992</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Diagonal $U_0L_1$</td>
<td>2L4×3½×⅜</td>
<td>A36</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>Web $U_1L_1$</td>
<td>2L3½×3×⅛</td>
<td>A36</td>
<td>36</td>
<td>58</td>
</tr>
<tr>
<td>Diagonal $U_1L_2$</td>
<td>2L3½×2½×⅛</td>
<td>A36</td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Manual Tables 2-3 and 2-4
Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Dimensions</th>
<th>Properties</th>
</tr>
</thead>
<tbody>
<tr>
<td>Top Chord</td>
<td>WT8×38.5</td>
<td>( t_w = 0.455 \text{ in.} )</td>
</tr>
<tr>
<td>Bottom Chord</td>
<td>WT8×28.5</td>
<td>( t_w = 0.430 \text{ in.} ) ( d = 8.22 \text{ in.} )</td>
</tr>
<tr>
<td>Diagonal ( U_0L_1 )</td>
<td>2L4×3( \frac{1}{2} )×( \frac{3}{8} )</td>
<td>( A = 5.35 \text{ in.}^2 ) ( y = 1.20 \text{ in.} )</td>
</tr>
<tr>
<td>Web ( U_1L_1 )</td>
<td>2L3( \frac{1}{2} )×3×( \frac{3}{16} )</td>
<td>( A = 3.91 \text{ in.}^2 )</td>
</tr>
<tr>
<td>Diagonal ( U_1L_2 )</td>
<td>2L3( \frac{1}{2} )×2( \frac{1}{2} )×( \frac{3}{8} )</td>
<td>( A = 3.58 \text{ in.}^2 )</td>
</tr>
</tbody>
</table>

Solution a:

Check shear yielding of bottom chord tee stem (on Section A-A)

\( R_s = 0.6F_yA_w = 0.6(50 \text{ ksi})(8.22 \text{ in.})(0.430 \text{ in.}) = 106 \text{ kips} \)  \( \text{Eqn. J4-3} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_s = 1.00(106 \text{ kips}) = 106 \text{ kips} )</td>
<td>( R_s / \Omega = \frac{106 \text{ kips}}{1.50} = 70.7 \text{ kips} )</td>
</tr>
<tr>
<td>106 kips &lt; 108 kips</td>
<td>n.g.</td>
</tr>
</tbody>
</table>

Additional shear area must be provided.

Try \( PL \frac{3}{16} \text{ in.} \times 4 \text{ in.} \) complete-joint-penetration groove welded to the stem of the WT.

\( R_s = 0.6(36 \text{ ksi})(4 \text{ in.})(\frac{3}{16} \text{ in.}) = 37.8 \text{ kips} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_s = 106 \text{ kips} + 1.00(37.8 \text{ kips}) = 144 \text{ kips} )</td>
<td>( R_s / \Omega = \frac{70.7 \text{ kips} + 37.8 \text{ kips}}{1.50} = 95.9 \text{ kips} )</td>
</tr>
<tr>
<td>144 kips &gt; 108 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>95.9 kips &gt; 72 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Design welds for member \( U_1L_1 \)

User Note: Specification Section J1.7 requiring that the center of gravity of the weld group coincide with the center of gravity of the member does not apply to end connections of statically loaded single angle, double angle and similar members.
The minimum weld size is \( w_{\text{min}} = \frac{3}{16} \text{ in.} \).

The maximum weld size is \( w_{\text{max}} = \) thickness - \( \frac{3}{16} \text{ in.} = \frac{3}{4} \text{ in.} \).

Calculate the minimum length of \( \frac{3}{16} \)-in. fillet weld.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{min}} = \frac{R_s}{1.392D} = \frac{108 \text{ kips}}{1.392(3 \text{ sixteenths})} )</td>
<td>( L_{\text{min}} = \frac{R_s}{0.928D} = \frac{72 \text{ kips}}{0.928(3 \text{ sixteenths})} )</td>
</tr>
<tr>
<td>= 25.9 in.</td>
<td>= 25.9 in.</td>
</tr>
</tbody>
</table>

Use 6\( \frac{1}{2} \) in. of \( \frac{3}{16} \)-in. weld at the heel and toe of both angles for a total of 26 in.

**Design welds for member \( U_{0L_1} \)**

The minimum weld size is \( w_{\text{min}} = \frac{3}{16} \text{ in.} \).

The maximum weld size is \( w_{\text{max}} = \) thickness - \( \frac{3}{16} \text{ in.} = \frac{3}{4} \text{ in.} \).

Calculate the minimum length of \( \frac{3}{16} \)-in. fillet weld.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{min}} = \frac{R_s}{1.392D} = \frac{165 \text{ kips}}{1.392(3 \text{ sixteenths})} )</td>
<td>( L_{\text{min}} = \frac{R_s}{0.928D} = \frac{110 \text{ kips}}{0.928(3 \text{ sixteenths})} )</td>
</tr>
<tr>
<td>= 39.5 in.</td>
<td>= 39.5 in.</td>
</tr>
</tbody>
</table>

Use 10 in. of \( \frac{3}{16} \)-in. weld at the heel and toe of both angles for a total of 40 in.

**Check tension yielding of diagonal \( U_{0L_1} \)**

\( R_u = F_y A_g = (36 \text{ ksi})(5.35 \text{ in.}^2) = 193 \text{ kips} \) \( \text{Eqn. J4-1} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 0.90(193 \text{ kips}) = 173 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{193 \text{ kips}}{1.67} = 115 \text{ kips} )</td>
</tr>
<tr>
<td>173 kips &gt; 165 kips \text{ o.k.}</td>
<td>115 kips &gt; 110 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>

**Check tension rupture of diagonal \( U_{0L_1} \)**

\( U = 1 - \frac{\bar{x}}{\bar{L}} = 1 - \frac{1.20 \text{ in.}}{10 \text{ in.}} = 0.880 \) \( \text{Table D3.1 Case 2} \)

\( R_u = F_y A_e = (58 \text{ ksi})(0.880)(5.35 \text{ in.}^2) = 273 \text{ kips} \) \( \text{Eqn. J4-2} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 0.75(273 \text{ kips}) = 205 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{273 \text{ kips}}{2.00} = 137 \text{ kips} )</td>
</tr>
<tr>
<td>205 kips &gt; 165 kips \text{ o.k.}</td>
<td>137 kips &gt; 110 kips \text{ o.k.}</td>
</tr>
</tbody>
</table>
Solution b:

Check shear yielding of top chord tee stem (on Section B-B)

\[ R_u = 0.6F_y A_w = 0.6(50 \text{ ksi})(8.26 \text{ in.})(0.455 \text{ in.}) = 113 \text{ kips} \]  

\[ \text{Eqn. J4-3} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 1.00(113 \text{ kips}) = 113 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{113 \text{ kips}}{1.50} = 75.3 \text{ kips} )</td>
</tr>
<tr>
<td>113 kips &gt; 78 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>75.3 kips &gt; 52 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Design welds for member \( U_1 L_1 \)

As calculated previously in Solution a, use 6½ in. of \( \frac{3}{16} \)-in. weld at the heel and toe of both angles for a total of 26 in.

Design welds for member \( U_1 L_2 \)

The minimum weld size is \( w_{\text{min}} = \frac{3}{16} \) in.

The maximum weld size is \( w_{\text{max}} = \frac{3}{8} \) in.

Calculate the minimum length of \( \frac{3}{4} \) in. fillet weld.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( L_{\text{min}} = \frac{R_u}{1.392D} = 114 \text{ kips} )</td>
<td>( L_{\text{min}} = \frac{R_u}{0.928D} = 76 \text{ kips} )</td>
</tr>
<tr>
<td>( = 20.5 \text{ in.} )</td>
<td>( = 20.5 \text{ in.} )</td>
</tr>
</tbody>
</table>

Use 7½ in. of \( \frac{3}{4} \)-in. fillet weld at the heel and 4 in. of fillet weld at the toe of each angle for a total of 23 in.

Check tension yielding of diagonal \( U_1 L_2 \)

\[ R_u = F_y A_e = (36 \text{ ksi})(3.58 \text{ in.}^2) = 129 \text{ kips} \]  

\[ \text{Eqn. J4-1} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 0.90(129 \text{ kips}) = 116 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{129 \text{ kips}}{1.67} = 77.2 \text{ kips} )</td>
</tr>
<tr>
<td>116 kips &gt; 114 kips</td>
<td>o.k.</td>
</tr>
<tr>
<td>77.2 kips &gt; 76 kips</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Check tension rupture of diagonal \( U_1 L_2 \)

\[ U = 1 - \frac{x}{l} = 1 - \frac{1.13 \text{ in.}}{\left( \frac{7 \frac{1}{2} \text{ in.} + 4 \text{ in.}}{2} \right)} = 0.803 \]  

\[ \text{Table D3.1 Case 2} \]

\[ R_u = F_y A_e = (58 \text{ ksi})(0.803)(3.58 \text{ in.}^2) = 167 \text{ kips} \]  

\[ \text{Eqn. J4-2} \]
Check block shear rupture

Because the cut end of the angle, the block shear rupture model presented in Part 9 does not directly apply. Conservatively, the block shear rupture strength will be based on the shear rupture strength of the WT stem along the length of the welds.

\[ R_u = 0.6 F_w A_w = 0.6 (65 \text{ ksi})(7/2 \text{ in.} + 4 \text{ in.})(0.455 \text{ in.}) = 204 \text{ kips} \quad \text{Eqn. J4-4} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 0.75 (167 \text{ kips}) = 125 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{167 \text{ kips}}{2.00} = 83.4 \text{ kips} )</td>
</tr>
<tr>
<td>125 kips &gt; 114 kips o.k.</td>
<td>83.4 kips &gt; 76 kips o.k.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_u = 0.75 (204 \text{ kips}) = 153 \text{ kips} )</td>
<td>( R_u / \Omega = \frac{204 \text{ kips}}{2.00} = 102 \text{ kips} )</td>
</tr>
<tr>
<td>153 kips &gt; 114 kips o.k.</td>
<td>102 kips &gt; 76 kips o.k.</td>
</tr>
</tbody>
</table>
Example II.C-5  HSS Chevron Brace Connection

Given:

The loads shown are actual loads from the bottom chevron from the example problem at the back of the book. The beam above has been design to carry its load with out the chevron, and the end connections have been design to all vertical and horizontal loads required. Check the HSS braces for tension and compression and design the connection including welding, shear lag requirements and check to see if stiffening is required.

Use E70 electrodes.

Material Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>$F_y$ (ksi)</th>
<th>$F_u$ (ksi)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×35</td>
<td>50</td>
<td>65</td>
</tr>
<tr>
<td>Brace</td>
<td>HSS6×6×½</td>
<td>46</td>
<td>58</td>
</tr>
<tr>
<td>Gusset Plate</td>
<td></td>
<td>36</td>
<td>58</td>
</tr>
</tbody>
</table>

Geometric Properties:

<table>
<thead>
<tr>
<th>Component</th>
<th>Section</th>
<th>$d$ (in.)</th>
<th>$t_w$ (in.)</th>
<th>$k_{des.}$ (in.)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Beam</td>
<td>W18×35</td>
<td>17.7</td>
<td>0.300</td>
<td>0.827</td>
</tr>
<tr>
<td>Brace</td>
<td>HSS6×6×½</td>
<td>6</td>
<td>6</td>
<td>9.74</td>
</tr>
</tbody>
</table>

Solution:

Determine required brace-to-gusset weld size

Since the brace loads are axial, the angle between the longitudinal brace axis and line of force is $\theta_w = 0^\circ$.

$$F_w = 0.60 F_{kxx} \left(1 + 0.5 \sin^{1.5} \theta_w \right) = 0.60 \left(70 \text{ ksi}\right) \left(1 + 0.5 \sin^{1.5} 0^\circ \right) = 42 \text{ ksi}$$
<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ W_{wreq'd} = \frac{P_u}{\phi F_y (0.707) L_w} + \frac{\gamma_d}{\gamma_{16}} \text{ in.} ]</td>
<td>[ W_{wreq'd} = \frac{P_u \Omega}{4 F_y (0.707) L_w} + \frac{\gamma_d}{\gamma_{16}} \text{ in.} ]</td>
</tr>
<tr>
<td>[ = \frac{158 \text{ kips}}{0.75(4)(42 \text{ ksi})(0.707)(6 \text{ in.})} + \frac{\gamma_d}{\gamma_{16}} \text{ in.} ]</td>
<td>[ = \frac{(105 \text{ kips})(2.00)}{(4)(42 \text{ ksi})(0.707)(6 \text{ in.})} + \frac{\gamma_d}{\gamma_{16}} \text{ in.} ]</td>
</tr>
<tr>
<td>[ = 0.296 \text{ in.} + \frac{\gamma_d}{\gamma_{16}} \text{ in.} = 0.358 \text{ in.} ]</td>
<td>[ = 0.295 \text{ in.} ]</td>
</tr>
</tbody>
</table>

Use 3/16-in. fillet weld

Use 3/8-in. fillet weld

Note: the \( \frac{\gamma_d}{\gamma_{16}} \text{ in.} \) added to the weld size is to account for the slot in HSS

The minimum weld size for this connection is 3/16 in. The required weld size is larger therefore, use 3/8-in. fillet welds.

**Determine required gusset plate thickness**

\[ W_e = W_w - \frac{\gamma_d}{\gamma_{16}} \text{ in.} = \frac{5}{16} \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ t_{req'd} = \frac{0.60 F_{Fy} W_e}{\phi (0.60 F_{Fy})} \left( \frac{0.707}{2} \right) ]</td>
<td>[ t_{req'd} = \frac{0.60 F_{Fy} W_e}{\Omega (0.60 F_{Fy})} \left( \frac{0.707}{2} \right) ]</td>
</tr>
<tr>
<td>[ = \frac{0.75(0.60)(70 \text{ ksi})(\frac{5}{16} \text{ in.})(0.707)(2)}{1.00(0.60)(36 \text{ ksi})} ]</td>
<td>[ = \frac{(1.50)(0.60)(70 \text{ ksi})(\frac{5}{16} \text{ in.})(0.707)(2)}{2.00(0.60)(36 \text{ ksi})} ]</td>
</tr>
<tr>
<td>[ = 0.644 \text{ in.} ]</td>
<td>[ = 0.644 \text{ in.} ]</td>
</tr>
</tbody>
</table>

Use a 3/8-in. gusset plate.

**Check gusset plate buckling** (compression brace)

\[ r = \frac{t_1}{\sqrt{12}} = \frac{\frac{5}{16} \text{ in.}}{\sqrt{12}} = 0.217 \text{ in.} \]

From the figure, the distance \( l_1 = 6\frac{1}{2} \text{ in.} \)

Since the gusset is attached by one edge only, the buckling mode could be a sidesway type as shown in Commentary Table C-C2.2. In this case use \( K = 1.2 \).

\[ \frac{K l_1}{r} = \frac{1.2(6\frac{1}{2} \text{ in.})}{0.217 \text{ in.}} = 36.0 \]

Limiting slenderness ratio \[ 4.71 \frac{F_y}{\sqrt{E}} = 4.71 \sqrt{\frac{29000 \text{ ksi}}{36 \text{ ksi}}} = 134 > 36.0 \]
\[
F_e = \frac{\pi^2 E}{\left(\frac{Kl}{r}\right)^2} = \frac{\pi^2 \left(29000 \text{ ksi}\right)}{(36.0)^2} = 221 \text{ ksi}
\]

\[
F_{cr} = \left[0.658 \frac{E_y}{E}ight] F_e = \left[0.658 \frac{36 \text{ ksi}}{221 \text{ ksi}}\right] (36 \text{ ksi}) = 33.6 \text{ ksi}
\]

\[
I_w = B + 2 \left(\text{connection length} \tan 30^\circ\right) = 6 \text{ in.} + 2(6 \text{ in.}) \tan 30^\circ = 12.9 \text{ in.}
\]

Note: Here, the Whitmore section is assumed to be entirely in the gusset. The Whitmore section can spread across the joint into adjacent connected material of equal or greater thickness or adjacent connected material of lesser thickness provided that a rational analysis is performed.

\[
A_w = l_w t_w = (12.9 \text{ in.}) \left(\frac{36}{2} \text{ in.}\right) = 9.675 \text{ in.}^2
\]

\[
P_e = F_{cr} A_w = (33.6 \text{ ksi}) (9.675 \text{ in.}^2) = 325 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi P_e = 0.90(325 \text{ kips}) = 293 \text{ kips})</td>
<td>(P_e / \Omega = \frac{325 \text{ kips}}{1.67} = 195 \text{ kips})</td>
</tr>
<tr>
<td>293 kips &gt; 158 kips (\text{o.k.})</td>
<td>204 kips &gt; 105 kips (\text{o.k.})</td>
</tr>
</tbody>
</table>

**Check tension yielding of gusset plate** (tension brace)

From above, \(A_w = 8.06 \text{ in.}^2\)

\[
R_e = F_{cr} A_w = (36 \text{ ksi}) (9.675 \text{ in.}^2) = 348 \text{ kips}
\]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_e = 0.90(348 \text{ kips}) = 313 \text{ kips})</td>
<td>(R_e / \Omega = \frac{348 \text{ kips}}{1.67} = 208 \text{ kips})</td>
</tr>
<tr>
<td>313 kips &gt; 158 kips (\text{o.k.})</td>
<td>208 kips &gt; 105 kips (\text{o.k.})</td>
</tr>
</tbody>
</table>

**Check shear strength at brace-to-gusset welds**

Try minimum weld length, \(L_w = 6 \text{ in.}\)

Effective Area, \(A_y = 4L_w t = 4(6 \text{ in.})(0.465 \text{ in.}) = 11.2 \text{ in.}^2\)

Nominal Shear Strength, \(V_y = 0.6F_y A_y = 0.6(46 \text{ ksi})(11.2 \text{ in.}^2) = 309 \text{ kips}\)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\phi R_y = 1.00(309 \text{ kips}) = 309 \text{ kips})</td>
<td>(R_y / \Omega = \frac{309 \text{ kips}}{1.50} = 206 \text{ kips})</td>
</tr>
<tr>
<td>309 kips &gt; 158 kips (\text{o.k.})</td>
<td>206 kips &gt; 105 kips (\text{o.k.})</td>
</tr>
</tbody>
</table>
Check shear lag fracture in HSS brace

\[ x = \frac{B^2 + 2BH}{4(B + H)} = \frac{(6 \text{ in.})^2 + 2(6 \text{ in.})(0.625 \text{ in.})}{4(6 \text{ in.} + 0.625 \text{ in.})} = 2.25 \text{ in.} \]

\[ U = 1 - \frac{x}{L_w} = 1 - \frac{2.25 \text{ in.}}{6 \text{ in.}} = 0.625 \text{ in.} \]

\[ A_e = A_g - 2rt_t = 9.74 - 2(0.9)(0.465 \text{ in.})(0.625 \text{ in.} + 0.625 \text{ in. gap}) = 9.01 \text{ in}^2 \]

\[ A_e = UA_h = 0.625(9.16 \text{ in}^2) = 5.72 \text{ in}^2 \]

\[ R_e = F_eA_e = (58 \text{ ksi})(5.63 \text{ in.}) = 327 \text{ kips} \]

Table D3.1
Case 6

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi R_e ) = 0.75(327 \text{ kips}) = 245 \text{ kips}</td>
<td>( R_e / \Omega = \frac{327 \text{ kips}}{2.00} = 164 \text{ kips} )</td>
</tr>
</tbody>
</table>

245 kips > 158 kips \textbf{o.k.} 164 kips > 105 kips \textbf{o.k.}

Calculate interface forces

Design the gusset-to-beam connection as if each brace were the only brace and locate each brace’s connection centroid at the ideal centroid locations to avoid inducing a moment on the gusset-beam interface, similarly to uniform force method special case 3.

\[ e_s = \frac{d}{2} = \frac{17.7 \text{ in.}}{2} = 8.85 \text{ in.} \]

\[ \theta = \tan^{-1} \left( \frac{12}{10 \tan 48^\circ} \right) = 48^\circ \]

Let \( \alpha = \alpha_s \) tan \( \theta = (8.85 \text{ in.}) \tan 48^\circ = 9.83 \text{ in.} \rightarrow \text{Use 10 in.} \)

\[ \beta = e_s = 0 \]

\[ r = \sqrt{(\alpha + e_s)^2 + (\beta + e_s)^2} = \sqrt{(10 \text{ in.} + 0)^2 + (0 + 8.85 \text{ in.})^2} = 13.4 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( H_{ab} = \frac{\alpha P_s}{r} = \frac{(10 \text{ in.})(158 \text{ kips})}{13.4 \text{ in.}} = 118 \text{ kips} )</td>
<td>( H_{ab} = \frac{\alpha P_s}{r} = \frac{(10 \text{ in.})(105 \text{ kips})}{13.4 \text{ in.}} = 78.6 \text{ kips} )</td>
</tr>
<tr>
<td>( V_{ab} = \frac{e_s P_s}{r} = \frac{(8.85 \text{ in.})(158 \text{ kips})}{13.4 \text{ in.}} = 105 \text{kips} )</td>
<td>( V_{ab} = \frac{e_s P_s}{r} = \frac{(8.85 \text{ in.})(105 \text{kips})}{13.4 \text{ in.}} = 69.6 \text{kips} )</td>
</tr>
</tbody>
</table>
Determine required gusset-to-beam weld size

The weld length is twice the horizontal distance from the work point to the centroid of the gusset-to-beam connection, \( \alpha \), for each brace. Therefore, \( l = 2\alpha = 2(10\text{ in.}) = 20\text{ in.} \).

Since the gusset straddles the work line of each brace, the weld is uniformly loaded. Therefore, the available strength is the average required strength and the fillet weld should be designed for 1.25 times the average strength.

\[
D_{req'} = \frac{1.25P_d}{1.392l} = \frac{1.25(158\text{ kips})}{1.392(20 \text{ in.})(2)} = 3.55
\]

\[
D_{req'} = \frac{1.25P_d}{0.928l} = \frac{1.25(105\text{ kips})}{0.928(20 \text{ in.})(2)} = 3.54
\]

The minimum fillet weld size is \( \frac{1}{4}\text{ in.} \). The required weld size is also \( \frac{1}{4}\text{ in.} \), use a \( \frac{1}{4}\text{ in.} \) fillet weld 40-in. long total.

Check gusset thickness (against weld size required for strength)

\[
t_{\text{min}} = \frac{6.19D}{F_u} = \frac{6.19(3.55)}{58 \text{ ksi}} = 0.379 \text{ in.} < \frac{3}{8}\text{ in.}
\]

\[\text{O.K.}\]

Check local web yielding of the beam

\[
R_\phi = (N + 5k) F_{tu} = \left[ 20 \text{ in.} + 5(0.827 \text{ in.}) \right] (50 \text{ ksi}) (0.300 \text{ in.}) = 362 \text{ kips}
\]

\[
\phi R_\phi = 1.00(662 \text{ kips}) = 662 \text{ kips}
\]

\[
158 \text{ kips} (\cos 48^\circ) = 105 \text{ kips}
\]

\[
662 \text{ kips} > 105 \text{ kips} \quad \text{O.K.}
\]

\[
R_n / \Omega = \frac{662 \text{ kips}}{1.50} = 441 \text{ kips}
\]

\[
105 \text{ kips} (\cos 48^\circ) = 70.2 \text{ kips}
\]

\[
441 \text{ kips} > 70.2 \text{ kips} \quad \text{O.K.}
\]
Example II.C-6  Heavy Wide Flange Compression Connection  
(flanges on the outside)

Given:

This truss has been designed with nominal 14 in. W-shapes, with the flanges to the outside of the truss. Beams framing into the top chord and lateral bracing are not shown but can be assumed to be adequate.

Based on multiple load cases, the critical dead and live load forces for this connection were determined. A typical top chord connection and the dead load and live load forces are shown below in detail A. Design this typical connection using 1-in. diameter ASTM A325 slip-critical bolts in oversized holes with a class-A faying surface.

Note: slip critical bolts in oversized holes were selected for this truss to facilitate field erection.
Material Properties:

Beam W-shapes: ASTM A992  
- $F_y = 50$ ksi  
- $F_u = 65$ ksi  
Gusset Plates: ASTM A36  
- $F_y = 36$ ksi  
- $F_u = 58$ ksi

Manual Tables 2-3 and 2-4

Geometric Properties:

W14×109  
- $d = 14.3$ in.  
- $b_y = 14.6$ in.  
- $t_y = 0.860$ in.
W14×61  
- $d = 13.9$ in.  
- $b_y = 9.99$ in.  
- $t_y = 0.645$ in.

Manual Table 1-1

Solution:

*Determine the required strength of the member*

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Left top chord:</td>
<td>1.2(262 kips) + 1.6(262 kips) = 734 kips</td>
<td>Left top chord: 262 kips + 262 kips = 524 kips</td>
</tr>
<tr>
<td>Right top chord:</td>
<td>1.2(345 kips) + 1.6(345 kips) = 966 kips</td>
<td>Right top chord: 345 kips + 345 kips = 690 kips</td>
</tr>
<tr>
<td>Vertical:</td>
<td>1.2(102 kips) + 1.6(102 kips) = 286 kips</td>
<td>Vertical: 102 kips + 102 kips = 204 kips</td>
</tr>
<tr>
<td>Diagonal:</td>
<td>1.2(113 kips) + 1.6(113 kips) = 316 kips</td>
<td>Diagonal: 113 kips + 113 kips = 226 kips</td>
</tr>
</tbody>
</table>

*Determine the single shear strength*

$d_{bolt} = 1$ in.  
ASTM A325-SC bolts  
Class A faying surface  
- $\mu = 0.35$ (class A)  
- $d_{hg} = 1.25$ in. (diameter of holes at gusset plates)  
- $h_{sc} = 0.85$ (oversized and short-slotted holes)  
- $T_b = 51.0$ kips  
- $D_a = 1.13$

Equation J3-4

Slip prevention up to the required strength level is selected in this case to prevent truss deflections that could permit a significant ponding condition for which the truss was not designed.

$R_n = \mu D_a h_{sc} T_b N_s = 0.35(1.13)(0.85)(51.0 \text{kips})(1) = 17.1 \text{kips}$

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>For connections designed to prevent slip at the required strength level;</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\phi = 0.85$</td>
<td>$\Omega = 1.76$</td>
<td>$\phi R_n = 0.85(17.1 \text{kips}) = 14.6 \text{kips/bolt}$</td>
</tr>
</tbody>
</table>
**Diagonal connection**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial force = 316 kips</td>
<td>Axial force = 226 kips</td>
</tr>
<tr>
<td>316 kips / 14.6 kips/bolt = 21.7 bolts</td>
<td>226 kips / 9.74 kips/bolt = 23.2 bolts</td>
</tr>
<tr>
<td>2 rows both sides = 21.7 bolts / 4 = 5.42</td>
<td>2 rows both sides = 23.2 bolts / 4 = 5.80</td>
</tr>
<tr>
<td>Therefore use 6 rows @ min. 3 in. spacing</td>
<td>Therefore use 6 rows @ min. 3 in. spacing</td>
</tr>
</tbody>
</table>

**Check the Whitmore section in the gusset plate (tension only)**

Whitmore section = gage of the bolts + tan 30° (length of bolt group) × 2

\[
= \left( 5 \frac{1}{2} \text{ in.} \right) + \tan 30° \left[ \left( 5 \text{ spaces} \right) \left( 3 \text{ in.} \right) \right] \times 2 \\
= 22.8 \text{ in.}
\]

Try \( \frac{3}{8} \) in. thick plate

\[ A_g = \left( 0.375 \text{ in.} \right) \left( 22.8 \text{ in.} \right) = 8.56 \text{ in}^2 \]

**Check tension yielding**

\[
\phi = 0.90 \\
\phi R_n = \phi F_y A_g \\
= 0.90 \times (36 \text{ ksi})(8.56 \text{ in}^2)(2) \\
= 555 \text{ kips} > 316 \text{ kips} \quad \text{o.k.}
\]

**Check shear yielding**

\[
\Omega = 1.67 \\
R_n / \Omega = F_y A_g / \Omega \\
= \left( 36 \text{ ksi} \right)(8.56 \text{ in}^2)(2) / 1.67 \\
= 369 \text{ kips} > 226 \text{ kips} \quad \text{o.k.}
\]

**Check block shear rupture in plate**

\[ \Omega = 2.00 \]

Tension stress is uniform,

Therefore; \( U_{bn} = 1.0 \)

\[ t_p = 0.375 \text{ in.} \]

\[ A_{sy}/\text{in.} = (2 \text{ rows}) \left[ \left( 6 \text{ bolts-1} \right) \left( 3 \text{ in.} \right) + 2 \text{ in.} \right] = 34.0 \text{ in.} \]

\[ A_{sv}/\text{in.} = 34.0 \text{ in.} \times (2 \text{ rows})(6 \text{ bolts -0.5}) \times (1.25 \text{ in.} + 0.0625 \text{ in.}) = 19.6 \text{ in.} \]

\[ A_{st}/\text{in.} = 5.50 \text{ in.} \times (1.25 \text{ in.} + 0.625 \text{ in.}) = 4.19 \text{ in.} \]
\[ \phi R_n = (0.6F_{uA_{mv}} + U_{bnF_{uA_{mv}}}) \leq (0.6F_{yA_{gv}} + U_{bnF_{uA_{mv}}}) / \Omega \]

**Tension Rupture Component**

\[ \phi U_{bn}(F_{uA_{mv}}) / t = 182 \text{ kips/in.} \]

**Shear Yielding Component**

\[ \phi(0.6F_{yA_{gv}}) / t = 551 \text{ kips/in.} \]

**Shear Rupture Component**

\[ \phi(0.6F_{uA_{mv}}) / t = 511 \text{ kips/in.} \]

\[ \phi R_n = (511 \text{ kips/in.} + 182 \text{ kips/in.})(0.375 \text{ in.}) \]
\[ = 260 \text{ kips} < 275 \text{ kips} \]
\[ 260 \text{ kips} > 316 \text{ kips} / 2 = 158 \text{ kips} \text{ o.k.} \]

*Check block shear rupture on beam flange*

By inspection, block shear rupture on the beam flange will not control.

**Bolt bearing on plate**

Based on Bolt Spacing = 3 in.; oversized holes, \( F_u = 58 \text{ ksi} \)

\[ \phi r_{n} = (91.4 \text{ kips/in.})(0.375 \text{ in.}) \]
\[ = 34.3 \text{ kips} > 14.6 \text{ kips} \]

Based on Edge Distance = 2 in.; oversized holes, \( F_u = 58 \text{ ksi} \)

\[ \phi r_{n} = (71.8 \text{ kips/in.})(0.375 \text{ in.}) \]
\[ = 26.9 \text{ kips} > 14.6 \text{ kips} \]

**Bolt bearing on flange**

Based on Bolt Spacing = 3 in.; standard holes, \( F_u = 65 \text{ ksi} \)

\[ \phi r_{n} = (113 \text{ kips/in.})(0.645 \text{ in.}) \]
\[ = 72.9 \text{ kips} > 14.6 \text{ kips} \]

Based on Edge Distance = 2 in.; standard holes, \( F_u = 65 \text{ ksi} \)

\[ r_{n} / \Omega = (60.9 \text{ kips/in.})(0.375 \text{ in.}) \]
\[ = 22.8 \text{ kips} > 9.74 \text{ kips} \]

Based on Edge Distance = 2 in.; oversized holes, \( F_u = 58 \text{ ksi} \)

\[ r_{n} / \Omega = (47.9 \text{ kips/in.})(0.375 \text{ in.}) \]
\[ = 18.0 \text{ kips} > 9.74 \text{ kips} \]

Based on Bolt Spacing = 3 in.; standard holes, \( F_u = 65 \text{ ksi} \)

\[ r_{n} / \Omega = (75.6 \text{ kips/in.})(0.645) \]
\[ = 48.8 \text{ kips} > 9.74 \text{ kips} \]

Based on Edge Distance = 2 in.; standard holes, \( F_u = 65 \text{ ksi} \)
Horizontal connection

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial force = 966 kips – 734 kips = 232 kips</td>
<td>Axial force = 690 kips – 524 kips = 166 kips</td>
</tr>
<tr>
<td>232 kips / 14.6 kips/bolt = 15.9 bolts</td>
<td>166 kips / 9.74 kips/bolt = 17.0 bolts</td>
</tr>
<tr>
<td>2 rows both sides = 15.9 bolts / 4 = 3.98</td>
<td>2 rows both sides = 17.0 bolts / 4 = 4.26</td>
</tr>
<tr>
<td>Use 6 bolts per row</td>
<td>Use 6 bolts per row</td>
</tr>
</tbody>
</table>

Check shear in plate

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>Try Plate with, ( t_p = 0.375 ) in.</td>
<td>Try Plate with, ( t_p = 0.375 ) in.</td>
</tr>
<tr>
<td>( A_{\text{gv}} / t_p = (1 \text{ row})(6 \text{ bolts -1})(6 \text{ in.}) + 4 \text{ in.} )</td>
<td>( A_{\text{gv}} / t_p = (1 \text{ row})(6 \text{ bolts -1})(6 \text{ in.}) + 4 \text{ in.} )</td>
</tr>
<tr>
<td>( = 34.0 \text{ in.} )</td>
<td>( = 34.0 \text{ in.} )</td>
</tr>
<tr>
<td>( A_{\text{sv}} / t_p = 34.0 \text{ in.} - (1 \text{ row})(6 \text{ bolts}) \times (1.25 \text{ in.} + 0.0625 \text{ in.}) = 26.1 \text{ in} )</td>
<td>( A_{\text{sv}} / t_p = 34.0 \text{ in.} - (1 \text{ row} (6 \text{ bolts} – 0.5) \times (1.25 \text{ in.} + 0.0625 \text{ in.}) = 26.8 \text{ in} )</td>
</tr>
</tbody>
</table>

Check shear yielding

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 1.00 )</td>
<td>( \Omega = 1.50 )</td>
</tr>
<tr>
<td>( \phi R_n = \phi(0.6F_yA_g) = 275 \text{ kips} )</td>
<td>( R_n / \Omega = (0.6F_yA_g) / \Omega = 184 \text{ kips} )</td>
</tr>
</tbody>
</table>

Check shear rupture

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi = 0.75 )</td>
<td>( \Omega = 2.00 )</td>
</tr>
<tr>
<td>( \phi R_n = \phi(0.6F_wA_m) = 256 \text{ kips} )</td>
<td>( R_n / \Omega = (0.6F_wA_m) / \Omega = 170 \text{ kips} )</td>
</tr>
<tr>
<td>( = 256 \text{ kips &lt; 275 kips} )</td>
<td>( = 170 \text{ kips &lt; 184 kips} )</td>
</tr>
<tr>
<td>256 kips &gt; 232 kips / 2 = 116 kips \textbf{o.k.}</td>
<td>170 kips &gt; 166 kips / 2 = 83.0 kips \textbf{o.k.}</td>
</tr>
</tbody>
</table>

Bolt bearing on plate

Manual
Table 7-6
<table>
<thead>
<tr>
<th>Based on Bolt Spacing = 6 in.; oversized holes, $F_u = 58$ ksi</th>
<th>Based on Bolt Spacing = 6 in.; oversized holes, $F_u = 58$ ksi</th>
<th>Manual Table 7-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{ts} = (104$ kips/in.$)(0.375$ in.$)$</td>
<td>$r_y / \Omega = (69.9$ kips/in.$)(0.375$ in.$)$</td>
<td></td>
</tr>
<tr>
<td>$= 39.0$ kips &gt; 14.6 kips</td>
<td>$= 26.1$ kips &gt; 9.74 kips</td>
<td></td>
</tr>
<tr>
<td>Based on Edge Distance = 2 in.; oversized holes, $F_u = 58$ ksi</td>
<td>Based on Edge Distance = 2 in.; oversized holes, $F_u = 58$ ksi</td>
<td>Manual Table 7-6</td>
</tr>
<tr>
<td>$\phi_{ts} = (71.8$ kips/in.$)(0.375$ in.$)$</td>
<td>$r_y / \Omega = (47.9$ kips/in.$)(0.375$ in.$)$</td>
<td></td>
</tr>
<tr>
<td>$= 26.9$ kips &gt; 14.6 kips</td>
<td>$= 18.0$ kips &gt; 9.74 kips</td>
<td></td>
</tr>
</tbody>
</table>

**Bolt bearing on flange**

<table>
<thead>
<tr>
<th>Based on Bolt Spacing = 6 in.; standard holes, $F_u = 65$ ksi</th>
<th>Based on Bolt Spacing = 6 in.; standard holes, $F_u = 65$ ksi</th>
<th>Manual Table 7-5</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi_{ts} = (117$ kips/in.$)(0.860$ in.$)$</td>
<td>$r_y / \Omega = (78.0$ kips/in.$)(0.860$ in.$)$</td>
<td></td>
</tr>
<tr>
<td>$= 101$ kips &gt; 14.6 kips</td>
<td>$= 67.0$ kips &gt; 9.74 kips</td>
<td></td>
</tr>
<tr>
<td>Based on edge distance = 2 in.; standard holes, $F_u = 65$ ksi</td>
<td>Based on edge distance = 2 in.; standard holes, $F_u = 65$ ksi</td>
<td>Manual Table 7-6</td>
</tr>
<tr>
<td>$\phi_{ts} = (85.9$ kips/in.$)(0.860$ in.$)$</td>
<td>$r_y / \Omega = (57.3$ kips/in.$)(0.860$ in.$)$</td>
<td></td>
</tr>
<tr>
<td>$= 73.9$ kips &gt; 14.6 kips</td>
<td>$= 49.3$ kips &gt; 9.74 kips</td>
<td></td>
</tr>
</tbody>
</table>

**Vertical connection**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Specification Section J4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Axial force = 286 kips</td>
<td>Axial force = 204 kips</td>
<td>Eqn. J4-3</td>
</tr>
<tr>
<td>286 kips / 14.6 kips/bolt = 19.6 bolts</td>
<td>204 kips / 9.74 kips/bolt = 20.9 bolts</td>
<td></td>
</tr>
<tr>
<td>2 rows both sides = 19.6 bolts / 4 = 4.91 Therefore use 5 rows</td>
<td>2 rows both sides = 20.9 bolts / 4 = 5.24 Therefore use 6 rows</td>
<td></td>
</tr>
</tbody>
</table>

**Check shear in plate**

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Specification Section J4.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>Try Plate with, $t_p = 0.375$ in.</td>
<td>Try Plate with, $t_p = 0.375$ in.</td>
<td></td>
</tr>
<tr>
<td>$A_{gy} / t_p = 33 \frac{1}{4}$ in. (from sketch)</td>
<td>$A_{gy} / t_p = 33 \frac{1}{4}$ in. (from sketch)</td>
<td></td>
</tr>
<tr>
<td>$A_{gy} / t_p = 33 \frac{1}{4}$ in. – (1 row)(7 bolts) $\times (1.25$ in. + 0.0625 in.$) = 24.6$ in</td>
<td>$A_{gy} / t_p = 33 \frac{1}{4}$ in. – (1 row)(7 bolts) $\times (1.25$ in. + 0.0625 in.$) = 24.6$ in</td>
<td></td>
</tr>
<tr>
<td>Check shear yielding</td>
<td>Check shear yielding</td>
<td></td>
</tr>
<tr>
<td>$\phi = 1.00$</td>
<td>$\Omega = 1.50$</td>
<td></td>
</tr>
<tr>
<td>$\phi R_n = \phi(0.6 F_y A_y) = 273$ kips</td>
<td>$R_n / \Omega = (0.6 F_y A_y) / \Omega = 182$ kips</td>
<td></td>
</tr>
</tbody>
</table>
Check shear rupture
Where \( \phi = 0.75 \)

\[ \phi R_n = \phi (0.6 F_u A_{nv}) = 240 \text{ kips} \]

\[ = 240 \text{ kips} < 273 \text{ kips} \]

\[ 240 \text{ kips} > 286 \text{ kips} / 2 = 143 \text{ kips} \quad \text{o.k.} \]

Bolt bearing on plate
Conservatively based on Bolt Spacing = 3 in.; oversized holes,

\[ \phi r_n = (91.4 \text{ kips/in.})(0.375 \text{ in.}) \]

\[ = 34.3 \text{ kips} > 14.6 \text{ kips} \]

Based on Edge Distance = 2 in.; oversized holes,

\[ \phi r_n = (71.8 \text{ kips/in.})(0.375 \text{ in.}) \]

\[ = 26.9 \text{ kips} > 14.6 \text{ kips} \]

Bolt bearing on flange
Conservatively based on Bolt Spacing = 3 in.; standard holes,

\[ \phi r_n = (104 \text{ kips/in.})(0.645 \text{ in.}) \]

\[ = 67.0 \text{ kips} > 14.6 \text{ kips} \]

Based on edge distance = 2 in.; standard holes,

\[ \phi r_n = (85.9 \text{ kips/in.})(0.645 \text{ in.}) \]

\[ = 55.4 \text{ kips} > 14.6 \text{ kips} \]

Bolt Shear Strength
Based on ASTM A325-N; single shear

\[ \phi r_s = 28.3 \text{ kips} > 14.6 \text{ kips} \]

Check shear rupture
Where \( \Omega = 2.00 \)

\[ R_n / \Omega = (0.6 F_u A_{nv}) / \Omega = 160 \text{ kips} \]

\[ = 160 \text{ kips} < 182 \text{ kips} \]

\[ 160 \text{ kips} > 204 \text{ kips} / 2 = 102 \text{ kips} \quad \text{o.k.} \]

Bolt bearing on plate
Conservatively based on Bolt Spacing = 3 in.; oversized holes,

\[ r_n / \Omega = (60.9 \text{ kips/in.})(0.375 \text{ in.}) \]

\[ = 22.8 \text{ kips} > 9.74 \text{ kips} \]

Based on Edge Distance = 2 in.; oversized holes,

\[ r_n / \Omega = (47.9 \text{ kips/in.})(0.375 \text{ in.}) \]

\[ = 18.0 \text{ kips} > 9.74 \text{ kips} \]

Bolt bearing on flange
Conservatively based on Bolt Spacing = 3 in.; standard holes,

\[ r_n / \Omega = (75.6 \text{ kips/in.})(0.645) \]

\[ = 48.8 \text{ kips} > 9.74 \text{ kips} \]

Based on edge distance = 2 in.; standard holes,

\[ r_n / \Omega = (57.3 \text{ kips/in.})(0.645) \]

\[ = 37.0 \text{ kips} > 9.74 \text{ kips} \]

Bolt Shear Strength
Based on ASTM A325-N; single shear

\[ r_s / \Omega = 18.8 \text{ kips} > 9.74 \text{ kips} \]

Eqn. J4-4
Manual Table 7-5
Manual Table 7-6
Manual Table 7-1
Note: The final layout for the connection is as follows:

Note that because the difference in depths between the top chord and the vertical and diagonal members, \( \frac{3}{8} \) in. loose shims are required on each side of the shallower members.
Chapter IID
Miscellaneous Connections

This section contains design examples on connections in the AISC Steel Construction Manual. that are not covered in other sections of AISC Design Examples.
Example II.D-1  Prying Action in Tees and in Single Angles

Given:

Design a WT tension-hanger connection between a 2L3×3×⅜ tension member and a W24×94 beam connection to support the following loads:

\[ P_D = 13.5 \text{ kips} \]
\[ P_L = 40 \text{ kips} \]

Use ¾-in. diameter ASTM A325-N bolts and 70 ksi electrodes.

Material Properties:

- **Hanger** (ASTM A992)  \( F_y = 50 \text{ ksi} \), \( F_u = 65 \text{ ksi} \)
- **Beam** (W24×94)  \( F_y = 50 \text{ ksi} \), \( F_u = 65 \text{ ksi} \)
- **Angles 2L3×3×⅜** (ASTM A36)  \( F_y = 36 \text{ ksi} \), \( F_u = 58 \text{ ksi} \)

Geometric Properties:

- **Beam** (W24×94)  \( d = 24.3 \text{ in.} \), \( t_w = 0.515 \text{ in.} \), \( b_y = 9.07 \text{ in.} \), \( t_y = 0.875 \text{ in.} \)
- **Angles 2L3×3×⅜**  \( A = 3.55 \text{ in.}^2 \), \( \bar{x} = 0.860 \text{ in.} \)

Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( P_u = 1.2(13.5 \text{ kips}) + 1.6(40 \text{ kips}) = 80.2 \text{ kips} )</td>
<td>( P_u = 13.5 \text{ kips} + 40 \text{ kips} = 53.5 \text{ kips} )</td>
</tr>
</tbody>
</table>
Check tension yielding of angles

\[ R_u = F_y A_y = (36 \text{ ksi})(3.55 \text{ in.}^2) = 128 \text{ kips} \]

\[ \phi R_u = 0.90(128 \text{ kips}) = 115 \text{ kips} > 80.2 \text{ kips} \]

Try \( \frac{3}{16} \)-in. fillet welds

\[ L_{\text{min}} = \frac{P_n}{1.392D} = \frac{80.2 \text{ kips}}{1.392(4 \text{ sixteenths})} = 14.4 \text{ in.} \]

Use four 4-in. welds (16 in. total), one at each toe and heel of each angle.

Check tension rupture of angles

Calculate the effective net area

\[ U = 1 - \frac{\pi}{L} = 1 - \frac{0.860 \text{ in.}}{4 \text{ in.}} = 0.785 \]

\[ A_v = A_u U = (3.55 \text{ in.}^2)(0.785) = 2.80 \text{ in.}^2 \]

\[ \phi = 0.75 \]

\[ R_u = F_y A_y = (58 \text{ ksi})(2.80 \text{ in.}^2) = 163 \text{ kips} \]

\[ \phi R_u = 0.75(163) = 122 \text{ kips} > 80 \text{ kips} \quad \text{o.k.} \]

\[ \Omega = 2.00 \]

\[ R_n / \Omega = 163 / 2.00 = 81.5 \text{ kips} > 53.5 \text{ kips} \quad \text{o.k.} \]

Select a preliminary WT using beam gage

\( g = 4 \text{ in.} \)

With four \( \frac{3}{16} \)-in. diameter ASTM A325-N bolts,

\[ T_u = r_u = \frac{P_n}{n} = \frac{80 \text{ kips}}{4} = 20 \text{ kips/bolt} \]

\[ B = \phi r_u = 29.8 \text{ kips} > 20 \text{ kips} \quad \text{o.k.} \]

Select a preliminary WT using beam gage

\( g = 4 \text{ in.} \)

With four \( \frac{3}{4} \)-in. diameter ASTM A325-N bolts,

\[ T_u = r_u = \frac{P_n}{n} = \frac{53.5 \text{ kips}}{4} = 13.4 \text{ kips/bolt} \]

\[ B = r_u / \Omega = 19.9 \text{ kips} > 13.4 \text{ kips} \quad \text{o.k.} \]
With four bolts, the maximum effective length is \(2g = 8\) in. Thus, there are 4 in. of tee length tributary to each pair of bolts and

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>2 bolts(20 kips/bolt)</td>
<td>(\frac{4\text{ in.}}{4\text{ in.}} = 10.0) kips/in.</td>
<td>2 bolts(13.4 kips/bolt)</td>
</tr>
</tbody>
</table>

The minimum depth \(WT\) that can be used is equal to the sum of the weld length plus the weld size plus the \(k\)-dimension for the selected section. From Manual Table 1-8 with an assumed \(b = 4\) in./2 = 2 in., \(t_o \approx \frac{1}{2}t_w\) in., and \(d_{min} = 4\) in. + \(\frac{1}{2}\) in. + \(k \approx 6\) in., appropriate selections include:

- \(WT6\times39.5\)
- \(WT8\times28.5\)
- \(WT7\times34\)
- \(WT9\times30\)

Try \(WT8\times28.5; b_f = 7.12\) in., \(t_f = 0.715\) in., \(t_w = 0.430\) in.

**Check prying action**

\[
b = \frac{g - t_w}{2} = \frac{(4\text{ in.} - 0.430\text{ in.})}{2} = 1.79\text{ in.} > 1\frac{1}{4}\text{-in. entering and tightening clearance, o.k.}
\]

\[
a = \frac{b_f - g}{2} = \frac{(7.12\text{ in.} - 4\text{ in.})}{2} = 1.56\text{ in.}
\]

Since \(a = 1.56\) in. < 1.25\(b = 2.24\) in., use \(a = 1.56\) in.

\[
b' = b - \frac{d}{2} = 1.79\text{ in.} - \left(\frac{\frac{1}{4}\text{ in.}}{2}\right) = 1.42\text{ in.}
\]

\[
a' = a + \frac{d}{2} = 1.56\text{ in.} + \left(\frac{\frac{1}{4}\text{ in.}}{2}\right) = 1.94\text{ in.}
\]

\[
\rho = \frac{b'}{a'} = \frac{1.42\text{ in.}}{1.94\text{ in.}} = 0.732
\]

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\beta = \frac{1}{\rho} \left(\frac{B}{T} - 1\right))</td>
<td>(\frac{1}{0.732} \left(\frac{29.8\text{ kips/bolt}}{20\text{ kips}} - 1\right))</td>
<td>(\frac{1}{0.732} \left(\frac{19.9\text{ kips/bolt}}{13.4\text{ kips}} - 1\right))</td>
</tr>
<tr>
<td></td>
<td>(= 0.669)</td>
<td>(= 0.663)</td>
</tr>
</tbody>
</table>

\[
\delta = 1 - \frac{d'}{p} = 1 - \left(\frac{\frac{1}{2}t_w}{4\text{ in.}}\right) = 0.797
\]
Since $\beta < 1.0$,

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha' = \frac{1}{\delta} \left( \frac{\beta}{1-\beta} \right) \leq 1.0$</td>
<td>$\alpha' = \frac{1}{\delta} \left( \frac{\beta}{1-\beta} \right) \leq 1.0$</td>
</tr>
<tr>
<td>$= \frac{1}{0.797} \left( \frac{0.669}{1-0.669} \right)$</td>
<td>$= \frac{1}{0.797} \left( \frac{0.663}{1-0.663} \right)$</td>
</tr>
<tr>
<td>$= 2.54 \therefore \alpha' = 1.0$</td>
<td>$= 2.47 \therefore \alpha' = 1.0$</td>
</tr>
<tr>
<td>$t_{\min} = \frac{4.44 Tb'}{\sqrt{pF_u(1+\delta\alpha')}}$</td>
<td>$t_{\min} = \frac{6.66 Tb'}{\sqrt{pF_u(1+\delta\alpha')}}$</td>
</tr>
<tr>
<td>$= \frac{4.44(20 \text{ kips/bolt})(1.42 \text{ in.})}{\sqrt{(4 \text{ in.})(65 \text{ ksi})[1+(0.797)(1.0)]}}$</td>
<td>$= \frac{6.66(13.4 \text{ kips/bolt})(1.42 \text{ in.})}{\sqrt{(4 \text{ in.})(65 \text{ ksi})[1+(0.797)(1.0)]}}$</td>
</tr>
<tr>
<td>$= 0.521 \text{ in.} &lt; t_f = 0.715 \text{ in.} \quad \text{o.k.}$</td>
<td>$= 0.521 \text{ in.} &lt; t_f = 0.715 \text{ in.} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>

Check tension yielding of the tee stem on the Whitmore section

The effective width of the tee stem (which cannot exceed the actual width of 8 in.) is

$$L_w = 3 \text{ in.} + 2(4 \text{ in.})(\tan 30^\circ) \leq 8 \text{ in.}$$

$$= 7.62 \text{ in.}$$

and the nominal strength is determined as

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.90$</td>
<td>$\Omega = 1.67$</td>
</tr>
<tr>
<td>$R_n = F_uA_{geff} = (50 \text{ ksi})(7.62 \text{ in.})(0.430 \text{ in.}) = 164 \text{ kips}$</td>
<td>$R_n = F_uA_{geff} = (50 \text{ ksi})(7.62 \text{ in.})(0.430 \text{ in.}) = 164 \text{ kips}$</td>
</tr>
<tr>
<td>$\phi R_n = 0.90(164) = 147 \text{ kips} &gt; 80 \text{ kips} \quad \text{o.k.}$</td>
<td>$R_n / \Omega = \frac{164}{1.67} = 98.2 \text{ kips} &gt; 53.5 \text{ kips} \quad \text{o.k.}$</td>
</tr>
</tbody>
</table>

Check shear rupture of the base metal along the toe and heel of each weld line

$$t_{\min} = \frac{6.19D}{F_u}$$

$$= 6.19 \left( \frac{4}{65 \text{ ksi}} \right)$$

$$= 0.381 \text{ in.} < 0.430 \text{ in.} \quad \text{o.k.}$$

Check block shear rupture of the tee stem

Since the angles are welded to the WT-hanger the gross area shear yielding will control.

$$A_{ge} = (2 \text{ welds})(4 \text{ in.})(0.430 \text{ in.}) = 3.44 \text{ in.}^2$$

Manual

Part 9

Eqn. D2-1

Manual

Part 9
Tension stress is uniform, therefore $U_{hs} = 1.0$.

\[ A_{st} = A_g = 1.0(3\text{ in.})(0.430\text{ in.}) = 1.29\text{ in.}^2 \]

\[
R_u = 0.6F_uA_{st} + U_{hs}F_uA_{st} \leq 0.6F_uA_{st} + U_{hs}F_uA_{st}
\]

\[
R_u = 0.6F_uA_{st} + U_{hs}F_uA_{st} = 0.6(50 \text{ ksi})(3.44 \text{ in.}^2) + 1.0(65 \text{ ksi})(1.29 \text{ in.}^2) = 187 \text{ kips}
\]

### Table D3.1

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\phi = 0.75$</td>
<td>$\Omega = 2.00$</td>
</tr>
<tr>
<td>$\phi R_u = 0.75(187) = 140 \text{ kips}$</td>
<td>$R_u / \Omega = \frac{187}{2.00} = 93.5 \text{ kips}$</td>
</tr>
<tr>
<td>$&gt; 80 \text{ kips}$</td>
<td>$&gt; 53.5 \text{ kips}$</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Note: Alternately, a WT tension hanger could be selected with a flange thickness to reduce the effect of prying action to an insignificant amount, i.e., $q_u \approx 0$. Assuming $b' = 1.42$ in.

### Table 1-8

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_{min} = \sqrt{\frac{4.44Tb'}{pF_u}}$</td>
<td>$t_{min} = \sqrt{\frac{6.66Tb'}{pF_u}}$</td>
</tr>
<tr>
<td>$= \frac{4.44(20 \text{ kips/bolt})(1.42 \text{ in.})}{(4 \text{ in./bolt})(65 \text{ ksi})}$</td>
<td>$= \frac{6.66(13.4 \text{ kips/bolt})(1.42 \text{ in.})}{(4 \text{ in./bolt})(65 \text{ ksi})}$</td>
</tr>
<tr>
<td>= 0.696 in.</td>
<td>= 0.698 in.</td>
</tr>
</tbody>
</table>

Try WT9×35.5

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$t_f = 0.810 \text{ in.} &gt; 0.696 \text{ in.}$</td>
<td>$t_f = 0.810 \text{ in.} &gt; 0.698 \text{ in.}$</td>
</tr>
<tr>
<td>$t_w = 0.495 \text{ in.} &gt; 0.430 \text{ in.}$</td>
<td>$t_w = 0.495 \text{ in.} &gt; 0.430 \text{ in.}$</td>
</tr>
<tr>
<td>$b_f = 7.64 \text{ in.} &gt; 7.12 \text{ in.}$</td>
<td>$b_f = 7.64 \text{ in.} &gt; 7.12 \text{ in.}$</td>
</tr>
</tbody>
</table>

Manual Table 1-8
Example II.D-2  Beam Bearing Plate

Given:

A W18×50 beam with a dead load end reaction of 15 kips and a live load end reaction of 45 kips is supported by a 10-in. thick concrete wall.

If the beam has $F_y = 50$ ksi, the concrete has $f'_c = 3$ ksi, and the bearing plate has $F_y = 36$ ksi, determine:

a. if a bearing plate is required if the beam is supported by the full wall thickness,
b. the bearing plate required if $N = 10$ in. (the full wall thickness),
c. the bearing plate required if $N = 6$ in. and the bearing plate is centered on the thickness of the wall.

Material Properties:

Beam W18×50  ASTM A992  $F_y = 50$ ksi  $F_u = 65$ ksi  Manual
Bearing Plate (if required)  ASTM A36  $F_y = 36$ ksi  $F_u = 58$ ksi  Tables 2-3 and 2-4
Concrete Wall  $f'_c = 3$ ksi

Geometric Properties:

Beam W18×50  $d = 18.0$ in.  $t_w = 0.355$ in.  $b_y = 7.50$ in.  $t_f = 0.570$ in.  Manual
  $k = 0.972$ in.  $k_f = 13/16$ in.
Concrete Wall  $h = 10$ in.
**Solution A:**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( R_u = 1.2(15 \text{kips}) + 1.6(45 \text{kips}) = 90 \text{kips} )</td>
<td>( R_u = 15 \text{kips} + 45 \text{kips} = 60 \text{kips} )</td>
</tr>
<tr>
<td><strong>Check local web yielding</strong></td>
<td>( N_{req} = \frac{R_u - \phi R_s}{\phi R_u} \geq k )</td>
<td><strong>Check local web yielding</strong></td>
</tr>
<tr>
<td></td>
<td>( = \frac{90 \text{kips} - 43.1 \text{kips}}{17.8 \text{kips/in.}} \geq 0.972 \text{ in.} )</td>
<td>( N_{req} = \frac{R_u - R_i / \Omega}{R_u / \Omega} \geq k )</td>
</tr>
<tr>
<td></td>
<td>( = 2.63 \text{ in.} &lt; 10 \text{ in.} \text{ o.k.} )</td>
<td>( = 2.64 \text{ in.} &lt; 10 \text{ in.} \text{ o.k.} )</td>
</tr>
<tr>
<td><strong>Check web crippling</strong></td>
<td>( N = \frac{10 \text{ in.}}{18.0 \text{ in.}} = 0.556 )</td>
<td><strong>Check web crippling</strong></td>
</tr>
<tr>
<td></td>
<td>Since ( \frac{N}{d} &gt; 0.2 ), ( N_{req} = \frac{R_u - \phi R_s}{\phi R_u} )</td>
<td>Since ( \frac{N}{d} &gt; 0.2 ), ( N_{req} = \frac{R_u - R_i / \Omega}{R_u / \Omega} )</td>
</tr>
<tr>
<td></td>
<td>( = \frac{90 \text{kips} - 52.0 \text{kips}}{6.30 \text{kips/in.}} \geq 0.972 \text{ in.} )</td>
<td>( = \frac{60 \text{kips} - 34.7 \text{kips}}{4.20 \text{kips/in.}} \geq 0.972 \text{ in.} )</td>
</tr>
<tr>
<td></td>
<td>( = 6.03 \text{ in.} &lt; 10 \text{ in.} \text{ o.k.} )</td>
<td>( = 6.03 \text{ in.} &lt; 10 \text{ in.} \text{ o.k.} )</td>
</tr>
<tr>
<td><strong>Verify</strong> ( \frac{N}{d} &gt; 0.2 ),</td>
<td>( \frac{N}{d} = 0.335 &gt; 0.2 \text{ o.k.} )</td>
<td><strong>Verify</strong> ( \frac{N}{d} &gt; 0.2 ),</td>
</tr>
<tr>
<td></td>
<td>( \frac{N}{d} = 0.335 &gt; 0.2 \text{ o.k.} )</td>
<td><strong>Check the bearing strength of concrete</strong></td>
</tr>
<tr>
<td></td>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( \phi P_p = \phi_c (0.85 f'_c) A_1 )</td>
<td>( P_p / \Omega_c = (0.85 f'_c) A_1 / \Omega_c )</td>
<td>( P_p / \Omega_c = (0.85 f'_c) A_1 / \Omega_c )</td>
</tr>
<tr>
<td></td>
<td>( = 0.60 (0.85)(3 \text{ ksi})(7.50 \text{ in.} \times 10 \text{ in.}) )</td>
<td>( = (0.85)(3 \text{ ksi})(7.50 \text{ in.} \times 10 \text{ in.}) / 2.50 )</td>
</tr>
<tr>
<td></td>
<td>( = 115 \text{kips} &gt; 90 \text{kips} \text{ o.k.} )</td>
<td>( = 76.5 \text{kips} &gt; 60 \text{kips} \text{ o.k.} )</td>
</tr>
</tbody>
</table>
### Check beam flange thickness

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Determine cantilever length</strong></td>
<td><strong>Determine cantilever length</strong></td>
</tr>
<tr>
<td>( n = \frac{b}{2} - k = \frac{7.50}{2} - 0.972 \text{ in.} = 2.78 \text{ in.} )</td>
<td>( n = \frac{b}{2} - k = \frac{7.50}{2} - 0.972 \text{ in.} = 2.78 \text{ in.} )</td>
</tr>
</tbody>
</table>

**Determine bearing pressure**

\( f_p = \frac{R_u}{A_t} \)

**Determine cantilever moment**

\[ M_u = \frac{R_u n^2}{2 A_t} \]

\[ Z = \frac{1}{4} t^2 \]

\[ M \leq F_y Z \leq F_y \left( \frac{t^2}{4} \right) \]

\[ t_{eq} = \sqrt{\frac{4 M_u}{\phi F_y}} = \sqrt{\frac{2 R_u n^2}{\phi A_t F_y}} \]

\( \phi = 0.90 \)

\[ t_{min} = \sqrt{\frac{2.22 R_u n^2}{A_t F_y}} \]

\[ = \sqrt{\frac{2.22(90 \text{ kips})(2.78 \text{ in.})^2}{(7.50 \text{ in.} \times 10 \text{ in.})(50 \text{ ksi})}} \]

\[ = 0.643 \text{ in.} > 0.570 \text{ in.} \quad \text{n.g.} \]

A bearing plate is required.

<table>
<thead>
<tr>
<th>Manual</th>
</tr>
</thead>
<tbody>
<tr>
<td>Part 14</td>
</tr>
</tbody>
</table>

**Determine cantilever length**

\( n = \frac{b}{2} - k = \frac{7.50}{2} - 0.972 \text{ in.} = 2.78 \text{ in.} \)

**Determine bearing pressure**

\( f_p = \frac{R_u}{A_t} \)

**Determine cantilever moment**

\[ M_a = \frac{R_a n^2}{2 A_t} \]

\[ Z = \frac{1}{4} t^2 \]

\[ M \leq F_y Z \leq F_y \left( \frac{t^2}{4} \right) \]

\[ t_{eq} = \sqrt{\frac{\Omega 4 M_u}{F_y}} = \sqrt{\frac{\Omega 2 R_u n^2}{A_t F_y}} \]

\( \Omega = 1.67 \)

\[ t_{min} = \sqrt{\frac{3.33 R_a n^2}{A_t F_y}} \]

\[ = \sqrt{\frac{3.33(60 \text{ kips})(2.78 \text{ in.})^2}{(7.50 \text{ in.} \times 10 \text{ in.})(50 \text{ ksi})}} \]

\[ = 0.643 \text{ in.} > 0.570 \text{ in.} \quad \text{n.g.} \]

A bearing plate is required.
Solution B:

\( N = 10 \text{ in.} \)

From Solution A, local web yielding and web crippling are not critical.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculate the required bearing-plate width.</strong></td>
<td><strong>Calculate the required bearing-plate width.</strong></td>
</tr>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( A_{1, \text{req}} = \frac{R_c}{\phi_c (0.85 f_y')} )</td>
<td>( A_{1, \text{req}} = \frac{R \Omega_c}{(0.85 f_y')} )</td>
</tr>
<tr>
<td>( = \frac{90 \text{ kips}}{0.60(0.85 \times 3 \text{ ksi})} )</td>
<td>( = \frac{60 \text{ kips}(2.50)}{(0.85 \times 3 \text{ ksi})} )</td>
</tr>
<tr>
<td>( = 58.8 \text{ in}^2 )</td>
<td>( = 58.8 \text{ in}^2 )</td>
</tr>
<tr>
<td>( B_{\text{req}} = \frac{A_{1, \text{req}}}{N} )</td>
<td>( B_{\text{req}} = \frac{A_{1, \text{req}}}{N} )</td>
</tr>
<tr>
<td>( = \frac{58.8 \text{ in}^2}{10 \text{ in.}} )</td>
<td>( = \frac{58.8 \text{ in}^2}{10 \text{ in.}} )</td>
</tr>
<tr>
<td>( = 5.88 \text{ in.} )</td>
<td>( = 5.88 \text{ in.} )</td>
</tr>
</tbody>
</table>

Use \( B = 8 \text{ in.} \) (least whole-inch dimension that exceeds \( b_y \)).

**Calculate required bearing-plate thickness.**

\( n = \frac{B}{2} - k \)

\( = \frac{8 \text{ in.}}{2} - 0.972 \text{ in.} \)

\( = 3.03 \text{ in.} \)

\( t_{\text{min}} = \sqrt{\frac{2.22 R_s n^2}{A_F y}} \)

\( = \sqrt{\frac{2.22(90 \text{ kips})(3.03 \text{ in.})^2}{(10 \text{ in.} \times 8 \text{ in.})(36 \text{ ksi})}} \)

\( = 0.799 \text{ in.} \)

Use PL1x10x0'-8

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Calculate the required bearing-plate width.</strong></td>
<td><strong>Calculate the required bearing-plate width.</strong></td>
</tr>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( A_{1, \text{req}} = \frac{R_c}{\phi_c (0.85 f_y')} )</td>
<td>( A_{1, \text{req}} = \frac{R \Omega_c}{(0.85 f_y')} )</td>
</tr>
<tr>
<td>( = \frac{60 \text{ kips}(2.50)}{(0.85 \times 3 \text{ ksi})} )</td>
<td>( = \frac{60 \text{ kips}(2.50)}{(0.85 \times 3 \text{ ksi})} )</td>
</tr>
<tr>
<td>( = 58.8 \text{ in}^2 )</td>
<td>( = 58.8 \text{ in}^2 )</td>
</tr>
<tr>
<td>( B_{\text{req}} = \frac{A_{1, \text{req}}}{N} )</td>
<td>( B_{\text{req}} = \frac{A_{1, \text{req}}}{N} )</td>
</tr>
<tr>
<td>( = \frac{58.8 \text{ in}^2}{10 \text{ in.}} )</td>
<td>( = \frac{58.8 \text{ in}^2}{10 \text{ in.}} )</td>
</tr>
<tr>
<td>( = 5.88 \text{ in.} )</td>
<td>( = 5.88 \text{ in.} )</td>
</tr>
</tbody>
</table>

Use \( B = 8 \text{ in.} \) (least whole-inch dimension that exceeds \( b_y \)).

**Calculate required bearing-plate thickness.**

\( n = \frac{B}{2} - k \)

\( = \frac{8 \text{ in.}}{2} - 0.972 \text{ in.} \)

\( = 3.03 \text{ in.} \)

\( t_{\text{min}} = \sqrt{\frac{3.33 R_s n^2}{A_F y}} \)

\( = \sqrt{\frac{3.33(60 \text{ kips})(3.03 \text{ in.})^2}{(10 \text{ in.} \times 8 \text{ in.})(36 \text{ ksi})}} \)

\( = 0.799 \text{ in.} \)

Use PL1x10x0'-8
Solution C:

\[ N = 6 \text{ in.} \]

From Solution A, local web yielding and web crippling are not critical.

Try \( B = 8 \text{ in.} \).

\[ A_1 = B \times N = (8 \text{ in.})(6 \text{ in.}) = 48 \text{ in.}^2 \]

To determine the dimensions of the area \( A_2 \), the load is spread into the concrete at a slope of 2:1 until an edge or the maximum condition \( \sqrt{A_2 / A_1} \leq 2 \) is met. There is also a requirement that the area \( A_2 \) be geometrically similar to \( A_1 \). The 6-in. dimension spreads 2 in. to each side to meet the concrete edge. Thus, the 8-in. dimension can also be spread 2 in. to each side. Therefore,

\[ N_1 = 6 \text{ in.} + 2(2 \text{ in.}) = 10 \text{ in.} \]

\[ B_1 = 8 \text{ in.} + 2(2 \text{ in.}) = 12 \text{ in.} \]

\[ A_2 = B_1 \times N_1 = 12 \text{ in.}(10 \text{ in.}) = 120 \text{ in.}^2 \]

Check \( \sqrt{A_2 / A_1} = 1.58 \leq 2 \) \( \text{o.k.} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \phi_c = 0.60 )</td>
<td>( \Omega_c = 2.50 )</td>
</tr>
<tr>
<td>( A_{1,req} = \frac{1}{A_2} \left( \frac{R}{\phi_c (0.85 f_y)} \right)^2 )</td>
<td>( A_{1,req} = \frac{1}{A_2} \left( \frac{R \Omega_c}{(0.85 f_y)} \right)^2 )</td>
</tr>
<tr>
<td>( = \frac{1}{120 \text{ in.}^2} \left( \frac{90 \text{ kips}}{0.60(0.85 \times 3 \text{ ksi})} \right)^2 )</td>
<td>( = \frac{1}{120 \text{ in.}^2} \left( \frac{60 \text{ kips}(2.50)}{0.85 \times 3 \text{ ksi}} \right)^2 )</td>
</tr>
<tr>
<td>( = 28.8 \text{ in.}^2 &lt; 48 \text{ in.}^2 ) ( \text{o.k.} )</td>
<td>( = 28.8 \text{ in.}^2 &lt; 48 \text{ in.}^2 ) ( \text{o.k.} )</td>
</tr>
</tbody>
</table>

**Calculate the required bearing-plate thickness.**

\[ n = \frac{B}{2} - k \]

\[ = \frac{8 \text{ in.}}{2} - 0.972 \text{ in.} \]

\[ = 3.03 \text{ in.} \]

\[ t_{min} = \sqrt{\frac{2.22 R_n n^2}{A_f f_y}} \]

\[ = \sqrt{\frac{2.22(90 \text{ kips})(3.03 \text{ in.})^2}{(6 \text{ in.})(8 \text{ in.})(36 \text{ ksi})}} \]

\[ = 1.03 \text{ in.} \]

Use PL1\( \frac{1}{4} \times 6 \times 0^* \)-8

---

**Manual**

**Part 14**

**Eqn. J8-2**

**Use PL1\( \frac{1}{4} \times 6 \times 0^* \)-8**
Example II.D-3  Slip-Critical Connection with Oversized Holes  
(Designed for Slip as a Serviceability Limit State)

Given:

Determine the number of bolts required to connect, 2L3×3×¼ tension member to a plate under a beam as shown. The angles have standard holes and the plate has oversized holes per Manual Table J3.3.

\[ R_D = 15 \text{ kips} \]
\[ R_L = 45 \text{ kips} \]

Use ¼-in. diameter ASTM A325-SC class A bolts. 
Assume that the strength of the beam, angles, and plate have been checked.

Material Properties:

<table>
<thead>
<tr>
<th>Material</th>
<th>ASTM</th>
<th>( F_y )</th>
<th>( F_u )</th>
</tr>
</thead>
<tbody>
<tr>
<td>L3×3×¼</td>
<td>A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
<tr>
<td>Plate Material</td>
<td>A36</td>
<td>36 ksi</td>
<td>58 ksi</td>
</tr>
</tbody>
</table>

Manual Table 2-3 and 2-4
Solution:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( R_u = (1.2)(15 \text{ kips}) + (1.6)(45 \text{ kips}) = 90 \text{ kips} )</td>
<td>( R_u = 15 \text{ kips} + 45 \text{ kips} = 60 \text{ kips} )</td>
</tr>
</tbody>
</table>

**Design of bolts**

Because of oversized holes the slip is a serviceability limit state, Class A faying surface, ¼ in. diameter ASTM A325-SC bolts in double shear

\[ \phi r_n = 18.8 \text{ kips/bolt} \]

\[ n = \frac{\phi R_n}{\phi r_n} = \frac{90 \text{ kips}}{18.8 \text{ kips/bolt}} = 4.78 \rightarrow 5 \text{ bolts} \]

Try (5) ½-in. dia. ASTM A325-SC bolts

**Check bolt shear strength**

\[ \phi r_n = \phi F_c A_b = 31.8 \text{ kips/bolt} \]

\[ \phi R_n = \phi r_n n = (31.8 \text{ kips/bolt})(5 \text{ bolts}) = 159 \text{ kips} > 90 \text{ kips} \quad \text{o.k.} \]

Try, PL ½ in.

**Check bolt bearing strength on plate**

\[ \phi r_n = \phi (1.2L_t F_u) \leq \phi (2.4dt F_u) \]

\[ \phi = 0.75 \]

Oversized holes, bolt spacing = 3 in.

\[ \phi r_n = (78.3 \text{ kips/in.})(\frac{3}{4} \text{ in.}) = 29.4 \text{ kips/bolt} \]

Oversized holes, edge distance = 1¼ in.

\[ \phi r_n = (40.8 \text{ kips/in.})(\frac{3}{4} \text{ in.}) = 15.3 \text{ kips/bolt} \]

15.3 kips/bolt < 29.4 kips/bolt

**Design of bolts**

Because of oversized holes the slip is a serviceability limit state, Class A faying surface, ¼ in. diameter ASTM A325-SC bolts in double shear

\[ \frac{r_n}{\Omega} = \frac{12.6 \text{ kips/bolt}}{40} = 0.315 \]

\[ n = \frac{R_u}{\frac{r_n}{\Omega}} = \frac{60 \text{ kips}}{12.6 \text{ kips/bolt}} = 4.76 \rightarrow 5 \text{ bolts} \]

Use (5) ½-in. dia. ASTM A325-SC bolts

**Check bolt shear strength**

\[ \frac{r_n}{\Omega} = \frac{F_c A_b}{\Omega} = 21.2 \text{ kips/bolt} \]

\[ R_u = \frac{r_n}{\Omega} n = (21.2 \text{ kips/bolt})(5 \text{ bolts}) = 106 \text{ kips} > 60 \text{ kips} \quad \text{o.k.} \]

Try, PL ½ in.

**Check bolt bearing strength on plate**

\[ \frac{r_n}{\Omega} = (1.2L_t F_u)/\Omega \leq (2.4dt F_u)/\Omega \]

\[ \Omega = 2.00 \]

Oversized holes, bolt spacing = 3 in.

\[ \frac{r_n}{\Omega} = (52.2 \text{ kips/in.})(\frac{3}{4} \text{ in.}) = 19.6 \text{ kips/bolt} \]

Oversized holes, edge distance = 1¼ in.

\[ \frac{r_n}{\Omega} = (27.2 \text{ kips/in.})(\frac{3}{4} \text{ in.}) = 10.2 \text{ kips/bolt} \]

10.2 kips/bolt < 19.6 kips/bolt

Manual Table 7-4

Manual Table 7-1

Eqn J3-6a

Manual Table 7-5

Manual Table 7-6
<table>
<thead>
<tr>
<th>φR_y = φr_y n</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (15.3 kips/bolt)(5 bolts)</td>
</tr>
<tr>
<td>= 76.5 kips &lt; 80 kips n.g.</td>
</tr>
<tr>
<td>Try 6 bolts</td>
</tr>
<tr>
<td>φR_y = φr_y n</td>
</tr>
<tr>
<td>= (15.3 kips/bolt)(6 bolts)</td>
</tr>
<tr>
<td>= 91.8 kips &gt; 80 kips o.k.</td>
</tr>
<tr>
<td>Use 6 bolts</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>R_y / Ω = r_y n / Ω</th>
</tr>
</thead>
<tbody>
<tr>
<td>= (10.2 kips/bolt)(5 bolts)</td>
</tr>
<tr>
<td>= 51.0 kips &lt; 80 kips n.g.</td>
</tr>
<tr>
<td>Try 6 bolts</td>
</tr>
<tr>
<td>R_y / Ω = r_y n / Ω</td>
</tr>
<tr>
<td>= (10.2 kips/bolt)(6 bolts)</td>
</tr>
<tr>
<td>= 61.2 kips &gt; 60 kips o.k.</td>
</tr>
<tr>
<td>Use 6 bolts</td>
</tr>
</tbody>
</table>
III. SYSTEM DESIGN EXAMPLES

DESIGN OF SELECTED MEMBERS
AND LATERAL ANALYSIS
OF A 4 STORY BUILDING

INTRODUCTION

This section illustrates the load determination and selection of members that are part of the gravity and lateral frame of a typical 4-story building. The design is completed in accordance with the 2005 AISC Specification for Structural Steel Buildings and the 13th Edition AISC Steel Construction Manual. Building code requirements are taken from International Building Code 2003 as the Design Code and loading criteria are based on SEI/ASCE 7-02.

This section includes:
- Analysis and design of a typical steel frame for gravity loads
- Analysis and design of a typical steel frame for lateral loads
- Examples illustrating each of the three methods for satisfying the stability provisions of AISC Specification Chapter C.

The building being analyzed in this design example is located in a Midwestern city with moderate wind and seismic loads. The criteria for these minimum loads are given in the description of the design example.

CONVENTIONS

The following conventions are used throughout this example:

1. Beams or columns that have similar, but not necessarily identical, loads are grouped together. This is done to simplify the selection process, because such grouping is generally a more economical practice for design, fabrication, and erection.

2. Certain calculations, such as design loads for snow drift, which might typically be determined using a spreadsheet or structural analysis program, are summarized and then incorporated into the analysis. This simplifying feature allows the design example to illustrate concepts relevant to the member selection process.

3. Two commonly used deflection calculations, for uniform loads, have been rearranged so that the conventional units in the problem can be directly inserted into the equation for steel design. They are as follows:

Simple Beam: $$\Delta = \frac{5 \text{ w kip/in.}(\text{l in.})^4}{384(29,000 \text{ ksi})(\text{l in.})^4} = \frac{\text{ w kip/ft}(\text{l ft})^4}{1290(\text{l in.})^4}$$

Beam Fixed at both Ends: $$\Delta = \frac{\text{ w kip/in.}(\text{l in.})^4}{384(29,000 \text{ ksi})(\text{l in.})^4} = \frac{\text{ w kip/ft}(\text{l ft})^4}{6450(\text{l in.})^4}$$
DESIGN SEQUENCE

The design sequence is presented as follows:

1. General description of the building including geometry, gravity loads, and lateral loads.
2. Roof member design and selection.
3. Floor member design and selection.
4. Column design and selection for gravity loads.
5. Wind load determination.
7. Horizontal force distribution to the lateral frames.
8. Preliminary column selection for the moment frames and braced frames.
9. Seismic load application to lateral systems.
10. Second order effects (P-delta) analysis
GENERAL DESCRIPTION OF THE BUILDING

Geometry

The design example is a 4-story building, comprised of 7 bays at 30 ft in the East-West (Numbered Grids) direction and bays of 45 ft, 30 ft and 45 ft in the North-South (Lettered Grids) direction. The floor-to-floor height for the 4 floors is 13'-6" and the height from the fourth floor to the roof (at the edge of the building) is 14'-6". Based on discussions with Fabricators, the same column size will be used for the whole height of the building.

The plans of these floors and the roof are shown on sheets S2.1 thru S2.3, found at the end of this Chapter. The exterior of the building is a ribbon window system with brick spandrels supported and back-braced with steel and in-filled with metal studs. The spandrel wall extends 2 ft above the elevation of the edge of the roof. The window and spandrel system is shown on design drawing Sheet S4.1.

The roof system is 1½-in. metal deck on bar joists. These bar joists are supported on steel beams as shown on Design Drawing Sheet S2.3. The roof slopes to interior drains. The middle 3 bays have a 6-ft-tall screen wall around them and house the mechanical equipment and the elevator overrun. This area has steel beams, in place of steel bar joists, to help carry the mechanical equipment.

The three elevated floors have 3 in. of normal weight concrete over 3-in. composite deck for a total slab thickness of 6 in. The supporting beams are spaced at 10 ft on center. These beams are carried by composite girders in the East-West direction to the columns. There is a 30 ft by 45 ft opening in the second floor, to create a 2-story atrium at the entrance. These floor layouts are shown on Drawings S2.1 and S2.2. The first floor is a slab on grade and the foundation consists of conventional spread footings.

The building includes both moment frames and braced frames for lateral resistance. The lateral system in the North-South direction consists of Chevron braces at the end of the building, located adjacent to the stairways. In the East-West direction there are no locations in which Chevron braces can be concealed. Consequently, the lateral system in the East-West direction is composed of moment frames at the North and South faces of the building.

This building is sprinklered and has large open spaces around it, and consequently does not require fire proofing for the floors.
Wind Forces

The Basic Wind Speed is 90 miles per hour (3 second gust). Because it is located in an Office Park with substantial parking areas around it, it will be analyzed as Wind Exposure Category C. Because it is an ordinary (Category II) office occupancy, the wind importance factor is 1.0.

Seismic Forces

This building is located in an area where the sub-soil has been evaluated and the site class has been determined to be Category D. The area has a short period $S_s = 0.121$ and a one-second period $S_1 = 0.060$. The seismic importance factor is 1.0, that of an ordinary office occupancy (Category II).

Roof and Floor Loads

Roof loads:

The ground snow load ($p_g$) for this building is 20 psf. The slope of the roof is ¼ in./ft or more at all locations, but not exceeding ½ in./ft. Consequently, 5 psf rain-on-snow surcharge is to be considered, but no ponding instability design calculations are not required. This roof can be designed as a fully exposed roof, but, per ASCE 7 Section 7.3, cannot be designed for less than $p_f = (I)p_g = 0.020$ kip/ft² snow live load. Snow drift will be applied at the edges of the roof and at the screen wall around the mechanical area.

Floor Loads:

The basic live load for the floor is 50 psf. An additional partition live load of 20 psf will be applied. However, because the locations of partitions and, consequently, corridors are not known, and will be subject to change, the entire floor will be designed for a live load of 80 psf. This live load will be reduced, based on type of member and area per the IBC provisions for live-load reduction.

ROOF MEMBER DESIGN AND SELECTION

Calculate dead load and live load

<table>
<thead>
<tr>
<th>Dead Load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Roofing</td>
<td>= 0.005 kip/ft²</td>
</tr>
<tr>
<td>Insulation</td>
<td>= 0.002 kip/ft²</td>
</tr>
<tr>
<td>Deck</td>
<td>= 0.002 kip/ft²</td>
</tr>
<tr>
<td>Beams</td>
<td>= 0.003 kip/ft²</td>
</tr>
<tr>
<td>Joists</td>
<td>= 0.003 kip/ft²</td>
</tr>
<tr>
<td>Misc.</td>
<td>= 0.005 kip/ft²</td>
</tr>
<tr>
<td>Total</td>
<td>= 0.020 kip/ft²</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Live Load</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Snow</td>
<td>= 0.020 kip/ft²</td>
</tr>
<tr>
<td>Rain on Snow</td>
<td>= 0.005 kip/ft²</td>
</tr>
<tr>
<td>Total</td>
<td>= 0.025 kip/ft²</td>
</tr>
</tbody>
</table>

Note: In this design, the Rain and Snow Load is greater than the Roof Live Load

The deck for this design example is 1½ in., wide rib, 22 gage, painted roof deck, placed in a pattern of 3 continuous spans minimum. The general joist layout is 6 ft on center. At 6 ft on center, this deck has an allowable total load capacity of 0.071 kip/ft². This will require changing 2 spaces at 6 ft to a 5 ft, 4 ft, 3 ft pattern at the ends of the Mechanical area. The roof diaphragm and roof loads extend 6 in. past the centerline of grid as shown on Drawing S4.1
Flat roof snow load = 0.020 kip/ft², Density $\gamma = 0.0166$ kip/ft², $h_b = 1.20$ ft

Summary of Drifts

<table>
<thead>
<tr>
<th></th>
<th>Length ($l_u$)</th>
<th>Height</th>
<th>Max. Drift Load</th>
<th>Max Drift Width ($W$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Side Parapet</td>
<td>121 ft</td>
<td>2 ft</td>
<td>0.0132 kip/ft²</td>
<td>6.36 ft</td>
</tr>
<tr>
<td>End Parapet</td>
<td>211 ft</td>
<td>2 ft</td>
<td>0.0132 kip/ft²</td>
<td>6.36 ft</td>
</tr>
<tr>
<td>Screen Wall</td>
<td>60.5 ft</td>
<td>6 ft</td>
<td>0.0305 kip/ft²</td>
<td>7.35 ft</td>
</tr>
</tbody>
</table>

**SELECT ROOF JOISTS**

*Layout loads and size joists*

User Note: Joists are normally designed by ASD and are designed and selected in this manner here.

The 45-ft side joist with the heaviest loads is shown below along with its end reactions and maximum moment:

Because the load is not uniform, select a 24KCS4 JOIST AT 16.5 plf, which has an allowable moment of 92.3 kip-ft and an allowable shear of 8.40 kips.

The standard 30 ft joist in the middle bay will have a uniform load of

$$w = (0.020 \text{ kip/ft}^2 + 0.025 \text{ kip/ft}^2)(6 \text{ ft}) = 0.270 \text{ kip/ft}.$$ Per joist catalog, select an 18K5 joist at 7.7 plf.

Note: the first joist away from the screen wall and the first joist away from the end of the building must account for snow drift. An 18K7 joist will be used in these locations.
SELECT ROOF BEAMS

Calculate loads and select beams in the mechanical area

For the beams in the mechanical area, the mechanical units could weigh as much as 0.060 kip/ft². Use 0.040 kip/ft² for additional live load, which will account for the mechanical units and any snow drift which could occur in the mechanical area. The beams in the mechanical area are at 6 ft on center.

Calculate minimum $I_x$ to limit deflection to 1/360 = 1 in., because plaster ceiling will be used in the lobby area. Use 0.040 kip/ft² as an estimate of the snow load, including some drifting that could occur in this area, for deflection calculations.

\[ I_{req} \text{ (Live Load)} = \frac{0.240 \text{ kip/ft}(30 \text{ ft})^4}{1290(1 \text{ in.})} = 151 \text{ in.}^4 \]

\[
\begin{align*}
W_0 &= 0.120 \text{ kips/ft} \\
W_l &= 0.390 \text{ kips/ft}
\end{align*}
\]

\[ \text{Beam Loading & Bracing Diagram (Full Lateral Support)} \]

Calculate the required strengths and select the beams in the mechanical area

\[
\begin{array}{|c|c|}
\hline
\text{LRFD} & \text{ASD} \\
\hline
W_a &= 6 \text{ ft}[1.2(0.020 \text{ kip/ft}^2) \\
& \quad +1.6(0.025 \text{ kip/ft}^2 + 0.040 \text{ kip/ft}^3)] \\
& = 0.768 \text{ kip/ft} \\
M_a &= \frac{0.768 \text{ kip/ft}(30 \text{ ft})^2}{8} = 86.4 \text{ kip-ft} \\
R_a &= \frac{30 \text{ ft}}{2}(0.768 \text{ kip/ft}) = 11.5 \text{ kips} \\
\phi_bV_a &= 94.8 \text{ kips} > 11.5 \text{ kips} \quad \text{o.k.} \\
\hline
W_a &= 6 \text{ ft}(0.020 \text{ kip/ft}^2 \\
& \quad +0.025 \text{ kip/ft}^2 + 0.040 \text{ kip/ft}^3) \\
& = 0.510 \text{ kip/ft} \\
M_a &= \frac{0.510 \text{ kip/ft}(30 \text{ ft})^2}{8} = 57.4 \text{ kip-ft} \\
R_a &= \frac{30 \text{ ft}}{2}(0.510 \text{ kip/ft}) = 7.65 \text{ kips} \\
V_a/\Omega_a &= 63.2 \text{ kips} > 7.65 \text{ kips} \quad \text{o.k.} \\
\hline
\end{array}
\]

Note: a W12×22 beam would also meet all criteria, but a 14 in. beam was selected so that consistent beam sizes could be used throughout the building.
SELECT BEAMS AT THE END OF THE BUILDING

The beams at the ends of the building carry the brick spandrel panel and a small portion of roof load. Because there is continuous glass underneath this window, limit live load deflection to ¼-in. total movement. In addition, per AISC Design Guide 3, Second Edition, limit deflection due to spandrel weight to the \( \frac{L}{480} \) or \( \frac{a}{2} \)-in. maximum. In calculating the wall loads, the brick weight is taken as 55 lb/ft, and the glass weight is taken as 90 lb/ft. The spandrel panel weight is approximately

\[ w = 7.50 \text{ ft} (0.055 \text{ kip/ft}^2) = 0.413 \text{ kip/ft}. \]

the dead load from the roof is equal to

\[ w = 3.5 \text{ ft} (0.020 \text{ kip/ft}^2) = 0.070 \text{ kip/ft} \]

and the live load from the roof can be conservatively taken as

\[ w = 3.5 \text{ ft} (0.025 \text{ kip/ft}^2 + 0.0132 \text{ kip/ft}^2) = 0.134 \text{ kip/ft}. \]

to account for the maximum snow drift as a uniform load

Assume the beams are simple spans of 22.5 ft.

Calculate minimum \( I \) to limit live load deflection to ¼ in.

\[ I_{req} = \frac{0.134 \text{ kip/ft}(22.5 \text{ ft})^4}{1290(0.25 \text{ in.})} = 106 \text{ in.}^4 \]

Calculate minimum \( I \) to limit spandrel weight deflection to \( \frac{a}{2} \) in.

\[ I_{req} = \frac{0.413 \text{ kip/ft}(22.5 \text{ ft})^4}{1290(0.375 \text{ in.})} = 218 \text{ in.}^4 \]

The loading diagram is as follows

\[ w_{db} = 0.413 + 0.070 = 0.483 \text{ kips/ft} \]

\[ w_{f} = 0.134 \text{ kips/ft} \]

22.5 ft

Calculate the required strengths and select the beams for the roof ends

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
</table>
| \[ W_u = 1.2 (0.070 \text{ kip/ft}^2 + 0.413 \text{ kip/ft}) \]  
+ \[ 1.6(0.134 \text{ kip/ft}^2) \]  
= 0.794 kip/ft | \[ W_u = (0.070 \text{ kip/ft}^2 + 0.413 \text{ kip/ft}) \]  
+ \[ 0.134 \text{ kip/ft}^2 \]  
= 0.617 kip/ft |
The beams along the side of the building carry the spandrel panel and a substantial roof dead load and live load. The spandrel beam is over glass, therefore, limit the live load deflection to \( \frac{1}{4} \) in. In addition, per AISC Design Guide 3, Second Edition, limit deflection due to spandrel weight to the \( \frac{L}{480} \) or \( \frac{a}{16} \) in. maximum. These beams will be part of the moment frames on the side of the building and therefore will be designed as fixed at both ends. The roof dead load and live load to this edge beam is equal to the joist end dead load and live load reaction. Treating this as a uniform load, divide this by the joist spacing.

\[
\begin{align*}
w_D &= 2.76 \text{ kips/ft} = 0.460 \text{ kip/ft}.
w_L &= 3.73 \text{ kips/ft} = 0.622 \text{ kip/ft}.
\end{align*}
\]

**Calculate minimum \( I_x \) to limit the live load deflection to \( \frac{1}{4} \) in.**

\[
I_{req} = \frac{(0.622 \text{ kip/ft})(30 \text{ ft})^4}{6450(0.25 \text{ in.})} = 312 \text{ in.}^4
\]

**Calculate minimum \( I_x \) to limit the spandrel weight deflection to \( \frac{a}{16} \) in.**

\[
I_{req} = \frac{(0.413 \text{ kip/ft})(30 \text{ ft})^4}{6450(0.375 \text{ in.})} = 138 \text{ in.}^4
\]

\[
\begin{align*}
w_D &= 0.873 \text{ kips/ft} \\
w_L &= 0.622 \text{ kips/ft}
\end{align*}
\]
Calculate the required strengths and select the beams for the roof sides

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_u = 1.2 \times (0.460 \text{ kip/ft}^2 + 0.413 \text{ kip/ft}) )</td>
<td>( W_u = (0.460 \text{ kip/ft}^2 + 0.413 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>( + 1.6 \times (0.622 \text{ kip/ft}^2) )</td>
<td>( + 0.622 \text{ kip/ft}^2 )</td>
</tr>
<tr>
<td>= 2.04 kip/ft</td>
<td>= 1.50 kip/ft</td>
</tr>
<tr>
<td>( M_u = \frac{2.04 \text{ kip/ft}(30 \text{ ft})^2}{12} = 153 \text{ kip-ft} )</td>
<td>( M_u = \frac{1.50 \text{ kip/ft}(30 \text{ ft})^2}{12} = 113 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

For \( L_b = 6 \text{ ft} \) and \( C_b = 1.0 \), select \( W_{16} \times 31 \), which has an available moment strength of 183 kip-ft and an \( I_x \) of 375 in.\(^4\)

For \( L_b = 6 \text{ ft} \) and \( C_b = 1.0 \), select \( W_{16} \times 31 \), which has an available moment strength of 122 kip-ft and an \( I_x \) of 375 in.\(^4\)

Manual
Table 3-10

Note: This roof beam may need to be upsized during the lateral load analysis to increase the stiffness and strength of the member and improve lateral frame drift performance.
SELECT THE BEAMS ALONG THE INTERIOR LINES OF THE BUILDING

There are 3 individual beam loadings that occur along grids C and D. The beams from 1 to 2 and 7 to 8 have a uniform snow load except for the snow drift at the end at the parapet. The snow drift from the far ends of the 45 foot joists is negligible. The beams from 2 to 3 and 6 to 7 are the same as the first group, except they have snow drift at the screen wall. The loading diagrams are shown below. A summary of the moments, left and right reactions, and required $I_x$ to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.

Note: by footnote G, for steel structural members, the dead load shall be taken as zero for D + L deflection calculation combination.

![Beam Loading & Bracing Diagram (Continuous Bracing) 1]

![Beam Loading & Bracing Diagram (Continuous Bracing) 2]

![Beam Loading & Bracing Diagram (Continuous Bracing) 3]

Note: by footnote G, for steel structural members, the dead load shall be taken as zero for D + L deflection calculation combination.
Summary of required strengths and required moment of inertia

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Grids 1 to 2 and 7 to 8 (opposite hand)</td>
<td>Grids 1 to 2 and 7 to 8 (opposite hand)</td>
</tr>
<tr>
<td>$R_u$ (left) = 1.2(11.6 kips) + 1.6(16.1 kips) = 38.8 kips</td>
<td>$R_a$ (left) = 11.6 kips + 15.6 kips = 27.2 kips</td>
</tr>
<tr>
<td>$R_u$ (right) = 1.2(11.3 kips) + 1.6(14.2 kips) = 36.1 kips</td>
<td>$R_a$ (right) = 11.3 kips + 14.2 kips = 25.5 kips</td>
</tr>
<tr>
<td>$M_a$ = 1.2(84.3 kip-ft) + 1.6(107 kip-ft) = 272 kip-ft</td>
<td>$M_a$ = 84.3 kip-ft + 107 kip-ft = 191 kip-ft</td>
</tr>
<tr>
<td>$I_{x\text{req'd}} = \frac{(0.938 \text{ klf})(30 \text{ ft})^4}{1290 (1.5 \text{ in.})}$ = 393 in.$^4$</td>
<td>$I_{x\text{req'd}} = \frac{(0.938 \text{ klf})(30 \text{ ft})^4}{1290 (1.5 \text{ in.})}$ = 393 in.$^4$</td>
</tr>
</tbody>
</table>

For $L_b = 6 \text{ ft}$ and $C_b = 1.0$, select W21×44 which has an available moment of 333 kip-ft and $I_x = 843 \text{ in.}^4$.

Grids 2 to 3 and 6 to 7 (opposite hand)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u$ (left) = 1.2(11.3 kips) + 1.6(14.3 kips) = 36.6 kips</td>
<td>$R_a$ (left) = 11.3 kips + 14.3 kips = 25.6 kips</td>
</tr>
<tr>
<td>$R_u$ (right) = 1.2(11.3 kips) + 1.6(17.9 kips) = 42.1 kips</td>
<td>$R_a$ (right) = 11.3 kips + 17.9 kips = 29.2 kips</td>
</tr>
<tr>
<td>$M_a$ = 1.2(84.4 kip-ft) + 1.6(109 kip-ft) = 278 kip-ft</td>
<td>$M_a$ = 84.4 kip-ft + 109 kip-ft = 195 kip-ft</td>
</tr>
<tr>
<td>$I_{x\text{req'd}} = \frac{(0.938 \text{ klf})(30 \text{ ft})^4}{1290 (1.5 \text{ in.})}$ = 393 in.$^4$</td>
<td>$I_{x\text{req'd}} = \frac{(0.938 \text{ klf})(30 \text{ ft})^4}{1290 (1.5 \text{ in.})}$ = 393 in.$^4$</td>
</tr>
</tbody>
</table>

For $L_b = 6 \text{ ft}$ and $C_b = 1.0$, select W21×44 which has an available moment of 222 kip-ft and $I_x = 843 \text{ in.}^4$.

The third individual beam loading occurs at the beams from 3 to 4, 4 to 5, and 5 to 6. This is the heaviest load.
SELECT THE BEAMS ALONG THE SIDES OF THE MECHANICAL AREA

The beams from 3 to 4, 4 to 5, and 5 to 6 have a uniform snow load outside the screen walled area, except for the snow drift at the parapet ends and the screen wall ends of the 45 foot joists. Inside the screen walled area the beams support the mechanical equipment. A summary of the moments, left and right reactions, and required $I_x$ to keep the live load deflection to equal or less than the span divided by 240 (or 1.50 in.) is given below.

Note: by footnote g, for steel structural members, the dead load shall be taken as zero for $D + L$ deflection calculation load combination.

![Beam Loading & Bracing Diagram](image)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_u = 1.2 \times (0.750 \text{ kip/ft}^2) + 1.6(1.57 \text{ kip/ft}^2)$</td>
<td>$W_u = 0.750 \text{ kip/ft} + 1.57 \text{ kip/ft}^2$</td>
</tr>
<tr>
<td>$M_u = \frac{3.41 \text{ kip/ft}(30 \text{ ft})^2}{8}$</td>
<td>$M_u = \frac{2.32 \text{ kip/ft}(30 \text{ ft})^2}{8}$</td>
</tr>
<tr>
<td>$R_u = \frac{30 \text{ ft}}{2} \times (3.41 \text{ kip/ft}) = 51.2 \text{ kips}$</td>
<td>$R_u = \frac{30 \text{ ft}}{2} \times (2.32 \text{ kip/ft}) = 34.8 \text{ kips}$</td>
</tr>
<tr>
<td>$I_x \text{ req'd} = \frac{1.57 \text{ kip/ft}(30.0 \text{ ft})^4}{1290(1.50 \text{ in.})} = 393 \text{ in.}^4$</td>
<td>$I_x \text{ req'd} = \frac{1.57 \text{ kip/ft}(30.0 \text{ ft})^4}{1290(1.50 \text{ in.})} = 393 \text{ in.}^4$</td>
</tr>
</tbody>
</table>

For $L_b = 6$ ft and $C_b = 1.0$, select W21×48, which has an available moment strength of 398 kip-ft, an available end shear of 217 kips and an $I_x$ of 959 in.$^4$
FLOOR MEMBER DESIGN AND SELECTION

Calculate dead load and live load

**Dead Load**

Slab and Deck = 0.057 kip/ft²  
Beams (est.) = 0.008 kip/ft²  
Misc. (ceiling, mechanical, etc.) = 0.010 kip/ft²  
Total = 0.075 kip/ft²  

---

**Steel Deck**  
Institute  
Diaphragm Design Manual  

**IBC Section 1607.9**

The floor and deck will be 3 in. of normal weight concrete, $f'_c = 4$ ksi, on 3 in. 20 gage, galvanized, composite deck, laid in a pattern of 3 or more continuous spans. The total depth of the slab is 6 in. The general layout for the floor beams is 10 ft on center. At 10 ft on center, this deck has an allowable superimposed live load capacity of 0.143 kip/ft². In addition, it can be shown that this deck can carry a 2,000 pound load over an area of 2.5 ft by 2.5 ft (per IBC Section 1607.4). The floor diaphragm and the floor loads extend 6 in. past the centerline of grid as shown on Drawing S4.1.
SELECT FLOOR BEAMS (composite and non-composite)

Note: There are two early and important checks in the design of composite beams. First, select a beam that either does not require camber, or establish a target camber and moment of inertia at the start of the design process. A reasonable approximation of the camber is between \( L/300 \) minimum and \( L/180 \) maximum (or a maximum of 1½ to 2 in.).

Second, check that the beam is strong enough to safely carry the wet concrete and a 0.020 kip/ft² construction live load (per ASCE 37-02), when designed by the ASCE 7 load combinations and the provisions of Chapter F of the Specification.

SELECT TYPICAL 45 FT COMPOSITE BEAM (10 FT ON CENTER)

Find a target moment of inertia for an un-shored beam

\[ I_{req} \approx \frac{0.65 \text{ kip}/\text{ft}(45 \text{ ft})^4}{1290(2.0 \text{ in.})} = 1030 \text{ in.}^4 \]

Determine minimum strength to carry wet concrete and construction live load

\[ w_{DL} = 0.065 \text{ kip/ft}^2(10 \text{ ft}) = 0.65 \text{ kip/ft} \]
\[ w_{LL} = 0.020 \text{ kip/ft}^2(10 \text{ ft}) = 0.20 \text{ kip/ft} \]

Determine the required moment strength due to wet concrete only

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_u = 1.4(0.65 \text{ kip}/\text{ft}) = 0.91 \text{ kip/ft} )</td>
<td>( W_u = 0.65 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{0.91 \text{ kip/ft}(45 \text{ ft})^2}{8} = 230 \text{ kip-ft} )</td>
<td>( M_u = \frac{0.65 \text{ kip/ft}(45 \text{ ft})^2}{8} = 165 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Determine the required moment strength due to wet concrete and construction live load

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_u = 1.2(0.65 \text{ kip}/\text{ft}) + 1.6(0.20 \text{ kip/ft}) = 1.10 \text{ kip/ft} )</td>
<td>( W_u = 0.65 \text{ kip/ft} + 0.20 \text{ kip/ft} = 0.85 \text{ kip/ft} )</td>
</tr>
<tr>
<td>( M_u = \frac{1.10 \text{ kip/ft}(45 \text{ ft})^2}{8} = 278 \text{ kip-ft} )</td>
<td>( M_u = \frac{0.85 \text{ kip/ft}(45 \text{ ft})^2}{8} = 215 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

Use Manual Table 3-2 to select a beam with \( I_x \geq 1030 \text{ in.}^4 \). Select W21×50, which has \( I_x = 984 \text{ in.}^4 \), close to our target value, and has an available moment strength of 413 kip-ft (LRFD) and 274 kip-ft (ASD).

Check for possible live load reduction due to area

For interior beams \( K_{LL} = 2 \)

The beams are at 10 ft on center, therefore the area \( A_T = 45 \times 10 = 450 \text{ ft}^2 \).

\[ L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL}A_T}} \right) = 0.060 \text{ kip/ft}^2 \geq 0.50L_o = 0.040 \text{ kip/ft}^2 \]
Therefore use 0.060 kip/ft².
The beam is continuously braced by the deck.

The beams are at 10 ft on center, therefore the loading diagram is as shown below.

\[ w_0 = 0.75 \text{ kips/ft} \]
\[ w_c = 0.60 \text{ kips/ft} \]

**Beam Loading & Bracing Diagram (Continuous Bracing)**

---

**Calculate the required moment strength**

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_u )</td>
<td>1.2(0.75 kip/ft) = 1.6 (0.60 kip/ft)</td>
<td>( W_a ) = 0.75 kip/ft + 0.60 kip/ft = 1.35 kip/ft</td>
</tr>
<tr>
<td>( M_u )</td>
<td>( \frac{1.86 \text{ kip/ft}(45 \text{ ft})}{8} = 471 \text{ kip-ft} )</td>
<td>( \frac{1.35 \text{ kip/ft}(45 \text{ ft})}{8} = 342 \text{ kip-ft} )</td>
</tr>
</tbody>
</table>

\( \gamma_2 = 6 \text{ in.} - \frac{1}{2} \text{ in.} = 5.5 \text{ in.} \)

Use Manual Table 3-19 to check W21×50 selected above. Using values of 471kip-ft (LRFD) or 342 kip-ft (ASD) and a \( \gamma_2 \) value of 5.5 in

<table>
<thead>
<tr>
<th></th>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select W21×50 beam, where</td>
<td>PNA = 7, ( Q_a = 184 \text{ kips} )</td>
<td>Select W21×50 beam, where</td>
</tr>
<tr>
<td>( \phi_0 M_p ) = 599 kip-ft &gt; 471 kip-ft</td>
<td>( M_p / \Omega_b = 399 \text{ kip-ft} &gt; 342 \text{ kip-ft} )</td>
<td>( \text{o.k.} )</td>
</tr>
</tbody>
</table>

---

**Determine \( b_{eff} \)**
The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline which shall not exceed:

1. one-eighth of the span of the beam, center to center of supports
   \( \frac{45 \text{ ft}}{8} \) (2 sides) = 11.3 ft.

2. one-half the distance to the center line of the adjacent beam
   \( \frac{10 \text{ ft}}{2} \) (2 sides) = 10.0 ft. **controls**

3. the distance to the edge of the slab
   Not applicable

---

Specification
Sec. I.3.1.1a
Check a

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ a = \frac{\sum Q_n}{0.85 f_y b} ]</td>
<td>[ a = \frac{\sum Q_n}{0.85 f_y b} ]</td>
</tr>
<tr>
<td>[ = \frac{184 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft})(12 \text{ in./ft})} ]</td>
<td>[ = \frac{184 \text{ kips}}{0.85(4 \text{ ksi})(10 \text{ ft})(12 \text{ in./ft})} ]</td>
</tr>
<tr>
<td>[ = 0.451 \text{ in.} &lt; 1.0 \text{ in.} \text{ o.k.} ]</td>
<td>[ = 0.451 \text{ in.} &lt; 1.0 \text{ in.} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

Check end shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R_u = \frac{45 \text{ ft}}{2} \left( \frac{1.86 \text{ kip/ft}}{2} \right) = 41.9 \text{ kips} ]</td>
<td>[ R_u = \frac{45 \text{ ft}}{2} \left( \frac{1.35 \text{ kip/ft}}{2} \right) = 30.4 \text{ kips} ]</td>
</tr>
<tr>
<td>[ \phi_b V_n = 217 \text{ kips} &gt; 41.9 \text{ kips} \text{ o.k.} ]</td>
<td>[ V_n/\Omega_b = 145 \text{ kips} &gt; 30.4 \text{ kips} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

Check live load deflection

\[ \Delta_{LL} = l/360 = ((45 \text{ ft})(12 \text{ in./ft}))/360 = 1.5 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ W21 \times 50: Y^2 = 5.5 \text{ in.}, \text{ PNA} = 7 ]</td>
<td>[ W21 \times 50: Y^2 = 5.5 \text{ in.}, \text{ PNA} = 7 ]</td>
</tr>
<tr>
<td>[ I_{LB} = 1640 \text{ in.}^4 ]</td>
<td>[ I_{LB} = 1640 \text{ in.}^4 ]</td>
</tr>
<tr>
<td>[ \Delta_{LL} = \frac{w_l I_l^4}{1290 I_{LB}^4} \left( \frac{0.60 \text{ kip/ft(45 ft)}^4}{1290(1730 \text{ in.}^4)} \right) ]</td>
<td>[ \Delta_{LL} = \frac{w_l I_l^4}{1290 I_{LB}^4} \left( \frac{0.60 \text{ kip/ft(45 ft)}^4}{1290(1730 \text{ in.}^4)} \right) ]</td>
</tr>
<tr>
<td>[ = 1.10 \text{ in.} &lt; 1.5 \text{ in.} \text{ o.k.} ]</td>
<td>[ = 1.16 \text{ in.} &lt; 1.5 \text{ in.} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

Determine the required shear stud connectors

Using perpendicular deck with one \( \frac{3}{8} \)-in. diameter stud per rib in normal weight, 4 ksi concrete, in weak position; \( Q_n = 17.2 \text{ kips/stud} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum Q_n = 184 \text{ kips} \quad \text{17.2 kips/stud} ]</td>
<td>[ \sum Q_n = 184 \text{ kips} \quad \text{17.2 kips/stud} ]</td>
</tr>
<tr>
<td>[ = 10.7 \text{ studs / side} ]</td>
<td>[ = 10.7 \text{ studs / side} ]</td>
</tr>
</tbody>
</table>

Therefore use 22 studs.

**SELECT TYPICAL 30 FT COMPOSITE (OR NON-COMPOSITE) BEAM (10 FT ON CENTER)**

Find a target moment of inertia for an un-shored beam

Hold deflection to around 1.5 in. maximum to facilitate concrete placement.

\[ I_{req} \approx \frac{0.65 \text{ kip/ft(30 ft)}^4}{1290(1.50 \text{ in.})} = 272 \text{ in.}^4 \]
Determine minimum strength to carry wet concrete and construction live load

\[ w_{DL} = 0.065 \text{kip/ft}^2(10 \text{ ft}) = 0.65 \text{ kip/ft} \]
\[ w_{LL} = 0.020 \text{kip/ft}^2(10 \text{ ft}) = 0.20 \text{ kip/ft} \]

Determine the required moment strength due to wet concrete only

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_u = 1.4(0.65 \text{ kip/ft}) = 0.91 \text{ kip/ft})</td>
<td>(W_u = 0.65 \text{ kip/ft})</td>
</tr>
<tr>
<td>(M_u = \frac{0.91 \text{ kip/ft}(30 \text{ ft})^2}{8} = 102 \text{ kip-ft})</td>
<td>(M_u = \frac{0.65 \text{ kip/ft}(30 \text{ ft})^2}{8} = 73.1 \text{ kip-ft})</td>
</tr>
</tbody>
</table>

Determine the required moment strength due to wet concrete and construction live load

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(W_u = 1.2(0.65 \text{ kip/ft}) + 1.6(0.20 \text{ kip/ft}) = 1.10 \text{ kip/ft})</td>
<td>(W_u = 0.65 \text{ kip/ft} + 0.20 \text{ kip/ft} = 0.85 \text{ kip/ft})</td>
</tr>
<tr>
<td>(M_u = \frac{1.10 \text{ kip/ft}(30 \text{ ft})^2}{8} = 124 \text{ kip-ft})</td>
<td>(M_u = \frac{0.85 \text{ kip/ft}(30 \text{ ft})^2}{8} = 95.6 \text{ kip-ft})</td>
</tr>
</tbody>
</table>

Controls

Use Manual Table 3-2 to find a beam with an \(I_x \geq 272 \text{ in.}^4\). Select W16×26, which has an \(I_x = 301 \text{ in.}^4\) which exceeds our target value, and has an available moment of 166 kip-ft (LRFD) and 110 kip-ft (ASD).

Check for possible live load reduction due to area

For interior beams \(K_{LL} = 2\)

The beams are at 10 ft on center, therefore the area \(A_T = 30 \text{ ft} \times 10 \text{ ft} = 300 \text{ ft}^2\).

\[ L = L_o \left(0.25 + \frac{15}{K_{LL} A_T}\right) = 0.069 \text{ kip/ft}^2 \geq 0.50 \quad L_o = 0.040 \text{ kip/ft}^2 \]

Therefore, use 0.069 kip/ft²

The beams are at 10 ft on center, therefore the loading diagram is as shown below.

\[ w_0 = 0.750 \text{ kips/ft} \]
\[ w_L = 0.690 \text{ kips/ft} \]
Calculate the required strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W_u = 1.2(0.75 \text{ kip/ft}) = 1.6 (0.69 \text{ kip/ft})$</td>
<td>$W_u = 0.750 \text{ kip/ft} + 0.690 \text{ kip/ft}$</td>
</tr>
<tr>
<td>$M_u = 2.00 \text{ kip/ft} (30 \text{ ft})^2 / 8 = 225 \text{ kip-ft}$</td>
<td>$M_u = 1.44 \text{ kip/ft} (30 \text{ ft})^2 / 8 = 162 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

From previous calculation, $Y_2 = 5.5 \text{ in.}$

Use Manual Table 3-19 to check the W16×26 selected above. Using required strengths of 225 kip-ft (LRFD) or 162 kip-ft (ASD) and a $Y_2$ value of 4.5 in

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select W16×26 beam, where</td>
<td>Select W16×26 beam, where</td>
</tr>
<tr>
<td>PNA = 7 and</td>
<td>PNA = 7 and</td>
</tr>
<tr>
<td>$\Sigma Q_u = 96.0 \text{ kips}$</td>
<td>$\Sigma Q_u = 96.0 \text{ kips}$</td>
</tr>
<tr>
<td>$\phi_b M_p = 248 \text{ kip-ft} &gt; 225 \text{ kip-ft}$ o.k.</td>
<td>$M_p / \Omega_b = 165 \text{ kip-ft} &gt; 162 \text{ kip-ft}$ o.k.</td>
</tr>
</tbody>
</table>

Determine $b_{eff}$

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline, which shall not exceed:

1. one-eighth of the span of the beam, center to center of supports
   $$\frac{30 \text{ ft}}{8} (2 \text{ sides}) = 7.50 \text{ ft.} \quad \text{Controls}$$

2. one-half the distance to the center line of the adjacent beam
   $$\frac{10 \text{ ft}}{2} (2 \text{ sides}) = 10.0 \text{ ft.}$$

3. the distance to the edge of the slab
   Not applicable

Check $a$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a = \frac{\Sigma Q_u}{0.85 f_{cy} b}$</td>
<td>$a = \frac{\Sigma Q_u}{0.85 f_{cy} b}$</td>
</tr>
<tr>
<td>$= \frac{96.0 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$</td>
<td>$= \frac{145 \text{ kips}}{0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})}$</td>
</tr>
<tr>
<td>$= 0.313 \text{ in.} &lt; 1.0 \text{ in.}$ o.k.</td>
<td>$= 0.473 \text{ in.} &lt; 1.0 \text{ in.}$ o.k.</td>
</tr>
</tbody>
</table>
Check end shear strength

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual Table 3-2</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ R_u = \frac{30 \text{ ft}}{2} (2.00 \text{ kip/ft}) = 30.0 \text{ kips} ]</td>
<td>[ R_u = \frac{30 \text{ ft}}{2} (1.44 \text{ kip/ft}) = 21.6 \text{ kips} ]</td>
<td>[ \phi_b V_n = 106 \text{ kips} &gt; 30.0 \text{ kips} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

Check live load deflection

\[ \Delta_{LL} = \frac{l}{360} = \frac{(30 \text{ ft})(12 \text{ in./ft})}{360} = 1.0 \text{ in.} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>IBC Table 1604.3</th>
</tr>
</thead>
<tbody>
<tr>
<td>W16×26</td>
<td>W16×26</td>
<td>Manual Table 3-20</td>
</tr>
<tr>
<td>[ Y2 = 5.5 \text{ in.} ]</td>
<td>[ Y2 = 5.5 \text{ in.} ]</td>
<td>[ PNA = 7 ]</td>
</tr>
<tr>
<td>[ \Delta_{LL} = \frac{w_{LL} I^4}{1290 I_{LB}} = \frac{0.69 \text{ kip/ft}(30 \text{ ft})^4}{1290(575 \text{ in.}^4)} ]</td>
<td>[ \Delta_{LL} = \frac{w_{LL} I^4}{1290 I_{LB}} = \frac{0.69 \text{ kip/ft}(30 \text{ ft})^4}{1290(575 \text{ in.}^4)} ]</td>
<td>[ = 0.753 \text{ in.} &lt; 1.0 \text{ in.} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>

Determine the required shear stud connectors

Using perpendicular deck with one ¾-in. diameter stud per rib in normal weight, 4 ksi concrete, in weak position; \( Q_n = 17.2 \text{ kips/stud} \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Manual Table 3-21</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ \sum Q_n = 96.0 \text{ kips} ]</td>
<td>[ \sum Q_n = 96.0 \text{ kips} ]</td>
<td>[ Q_n = 17.2 \text{ kips/stud} ]</td>
</tr>
<tr>
<td>[ \frac{\sum Q_n}{Q_n} = 5.58 \text{ studs / side} ]</td>
<td>[ \frac{\sum Q_n}{Q_n} = 5.58 \text{ studs / side} ]</td>
<td>[ \text{Use 12 studs at about 3’-0” o.c.} ]</td>
</tr>
</tbody>
</table>

Note: There is a maximum spacing of 8(6 in.) = 4’-0” or 3’-0” between studs.

Therefore use 12 studs, at no more than 3’-0” on center.

Note: this beam could also be designed as a non-composite beam. Use Manual Table 3-2 with previous moments and shears:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
<th>Specification Sec. 13.2d(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select W18×35</td>
<td>Select W18×35</td>
<td></td>
</tr>
<tr>
<td>[ \phi_b M_p = 249 \text{ kip-ft} &gt; 225 \text{ kip-ft} \text{ o.k.} ]</td>
<td>[ M_p/\Omega_b = 166 \text{ kip-ft} &gt; 162 \text{ kip-ft} \text{ o.k.} ]</td>
<td>[ \phi_b V_n = 159 &gt; 30.0 \text{ kips} \text{ o.k.} ]</td>
</tr>
</tbody>
</table>
Check beam deflections

Check W18×35 with an \( I_x = 510 \text{ in.}^4 \), with only wet concrete on beams. Manual Table 3-1

Note: Because this beam is stronger than the W16×26 composite beam, no wet concrete strength checks are required.

\[
\Delta_{dc} (\text{wet concrete}) = \frac{0.65 \text{ kip}/\text{ft}(30 \text{ ft})^4}{1290(510 \text{ in.}^4)} = 0.80 \text{ in.} \quad \text{o.k.}
\]

Note: A good break point to eliminate camber is \( \frac{3}{4} \text{ in.} \). If a W18×40 (\( I_x = 612 \text{ in.}^4 \)), \( \Delta_{dc} (\text{wet concrete}) = 0.615 \text{ in.} \) was selected, no camber is required.

Per previous smaller beam calculation, the live load deflection is o.k.

Therefore selecting a W18×40 would eliminate both shear studs and cambering. The cost of the extra steel weight may be offset by the elimination of studs and cambering. Local labor and material costs should be checked in making this determination.

**SELECT TYPICAL END BEAM**

The influence area for these beams is less than 400 ft\(^2\), therefore no live load reduction can be taken.

These beams carry 5.5 ft of dead load and live load as well as a wall load.

The dead load is

\[
w = 5.5 \text{ ft}(0.075 \text{ kips/ft}) = 0.413 \text{ kip/ft}.
\]

The dead load of the wall system at the floor is

\[
\begin{align*}
w &= 7.5 \text{ ft}(0.055 \text{ kip/ft} + 6.0 \text{ ft}(0.015 \text{ kip/ft}) \\
&= 0.413 \text{ kip/ft} + 0.090 \text{ kip/ft} \\
&= 0.503 \text{ kip/ft}.
\end{align*}
\]

The total dead load is \( w = 0.916 \text{ kip/ft} \).

The live load is \( w = 5.5 \text{ ft}(0.080 \text{ kip/ft}) = 0.440 \text{ kip/ft} \).
The loading diagram is as follows

\[ w_0 = 0.916 \text{ kips/ft} \]
\[ w_L = 0.440 \text{ kips/ft} \]

Calculate the required strengths

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( W_a = 1.2(0.916 \text{ kip/ft}) + 1.6 (0.440 \text{ kip/ft}) )</td>
<td>( W_a = 0.916 \text{ kip/ft} + 0.440 \text{ kip/ft} )</td>
</tr>
<tr>
<td>= 1.80 kip/ft</td>
<td>= 1.36 kip/ft</td>
</tr>
<tr>
<td>( M_a = \frac{1.80 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
<td>( M_a = \frac{1.36 \text{ kip/ft}(22.5 \text{ ft})^2}{8} )</td>
</tr>
<tr>
<td>= 114 kip-ft</td>
<td>= 85.8 kip-ft</td>
</tr>
<tr>
<td>( R_a = \frac{22.5 \text{ ft}}{2} (1.80 \text{ kip/ft}) )</td>
<td>( R_a = \frac{22.5 \text{ ft}}{2} (1.36 \text{ kip/ft}) )</td>
</tr>
<tr>
<td>= 20.3 kips</td>
<td>= 15.3 kips</td>
</tr>
</tbody>
</table>

Because these beams are less than 25-ft long, they will be most efficient as non-composite beams. The beams at the ends of the building carry a brick spandrel panel. Because there is continuous glass underneath this spandrel, limit live load deflection to \( \frac{1}{4} \) in. total movement. In addition, per AISC Design Guide 3, Second Edition, limit deflection due to spandrel weight to \( L/480 \) or \( \frac{3}{8} \)-in. maximum.

**Calculate minimum \( I_x \) to limit live load deflection to \( \frac{1}{4} \) in.**

\[ I_{req} = \frac{0.440 \text{ kip/ft}(22.5 \text{ ft})^4}{1290(0.25 \text{ in.})} = 350 \text{ in.}^4 \text{ controls} \]

**Calculate minimum \( I_x \) to limit spandrel weight deflection to \( \frac{3}{8} \) in.**

\[ I_{req} = \frac{0.413 \text{ kip/ft}(22.5 \text{ ft})^4}{1290(0.375 \text{ in.})} = 218 \text{ in.}^4 \]

Select Beam from Manual Table 3-2

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select W16×31 with an ( I_x = 375 \text{ in.}^4 )</td>
<td>Select W16×31 with an ( I_x = 375 \text{ in.}^4 )</td>
</tr>
<tr>
<td>( \phi_M p = 203 \text{ kip-ft} \geq 114 \text{ kip-ft} )</td>
<td>( M_p/\Omega_v = 135 \text{ kip-ft} &gt; 85.8 \text{ kip-ft} )</td>
</tr>
<tr>
<td>( \phi V_a = 131 &gt; 20.3 \text{ kips} )</td>
<td>( V_a/\Omega_v = 87.3 \text{ kips} &gt; 15.3 \text{ kips} )</td>
</tr>
<tr>
<td>o.k.</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
SELECT TYPICAL SIDE BEAM

The beams along the side of the building carry the spandrel panel and glass, and dead load and live load from the intermediate floor beams. The spandrel beam is over glass below, therefore, limit the live load deflection to ¼ in. In addition, per Steel Design Guide 3, Second Edition, limit deflection due to spandrel weight to the $L/480$ or $\frac{L}{30}$ in. maximum. These beams will be part of the moment frames on the side of the building and therefore will be designed as fixed at both ends.

Establish loads

The dead load reaction from the floor beams is

$$P = 0.75 \text{ kip/ft (45 ft / 2)} = 16.9 \text{ kips}$$

The uniform dead load along the beam is

$$w = 0.50 \text{ ft (0.075 kip/ft}^2) + 0.503 \text{ kip/ft} = 0.541 \text{ kip/ft}.$$  

Select typical 30 foot composite(or non-composite) girders

Check for possible live load reduction

For edge beams with cantilevered slabs, $K_{LL} = 1$, per ASCE Table 4-2. However, it is also permissible to calculate the value of $K_{LL}$ based upon influence area. Because the cantilever dimension is small, $K_{LL}$ will be closer to 2 than 1. The calculated value of $K_{LL}$ based upon the influence area is

$$K_{LL} = 1.98.$$  

The $A_T = 30 \text{ ft} \times (22.5 \text{ ft} + 0.5 \text{ ft}) = 690 \text{ ft}^2.$$

$$L = L_o \left(0.25 + \frac{15}{K_{LL} A_T}\right) = 0.0523 \text{ kip/ft}^2 \geq 0.50 L_o = 0.040 \text{ kip/ft}^2$$

Therefore, use 0.0523 kip/ft$^2$.

The live load from the floor beams is $P = 0.523 \text{ kip/ft(45 ft / 2)} = 11.8 \text{ kips}$

The uniform live load along the beam is $w = 0.50 \text{ ft (0.0523 kip/ft}^2) = 0.025 \text{ kip/ft}.$

The loading diagram is shown below.
A summary of the moments, reactions and required moments of inertia, determined from a structural analysis of a fixed-end beam, is as follows:

*Calculate the required strengths and select the beams for the floor side beams*

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical side beam</td>
<td>Typical side beam</td>
</tr>
<tr>
<td>$R_u = 49.5$ kips</td>
<td>$R_u = 37.2$ kips</td>
</tr>
<tr>
<td>$M_u$ at ends = 313 kip-ft</td>
<td>$M_u$ at ends = 234 kip-ft</td>
</tr>
<tr>
<td>$M_u$ at ctr. = 156 kip-ft</td>
<td>$M_u$ at ctr. = 117 kip-ft</td>
</tr>
<tr>
<td>$\Delta_{u} = 0.177$ in. &lt; 1.0 in. o.k.</td>
<td>$\Delta_{u} = 0.177$ in. &lt; 1.0 in. o.k.</td>
</tr>
</tbody>
</table>

Calculate minimum $I_x$ to limit the spandrel weight deflection to $\frac{1}{8}$ in.

$$I_{req} = \frac{(0.503 \text{ kip/ft})(30 \text{ ft})^4}{6450(0.375 \text{ in.})} = 168 \text{ in.}^4$$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical side beam</td>
<td>Typical side beam</td>
</tr>
<tr>
<td>Select $W21 \times 44$ which has an available moment of 358 kip-ft and an $I_x$ of 843 in.$^4$</td>
<td>Select $W21 \times 44$ which has an available moment of 238 kip-ft and an $I_x$ of 843 in.$^4$</td>
</tr>
</tbody>
</table>

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced at 10 ft on center by the intermediate beams. Also, during concrete placement, a construction live load of 0.020 kip/ft$^2$ will be present. This load pattern and a summary of the moments and reactions is shown below.
### LRFD

<table>
<thead>
<tr>
<th>Typical side beam with wet concrete only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 21.1$ kips</td>
</tr>
<tr>
<td>$M_u$ at ends = 140 kip-ft</td>
</tr>
<tr>
<td>$M_u$ at ctr. = 70.0 kip-ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Typical side beam with wet concrete and construction load</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 25.6$ kips</td>
</tr>
<tr>
<td>$M_u$ at ends = 169 kip-ft controls</td>
</tr>
<tr>
<td>$M_u$ at ctr. = 84.5 kip-ft</td>
</tr>
</tbody>
</table>

\[ \Delta_{LL} = 0.220 \text{ in.} < 1.0 \text{ in.} \quad \text{o.k.} \]

### ASD

<table>
<thead>
<tr>
<th>Typical side beam with wet concrete only</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 15.1$ kips</td>
</tr>
<tr>
<td>$M_u$ at ends = 99.9 kip-ft</td>
</tr>
<tr>
<td>$M_u$ at ctr. = 50.0 kip-ft</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Typical side beam with wet concrete and construction load controls</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_u = 19.8$ kips</td>
</tr>
<tr>
<td>$M_u$ at ends = 131 kip-ft controls</td>
</tr>
<tr>
<td>$M_u$ at ctr. = 65.3 kip-ft</td>
</tr>
</tbody>
</table>

\[ \Delta_{LL} = 0.288 \text{ in.} < 1.0 \text{ in.} \quad \text{o.k.} \]

### LRFD

<table>
<thead>
<tr>
<th>The available strength of a W21×44 with an unbraced length of 10 ft, and a conservative value of $C_b = 1.0$ is 262 kip-ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>262 kip-ft &gt; 169 kip-ft controls</td>
</tr>
</tbody>
</table>

### ASD

<table>
<thead>
<tr>
<th>The available strength of a W21×44 with an unbraced length of 10 ft, and a conservative value of $C_b = 1.0$ is 175 kip-ft.</th>
</tr>
</thead>
<tbody>
<tr>
<td>175 kip-ft &gt; 131 kip-ft controls</td>
</tr>
</tbody>
</table>

Note: The W21×44 is adequate for strength and deflection, but may be increased in size to help with moment frame strength or drift control.
SELECT TYPICAL INTERIOR BEAM

Establish loads

The dead load reaction from the floor beams is $P = 0.75 \text{ kip/ft} (37.5 \text{ ft}) = 28.1 \text{ kips}$

Check for live load reduction due to area

For interior beams, $K_{LL} = 2$

The $A_T = 30 \text{ ft} \times 37.5 \text{ ft} = 1125 \text{ ft}^2$.

$L = L_o \left(0.25 + \frac{15}{\sqrt{K_{LL} A_T}}\right) = 0.0453 \text{ kip/ft}^2 \geq 0.50 L_o = 0.040 \text{ kip/ft}^2$

Therefore, use 0.0453 kip/ft$^2$

The live load from the floor beams is $P_L = 0.0453 \text{ kip/ft}^2 (10 \text{ ft})(37.5 \text{ ft}) = 17.0 \text{ kips}$

Note: The dead load for this beam is included in the assumed overall dead load.

A summary of the simple moments and reactions is shown below:

Calculate the total loads, moments and shears, and select the beams for the interior beams

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical interior beam $R_u = 60.9 \text{ kips}$ $M_u = 609 \text{ kip-ft}$</td>
<td>Typical interior beam $R_u = 45.1 \text{ kips}$ $M_u = 451 \text{ kip-ft}$</td>
</tr>
</tbody>
</table>

Check for beam requirements when carrying wet concrete

Note: During concrete placement, because the deck is parallel to the beam, the beam will not have continuous lateral support. It will be braced, at 10 ft on center by the intermediate beams. Also, during concrete placement, a construction live load of $0.020 \text{ kip/ft}^2$ will be present. This load pattern and a summary of the moments and reactions, and deflection requirements is shown below. Limit wet concrete deflection to 1.5 in.
**III-26**

\[ P_o = 24.4 \text{ kips} \]
\[ P_e = 7.50 \text{ kips} \]

![Beam Loading & Bracing Diagram](Braced atThird Points)

\[ 30 \text{ ft} \]

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Interior Beam with wet concrete only</td>
<td>Typical interior Beam with wet concrete only</td>
</tr>
<tr>
<td>( R_u = 34.1 ) kips</td>
<td>( R_u = 24.4 ) kips</td>
</tr>
<tr>
<td>( M_u = 341 ) kip-ft</td>
<td>( M_u = 207 ) kip-ft</td>
</tr>
</tbody>
</table>

Target \( I_x \geq 658 \text{ in.}^4 \)

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Interior Beam with wet concrete and construction load</td>
<td>Typical Interior Beam with wet concrete and construction load</td>
</tr>
<tr>
<td>( R_u = 41.3 ) kips</td>
<td>( R_u = 31.9 ) kips</td>
</tr>
<tr>
<td>( M_u ) (midspan) =413 kip-ft</td>
<td>( M_u ) (midspan) = 319 kip-ft</td>
</tr>
<tr>
<td>Select beam with an unbraced length of 10 ft and a conservative ( C_b = 1.0 )</td>
<td>Select beam with an unbraced length of 10 ft and a conservative ( C_b = 1.0 )</td>
</tr>
<tr>
<td>Select ( W^{21} \times 68 ), which has an available moment strength of 532 kip-ft</td>
<td>Select ( W^{21} \times 68 ), which has an available moment strength of 354 kip-ft</td>
</tr>
<tr>
<td>( \phi M_p = 532 ) kip-ft &gt; 413 kip-ft <strong>o.k.</strong></td>
<td>( M_p/\Omega_b = 354 ) kip-ft ≈ 319 kip-ft <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Check \( W^{21} \times 68 \) as a composite beam**

From previous calculations:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Typical Interior Beam</td>
<td>Typical Interior beam</td>
</tr>
<tr>
<td>( R_u = 60.9 ) kips</td>
<td>( R_u = 45.1 ) kips</td>
</tr>
<tr>
<td>( M_u ) (midspan) =609 kip-ft</td>
<td>( M_u ) (midspan) = 451 kip-ft</td>
</tr>
</tbody>
</table>

\( Y_2 \) (from previous calculations) = 5.5 in.

Use Manual Table 3-19, check \( W^{21} \times 68 \), using required strengths of 609 kip-ft (LRFD) and 451 kip-ft (ASD) and \( Y_2 \) value of 5.5 in.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>Select a ( W^{21} \times 68 )</td>
<td>Select a ( W^{21} \times 68 )</td>
</tr>
<tr>
<td>Where ( PNA = 7, \sum Q_n = 251 ) kips</td>
<td>Where ( PNA = 7, \sum Q_n = 251 ) kips</td>
</tr>
<tr>
<td>( \phi M_u = 847 ) kip-ft &gt; 609 kip-ft <strong>o.k.</strong></td>
<td>( M_u/\Omega_b = 564 ) kip-ft &gt; 461 kip-ft <strong>o.k.</strong></td>
</tr>
</tbody>
</table>

---

Manual Table 3-10

Manual Table 3-19
This beam has an $I_x = 1480 \text{ in.}^4 > 658 \text{ in.}^4$
Therefore, do not camber.

**Determine $b_{eff}$**

The effective width of the concrete slab is the sum of the effective widths for each side of the beam centerline which shall not exceed:

1. one-eighth of the span of the beam, center to center of supports
   \[
   \frac{30 \text{ ft}}{8} \times 2 = 7.50 \text{ ft.}
   \]

2. one-half the distance to the center line of the adjacent beam
   \[
   \left(\frac{45 \text{ ft}}{2} + \frac{30 \text{ ft}}{2}\right) = 37.5 \text{ ft.}
   \]

3. the distance to the edge of the slab
   Not applicable.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Check $a$</strong></td>
<td><strong>Check $a$</strong></td>
</tr>
<tr>
<td>$a = \frac{\sum Q_a}{0.85 f_c b}$</td>
<td>$a = \frac{\sum Q_a}{0.85 f_c b}$</td>
</tr>
<tr>
<td>251 kips</td>
<td>251 kips</td>
</tr>
<tr>
<td>$0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})$</td>
<td>$0.85(4 \text{ ksi})(7.50 \text{ ft})(12 \text{ in./ft})$</td>
</tr>
<tr>
<td>= 0.820 in. &lt; 1.0 in.</td>
<td>= 0.820 in. &lt; 1.0 in.</td>
</tr>
</tbody>
</table>

**Determine the required shear stud connectors**

Using parallel deck with one $\frac{3}{4}$-in. diameter stud in normal weight, 4 ksi concrete, in weak position; $Q_n = 21.5 \text{ kips/stud}$

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\frac{\sum Q_a}{Q_n} = \frac{251 \text{ kips}}{21.5 \text{ kips/stud}} = 11.7 \text{ studs / side}$</td>
<td>$\frac{\sum Q_a}{Q_n} = \frac{251 \text{ kips}}{21.5 \text{ kips/stud}} = 11.7 \text{ studs / side}$</td>
</tr>
<tr>
<td>Therefore, use a minimum 24 studs for horizontal shear.</td>
<td>Therefore, use a minimum 24 studs for horizontal shear.</td>
</tr>
<tr>
<td>The maximum stud spacing is 3’-0”</td>
<td>The maximum stud spacing is 3’-0”</td>
</tr>
<tr>
<td>Since the load is concentrated at ½ points, the studs are to be arranged as follows:</td>
<td>Since the load is concentrated at ½ points, the studs are to be arranged as follows:</td>
</tr>
<tr>
<td>Use 12 studs between supports and supported beams at ½ points. Between supported beams middle ¼ of span, use 4 studs to satisfy minimum spacing requirements.</td>
<td>Use 12 studs between supports and supported beams at ½ points. Between supported beams middle ¼ of span, use 4 studs to satisfy minimum spacing requirements.</td>
</tr>
<tr>
<td>Thus, 28 studs are required in a 12:4:12 arrangement.</td>
<td>Thus, 28 studs are required in a 12:4:12 arrangement.</td>
</tr>
</tbody>
</table>
Note: This W21×68 beam, with full lateral support, is very close to having sufficient available strength to support the imposed loads without composite action. Since design was completed using the available Manual tables, a PNA location 7 was selected. A more economical solution would result if a PNA lower in the beam web could have been selected. In that case, very few studs would have been required and a larger non-composite beam might be a better solution. It is important to note the significant impact of properly designing for construction.
COLUMNS MEMBER DESIGN SELECTION FOR GRAVITY COLUMNS

Estimate column loads

<table>
<thead>
<tr>
<th>Roof</th>
<th>(from previous calculations)</th>
<th></th>
<th>0.020 kips/ft²</th>
<th>0.025 kips/ft²</th>
<th>0.045 kips/ft²</th>
</tr>
</thead>
<tbody>
<tr>
<td>Dead Load</td>
<td></td>
<td></td>
<td>0.020 kips/ft²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Live (Snow)</td>
<td></td>
<td></td>
<td>0.025 kips/ft²</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total</td>
<td></td>
<td></td>
<td>0.045 kips/ft²</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Snow drifting loads at the perimeter of the roof and at the mechanical screen wall from previous calculations

Reaction to column (side parapet)

\[ w = (3.73 \text{ kips} / 6 \text{ ft}) - (0.025 \text{ ksf})(23.0 \text{ ft}) = 0.047 \text{ kips/ft} \]

Reaction to column (end parapet)

\[ w = (0.43 \text{ klf}) - (0.025 \text{ ksf})(15.5 \text{ ft}) = 0.043 \text{ kips/ft} \]

Reaction to column (screen wall)

\[ w = (0.48 \text{ klf}) - (0.025 \text{ ksf})(15.5 \text{ ft}) = 0.105 \text{ kips/ft} \]

Mechanical equipment (average)

\[ w = 0.040 \text{ kips/ft} \]
<table>
<thead>
<tr>
<th>Col Description</th>
<th>Width</th>
<th>Length</th>
<th>Area</th>
<th>DL</th>
<th>$P_D$</th>
<th>LL</th>
<th>$P_L$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>ft</td>
<td>ft</td>
<td>ft²</td>
<td>kips/ft²</td>
<td>kips</td>
<td>kips/ft²</td>
<td>kips</td>
</tr>
<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F snow drifting side exterior wall</td>
<td>23.0</td>
<td>30.0</td>
<td>690</td>
<td>0.020</td>
<td>13.8</td>
<td>0.025</td>
<td>17.3</td>
</tr>
<tr>
<td>snow drifting side exterior wall</td>
<td>30.0</td>
<td>30.0</td>
<td>0.413 klf</td>
<td>12.4</td>
<td>26.2</td>
<td>18.7</td>
<td></td>
</tr>
<tr>
<td>1B, 1E, 8B, 8E snow drifting end exterior wall</td>
<td>3.50</td>
<td>22.5</td>
<td>79</td>
<td>0.020</td>
<td>1.58</td>
<td>0.025</td>
<td>1.97</td>
</tr>
<tr>
<td>snow drifting side exterior wall</td>
<td>22.5</td>
<td>22.5</td>
<td>0.413 klf</td>
<td>9.29</td>
<td>10.9</td>
<td>2.94</td>
<td></td>
</tr>
<tr>
<td>1A, 1F, 8A, 8F snow drifting end exterior wall</td>
<td>23.0</td>
<td>15.5</td>
<td>345</td>
<td>0.020</td>
<td>6.34</td>
<td>0.025</td>
<td>7.93</td>
</tr>
<tr>
<td>snow drifting end exterior wall</td>
<td>11.8</td>
<td></td>
<td></td>
<td>0.047 klf</td>
<td>0.51</td>
<td></td>
<td></td>
</tr>
<tr>
<td>snow drifting side exterior wall</td>
<td>15.5</td>
<td></td>
<td></td>
<td>0.413 klf</td>
<td>0.73</td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior wall</td>
<td>26.3</td>
<td></td>
<td></td>
<td>0.413 klf</td>
<td>10.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1C, 1D, 8C, 8D snow-drifting end exterior wall</td>
<td>37.5</td>
<td>15.5</td>
<td>581</td>
<td>0.020</td>
<td>10.8</td>
<td>0.025</td>
<td>13.5</td>
</tr>
<tr>
<td>snow-drifting end exterior wall</td>
<td>26.3</td>
<td></td>
<td></td>
<td>0.413 klf</td>
<td>10.8</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2C, 2D, 7C, 7D</td>
<td>37.5</td>
<td>30.0</td>
<td>1,125</td>
<td>0.020</td>
<td>22.5</td>
<td>0.025</td>
<td>28.1</td>
</tr>
<tr>
<td>3C, 3D, 4C, 4D snow-drifting mechanical area</td>
<td>22.5</td>
<td>30.0</td>
<td>675</td>
<td>0.020</td>
<td>13.5</td>
<td>0.025</td>
<td>16.9</td>
</tr>
<tr>
<td>snow-drifting mechanical area</td>
<td>30.0</td>
<td></td>
<td></td>
<td>0.105 klf</td>
<td>3.15</td>
<td></td>
<td></td>
</tr>
<tr>
<td>mechanical area</td>
<td>15.0</td>
<td>30.0</td>
<td>0.060</td>
<td>0.025</td>
<td>11.3</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
Floor Loads (from previous calculations)

Dead load: 0.075 kips/ft²
Live load: 0.080 kips/ft²
Total load: 0.155 kips/ft²

Reduction in live loads, analyzed at the base of 3 floors

Columns: 2A, 2F, 3A, 3F, 4A, 4F, 5A, 5F, 6A, 6F, 7A, 7F
Exterior column without cantilever slabs

\[
K_{LL} = 4 \quad L_o = 0.080 \text{ ksf} \quad n = 3
\]

\[
A_T = (23 \text{ ft})(30 \text{ ft}) = 690 \text{ ft}^2
\]

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right)
\]

\[
= 0.080 \text{ ksf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(690 \text{ ft}^2)}} \right)
\]

\[
= 0.033 \text{ ksf} \geq 0.4L_o = 0.032 \text{ ksf}
\]

use \( L = 0.033 \text{ ksf} \)

Columns: 1B, 1E, 8B, 8E
Exterior column without cantilever slabs

\[
K_{LL} = 4 \quad L_o = 0.080 \text{ ksf} \quad n = 3
\]

\[
A_T = (5.5 \text{ ft})(22.5 \text{ ft}) = 124 \text{ ft}^2
\]

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right)
\]

\[
= 0.080 \text{ ksf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(124 \text{ ft}^2)}} \right)
\]

\[
= 0.051 \text{ ksf} \geq 0.4L_o = 0.032 \text{ ksf}
\]

use \( L = 0.051 \text{ ksf} \)
Columns: 1A, 1F, 8A, 8F
Corner column without cantilever slabs

Conservatively assume \( K_{LL} = 1 \) \( L_o = 0.080 \text{ ksf} \) \( n = 3 \)

\( A_T = (15.5 \text{ ft})(23 \text{ ft}) - (124 \text{ ft}^2 / 2) = 295 \text{ ft}^2 \)

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right) \geq 0.4L_o
\]

\[
= 0.080 \text{ ksf} \left( 0.25 + \frac{15}{\sqrt{(1)(3)(295 \text{ ft}^2)}} \right) \geq 0.4(0.08 \text{ ksf})
\]

\[
= 0.060 \text{ ksf} \geq 0.032 \text{ ksf}
\]

use \( L = 0.060 \text{ ksf} \)

Columns: 1C, 1D, 8C, 8D
Exterior column without cantilever slabs

\( K_{LL} = 4 \) \( L_o = 0.080 \text{ ksf} \) \( n = 3 \)

\( A_T = (15.5 \text{ ft})(37.5 \text{ ft}) - (124 \text{ ft}^2 / 2) = 519 \text{ ft}^2 \)

\[
L = L_o \left( 0.25 + \frac{15}{\sqrt{K_{LL} n A_T}} \right) \geq 0.4L_o
\]

\[
= 0.080 \text{ ksf} \left( 0.25 + \frac{15}{\sqrt{(4)(3)(519 \text{ ft}^2)}} \right) \geq 0.4(0.08 \text{ ksf})
\]

\[
= 0.035 \text{ ksf} \geq 0.032 \text{ ksf}
\]

use \( L = 0.035 \text{ ksf} \)
Columns: 2C, 2D, 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D, 7C, 7D
Interior column

\[ K_{LL} = 4 \quad L_o = 0.080 \text{ ksf} \quad n = 3 \]

\[ A_T = 1,125 \text{ ft}^2 \]

\[ L = L_o \left(0.25 + \frac{15}{\sqrt{K_{LL}nA_T}}\right) \geq 0.4L_o \]

\[ = 0.080 \text{ ksf} \left(0.25 + \frac{15}{\sqrt{(4)(3)(1,125 \text{ ft}^2)}}\right) \geq 0.4(0.08 \text{ ksf}) \]

\[ = 0.030 \text{ ksf} \geq 0.032 \text{ ksf} \]

use \( L = 0.032 \text{ ksf} \)
<table>
<thead>
<tr>
<th>Column</th>
<th>Width ft</th>
<th>Length ft</th>
<th>Area ft²</th>
<th>DL kips/ft²</th>
<th>P₀ kips</th>
<th>LL kips/ft²</th>
<th>P₀ kips</th>
</tr>
</thead>
<tbody>
<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F</td>
<td>23.0</td>
<td>30.0</td>
<td>690</td>
<td>0.075</td>
<td>51.8</td>
<td>0.033</td>
<td>22.9</td>
</tr>
<tr>
<td>5A, 5F, 6A, 6F, 7A, 7F</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>exterior wall</td>
<td>30.0</td>
<td></td>
<td></td>
<td>0.503 klf</td>
<td>15.1</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
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## Column load summary

<table>
<thead>
<tr>
<th>Col</th>
<th>Floor</th>
<th>$P_D$</th>
<th>$P_L$</th>
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<tbody>
<tr>
<td></td>
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<td>kips</td>
<td>kips</td>
</tr>
<tr>
<td>2A, 2F, 3A, 3F, 4A, 4F</td>
<td>Roof</td>
<td>26.2</td>
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</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>66.8</td>
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<td></td>
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<td>87.4</td>
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<tr>
<td>5A, 5F, 6A, 6F, 7A, 7F</td>
<td>Roof</td>
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<td>1B, 1E, 8B, 8E</td>
<td>Roof</td>
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<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
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<td>1A, 1F, 8A, 8F</td>
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<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
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<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>50.3</td>
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<td>Total</td>
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<td>69.5</td>
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<tr>
<td>1C, 1D, 8C, 8D</td>
<td>Roof</td>
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<td>28.1</td>
</tr>
<tr>
<td></td>
<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>84.4</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
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<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
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</tr>
<tr>
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<td>Total</td>
<td>276</td>
<td>136</td>
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<td>Roof</td>
<td>40.5</td>
<td>31.3</td>
</tr>
<tr>
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<td>4&lt;sup&gt;th&lt;/sup&gt;</td>
<td>84.4</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
<td>3&lt;sup&gt;rd&lt;/sup&gt;</td>
<td>84.4</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
<td>2&lt;sup&gt;nd&lt;/sup&gt;</td>
<td>84.4</td>
<td>36.0</td>
</tr>
<tr>
<td></td>
<td>Total</td>
<td>294</td>
<td>139</td>
</tr>
</tbody>
</table>
Selection of interior column

Columns: 3C, 3D, 4C, 4D, 5C, 5D, 6C, 6D

Elevation of second floor slab:  113.5 ft
Elevation of first floor slab:   100 ft
Column unbraced length:   13.5 ft

\[
P_u = 1.2(294 \text{ kips}) + 1.6(139 \text{ kips}) = 575 \text{ kips}
\]
\[
P_a = 294 \text{ kips} + 139 \text{ kips} = 433 \text{ kips}
\]

From the tables, enter with the effective length of 13.5 ft, and proceed across the table until reaching the lightest size that has sufficient available strength at the required unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12×65</td>
<td>W12×65</td>
</tr>
<tr>
<td>(\phi P_u = 696 \text{ kips} &gt; 575 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>(P_u/\Omega_c = 463 \text{ kips} &gt; 433 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>W14×68</td>
<td>W14×68</td>
</tr>
<tr>
<td>(\phi P_u = 655 \text{ kips} &gt; 575 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>(P_u/\Omega_c = 436 \text{ kips} &gt; 433 \text{ kips})</td>
<td>o.k.</td>
</tr>
</tbody>
</table>

Selection of interior column

Columns: 2C, 2D, 7C, 7D

Elevation of second floor slab:  113.5 ft
Elevation of first floor slab:   100.0 ft
Column unbraced length: 13.5 ft

\[
P_u = 1.2(276 \text{ kips}) + 1.6(136 \text{ kips}) = 549 \text{ kips}
\]
\[
P_a = 276 \text{ kips} + 136 \text{ kips} = 412 \text{ kips}
\]

From the tables, enter with the effective length of 13.5 ft, and proceed until reaching the lightest size that has sufficient available strength at the required unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12×65</td>
<td>W12×65</td>
</tr>
<tr>
<td>(\phi P_u = 696 \text{ kips} &gt; 549 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>(P_u/\Omega_c = 463 \text{ kips} &gt; 412 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>W14×68</td>
<td>W14×68</td>
</tr>
<tr>
<td>(\phi P_u = 655 \text{ kips} &gt; 549 \text{ kips})</td>
<td>o.k.</td>
</tr>
<tr>
<td>(P_u/\Omega_c = 436 \text{ kips} &gt; 412 \text{ kips})</td>
<td>o.k.</td>
</tr>
</tbody>
</table>
**Selection of exterior column**

Columns: 1B, 1E, 8B, 8E

- Elevation of second floor slab: 113.5 ft
- Elevation of first floor slab: 100.0 ft
- Column unbraced length: 13.5 ft

\[
P_u = 1.2(72.7 \text{ kips}) + 1.6(21.9 \text{ kips}) = 122 \text{ kips}
\]

\[
P_a = 72.7 \text{ kips} + 21.9 \text{ kips} = 95 \text{ kips}
\]

From the tables, enter with the effective length of 13.5 ft, and proceed down the sizes (across the page) until a size is found that satisfies the total load at the unbraced length.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>(P_u = 1.2(72.7 \text{ kips}) + 1.6(21.9 \text{ kips}))</td>
<td>(P_u = 72.7 \text{ kips} + 21.9 \text{ kips})</td>
</tr>
<tr>
<td>= 122 kips</td>
<td>= 95 kips</td>
</tr>
</tbody>
</table>

**Manual**

<table>
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<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12×40</td>
<td>W12×40</td>
</tr>
<tr>
<td>(\phi P_n = 316 \text{ kips} &gt; 139 \text{ kips})</td>
<td>(P_n / \Omega_c = 210 \text{ kips} &gt; 105 \text{ kips})</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
<td><strong>o.k.</strong></td>
</tr>
</tbody>
</table>

**Manual**

<table>
<thead>
<tr>
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<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>W12×40</td>
<td>W12×40</td>
</tr>
<tr>
<td>(P_n / \Omega_c = 210 \text{ kips} &gt; 105 \text{ kips})</td>
<td>-</td>
</tr>
<tr>
<td><strong>o.k.</strong></td>
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</tbody>
</table>

Note: A 12 in. column was selected above for ease of erection.
WIND LOAD DETERMINATION

Simplified wind load method (mean roof height less than 60 ft)  

To qualify for the simplified wind load method, the following must be true.

1. simple diaphragm building  \textbf{o.k.}  
2. not a flexible building  \textbf{o.k.}  
3. does not have response characteristics requiring special considerations  \textbf{o.k.}  
4. regular shape and symmetrical cross section  \textbf{o.k.}  

Define input parameters

1. Occupancy category: II  
2. Basic wind speed $V$, 90 mph  
3. Importance factor $I_w$, 1.0  
4. Exposure category: C  
5. Height and exposure adjustment $\lambda$: 1.59

\[ p_s = \lambda I_w p_{s30} = \left\{ \begin{array}{ll} (1.59)(1.0)(12.8 \text{ psf}) = 20.4 \text{ psf} & \text{Horizontal pressure zone A} \\ (1.59)(1.0)(10.2 \text{ psf}) = 13.5 \text{ psf} & \text{Horizontal pressure zone C} \\ (1.59)(1.0)(-15.4 \text{ psf}) = -24.5 \text{ psf} & \text{Vertical pressure zone E} \\ (1.59)(1.0)(-8.8 \text{ psf}) = -14.0 \text{ psf} & \text{Vertical pressure zone F} \\ (1.59)(1.0)(-10.7 \text{ psf}) = -17.0 \text{ psf} & \text{Vertical pressure zone G} \\ (1.59)(1.0)(-6.8 \text{ psf}) = -10.8 \text{ psf} & \text{Vertical pressure zone H} \end{array} \]

$a = 10\%$ of least horizontal dimension or $0.4h$, whichever is smaller, but not less than either $4\%$ of least dimension or 3 ft.

$10\%$ of least dimension: 12.0 ft  
$40\%$ of eave height: 22.0 ft  
$4\%$ of least dimension or 3 ft: 4.8 ft  
\[ a = 12.0 \text{ ft} \]  
\[ 2a = 24.0 \text{ ft} \]

Zone A - End zone of wall (width = $2a$)  
Zone C – Interior zone of wall  
Zone E – End zone of windward roof (width = $2a$)  
Zone F – End zone of leeward roof (width = $2a$)  
Zone G – Interior zone of windward roof  
Zone H – Interior zone of leeward roof
Calculate load to roof diaphragm

Assume mechanical screen wall height: 6 ft
Wall height: \( \frac{1}{2} (55.0 \text{ ft} - 40.5 \text{ ft}) = 7.25 \text{ ft} \)
Parapet wall height: 2 ft.
Total wall height at roof at screen wall: 6 ft + 7.25 ft = 13.3 ft
Total wall height at roof at parapet: 2 ft + 7.25 ft = 9.25 ft

Calculate load to 4th floor diaphragm

Wall height: \( \frac{1}{2} (55.0 \text{ ft} - 40.5 \text{ ft}) = 7.25 \text{ ft} \)
\( \frac{1}{2} (40.5 \text{ ft} - 27.0 \text{ ft}) = 6.75 \text{ ft} \)
Total wall height at floor: 6.75 ft + 7.25 ft = 14.0 ft

Calculate load to third floor diaphragm

Wall height: \( \frac{1}{2} (40.5 \text{ ft} - 27.0 \text{ ft}) = 6.75 \text{ ft} \)
\( \frac{1}{2} (27.0 \text{ ft} - 13.5 \text{ ft}) = 6.75 \text{ ft} \)
Total wall height at floor: 6.75 ft + 6.75 ft = 13.5 ft

Calculate load to second floor diaphragm

Wall height: \( \frac{1}{2} (27.0 \text{ ft} - 13.5 \text{ ft}) = 6.75 \text{ ft} \)
\( \frac{1}{2} (13.5 \text{ ft} - 0.0 \text{ ft}) = 6.75 \text{ ft} \)
Total wall height at floor: 6.75 ft + 6.75 ft = 13.5 ft
Total load to diaphragm:

Load to diaphragm: 

\[ w_{s(A)} = (20.4 \text{ psf})(9.25 \text{ ft}) = 189 \text{ plf} \]

\[ w_{s(C)} = (13.5 \text{ psf})(9.25 \text{ ft}) = 125 \text{ plf} \]

\[ w_{s(C)} = (13.5 \text{ psf})(13.3 \text{ ft}) = 179 \text{ plf} \]

Load to diaphragm: 

\[ w_{s(A)} = (20.4 \text{ psf})(14.0 \text{ ft}) = 286 \text{ plf} \]

\[ w_{s(C)} = (13.5 \text{ psf})(14.0 \text{ ft}) = 189 \text{ plf} \]

Load to diaphragm: 

\[ w_{s(A)} = (20.4 \text{ psf})(13.5 \text{ ft}) = 275 \text{ plf} \]

\[ w_{s(C)} = (13.5 \text{ psf})(13.5 \text{ ft}) = 182 \text{ plf} \]

\[ l = \text{length of structure, ft} \]

\[ b = \text{width of structure, ft} \]

\[ h = \text{height of wall at building element, ft} \]

Wind from a north – south direction

Total load to diaphragm: 

\[ P_{W(n-s)} = 2a w_{s(A)} + w_{s(C)} (l - 2a) \]

Wind from a east – west direction

Total load to diaphragm: 

\[ P_{W(e-w)} = 2a w_{s(A)} + w_{s(C)} (b - 2a) \]

\begin{tabular}{|c|c|c|c|c|c|c|c|c|c|}
\hline
\textbf{l} & \textbf{b} & \textbf{2a} & \textbf{h} & \textbf{p(A)} & \textbf{p(C)} & \textbf{w(A)} & \textbf{w(C)} & \textbf{P_{W(n-s)}} & \textbf{P_{W(e-w)}} \\
\hline
\text{Roof}  & 213 & 123 & 24 & 9.25 & 20.4 & 189 & 9.06 & 9.06 \\
& & & & 13.3 & 13.5 & 179 & 16.1 & 5.37 \\
\text{Fourth} & 213 & 123 & 24 & 14.0 & 20.4 & 13.5 & 286 & 189 & 42.6 & 25.6 \\
\text{Third} & 213 & 123 & 24 & 13.5 & 20.4 & 13.5 & 275 & 182 & 41.1 & 24.7 \\
\text{Second} & 213 & 123 & 24 & 13.5 & 20.4 & 13.5 & 275 & 182 & 41.1 & 24.7 \\
\text{Base} & & & & & & & & & \textbf{159} & \textbf{95.0} \\
\hline
\end{tabular}
SEISMIC LOAD DETERMINATION

The floor plan area: 120 ft, column center line to column center line, by 210 ft, column center line to column center line, with the edge of floor slab or roof deck 6 in. beyond the column center line.

Area = (121 ft)(211 ft) = 25,531 ft²

The perimeter cladding system length:

Length = (2)(122 ft) + (2)(212 ft) = 668 ft

The perimeter cladding weight at floors:

<table>
<thead>
<tr>
<th>Material</th>
<th>Area (ft²)</th>
<th>Weight (klf)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Brick spandrel panel with metal stud backup</td>
<td>(7.5 ft)(0.055 ksf) = 0.413 klf</td>
<td></td>
</tr>
<tr>
<td>Window wall system</td>
<td>(6.0 ft)(0.015 ksf) = 0.090 klf</td>
<td></td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>0.503 klf</strong></td>
<td></td>
</tr>
</tbody>
</table>

Typical roof dead load (from previous calculations):

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof Area</td>
<td>(25,531 ft²)(0.020 ksf) = 511 kips</td>
</tr>
<tr>
<td>Wall perimeter</td>
<td>(668 ft)(0.413 klf) = 276 kips</td>
</tr>
<tr>
<td>Mechanical equipment</td>
<td>= 32 kips</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>818 kips</strong></td>
</tr>
</tbody>
</table>

Typical third and fourth floor dead load:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor Area</td>
<td>(25,531 ft²)(0.085 ksf) = 2,170 kips</td>
</tr>
<tr>
<td>Wall perimeter</td>
<td>(668 ft)(0.503 klf) = 336 kips</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2,506 kips</strong></td>
</tr>
</tbody>
</table>

Second floor dead load: the floor area is reduced because of the open atrium

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Floor Area</td>
<td>(24,226 ft²)(0.085 ksf) = 2,059 kips</td>
</tr>
<tr>
<td>Wall perimeter</td>
<td>(668 ft)(0.503 klf) = 336 kips</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>2,395 kips</strong></td>
</tr>
</tbody>
</table>

Total dead load of the building:

<table>
<thead>
<tr>
<th>Component</th>
<th>Weight (kips)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof</td>
<td>818 kips</td>
</tr>
<tr>
<td>Fourth floor</td>
<td>2,506 kips</td>
</tr>
<tr>
<td>Third floor</td>
<td>2,506 kips</td>
</tr>
<tr>
<td>Second floor</td>
<td>2,395 kips</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>8,225 kips</strong></td>
</tr>
</tbody>
</table>
Calculate the seismic forces

Office Building        Category II
Seismic Use Group    I
Importance Factor $I_e = 1.00$

$S_S = 0.121$
$S_I = 0.06$

Soil, site class D

$F_a @ S_S \leq 0.25 = 1.6$
$F_v @ S_I \leq 0.1 = 2.4$

$S_{MS} = F_a S_S = (1.6)(0.121) = 0.194$
$S_{MI} = F_v S_I = (2.4)(0.06) = 0.144$

$S_{DS} = \gamma_S S_{MS} = 0.129$
$S_{DI} = \gamma_S S_{MI} = 0.096$

$S_{DS} \leq 0.167g$, Seismic Use Group I: Seismic Design Category: A

$0.067g \leq S_{DI} \leq 0.133g$, Seismic Use Group I: Seismic Design Category: B

Building Height, $h_n = 55$ ft

$C_r = 0.02$: $x = 0.75$

$T_a = C_r (h_n x)^{0.75} = (0.02)(55 \text{ ft})^{0.75} = 0.404$

$0.8 T_s = 0.8 (S_{DI} / S_{DS}) = 0.8(0.129 / 0.096) = 0.595$

Since $T_a < 0.8 T_s$: Seismic Design Category A may be used and it is therefore permissible to select a structural steel system not specifically detailed for seismic resistance, for which the Seismic Response Modification Coefficient $R = 3$

Overstrength Factor: $\Omega_o = 3$

Seismic Base Shear: $V = C_s W$

$$C_s = \frac{S_{DS}}{R / I_e} = \frac{0.129}{0.3 / 1} = 0.043 \quad \text{Controls}$$
III-43

\[ C_s = \frac{S_{DI}}{R/I_E} \leq 0.096 \left( \frac{S_{D1}}{R} \right) \left( \frac{I_E}{I_1} \right) \leq 0.079 \]

\[ \text{Min } C_s = 0.44 \cdot S_{D5IE} = 0.44(0.129)(1.0) = 0.006 \]

\[ V = 0.043(8225 \text{ kips}) = 354 \text{ kips} \]

**Calculate vertical distribution of seismic forces**

\[ F_x = C_{vx}V = C_{vx}(0.043)W \]

\[ C_{vx} = \frac{w_i h_x^k}{\sum w_i h_x^k} \]

for structures having a period of 0.5 sec or less, \( k = 1 \)

**Calculate horizontal shear distribution and torsion**

\[ V_x = \sum F_i \]

**Calculate Overturning Moment**

\[ M_x = \sum F_i (h_i - h_x) \]

<table>
<thead>
<tr>
<th>( w_x ) (kips)</th>
<th>( h_x ) (ft)</th>
<th>( w_x h_x^k ) (kip-ft)</th>
<th>( C_{vx} )</th>
<th>( F_x ) (kips)</th>
<th>( V_x ) (kips)</th>
<th>( M_x ) (kip-ft)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Roof 818</td>
<td>55.0</td>
<td>45,000</td>
<td>0.18</td>
<td>64.6</td>
<td>64.6</td>
<td></td>
</tr>
<tr>
<td>Fourth 2,506</td>
<td>40.5</td>
<td>101,500</td>
<td>0.41</td>
<td>146</td>
<td>210</td>
<td>936</td>
</tr>
<tr>
<td>Third 2,506</td>
<td>27.0</td>
<td>67,700</td>
<td>0.27</td>
<td>97.1</td>
<td>307</td>
<td>3,774</td>
</tr>
<tr>
<td>Second 2,395</td>
<td>13.5</td>
<td>32,300</td>
<td>0.13</td>
<td>46.4</td>
<td>354</td>
<td>7,922</td>
</tr>
<tr>
<td>Base 8,225</td>
<td>13.5</td>
<td>246,500</td>
<td>0.13</td>
<td>46.4</td>
<td>354</td>
<td>12,696</td>
</tr>
</tbody>
</table>

**Determine rigidity of diaphragms**

**Roof**

Roof deck: 1½ in. deep, 22 gage, Type B, support fasteners; ½ in. puddle welds and sidelap fasteners; #10 TEK screws

Joist spacing = 6 ft
Diaphragm length = 210 ft
Diaphragm width = 120 ft

Load to diaphragm = \( \frac{(64.6 \text{ kips})}{(210 \text{ ft})} = 0.308 \text{ klf} \)

Diaphragm shear load = \( \frac{(308 \text{ plf})(210 \text{ ft})}{2(120 \text{ ft})} = 269 \text{ plf} \)
Fastener layout = 36 / 5; sidelap = 3 #10 TEK  

\[ DB = 758 \quad K1 = 0.287 \quad K2 = 870 \quad \text{Shear strength} = 298 \text{ plf} \]

\[ G' = \frac{K2}{3.78 + \frac{0.3DB}{\text{span}} + 3K1 \text{ span}} = \frac{870}{3.78 + \frac{0.3(758)}{6} + 3(0.287)(6)} = 18.6 \text{ k/in} \]

\[ \Delta = \frac{wL^2}{8BG'} = \left( \frac{0.308 \text{ klf}}{210 \text{ ft}} \right)^2 = 0.761 \text{ in} \]

Allowable story drift: 0.025 \( h_{sx} = 0.025(14.5 \text{ ft}) = 0.363 \text{ in.} \)  

Diaphragm, rigid. Lateral deformation \( \leq 2 \) (story drift)  

\[ \Delta = 0.761 \text{ in.} > 2(0.363 \text{ in.}) = 0.725 \text{ in.} \]

Therefore, the roof diaphragm is flexible, but using a rigid diaphragm distribution is more conservative.

**Fourth floor**

Floor deck: 3 in. deep, 22 gage, Composite deck with normal weight concrete, support fasteners; ½ in. puddle welds and button punched sidelap fasteners  
Beam spacing = 10 ft  
Diaphragm length = 210 ft  
Diaphragm width = 120 ft  
Load to diaphragm = (146 kips) / (210 ft) = 0.695 klf  
Diaphragm shear load = \( \frac{(695 \text{ plf})(210 \text{ ft})}{2(120 \text{ ft})} = 608 \text{ plf} \)

Fastener layout = 36 / 4; 1 button punched sidelap fastener  

\[ K1 = 0.178 \quad K2 = 870 \quad K3 = 2,380 \quad \text{Shear strength} = 1,588 \text{ plf} \]

\[ G' = \left( \frac{K2}{3.590 + 3K1 \text{ span}} \right) + K3 = \left( \frac{870}{3.50 + 3(0.729)(6)} \right) + 2,380 = 2,533 \text{ k/in} \]

\[ \Delta = \frac{wL^2}{8BG'} = \left( \frac{0.695 \text{ klf}}{210 \text{ ft}} \right)^2 = 0.0126 \text{ in} \]
Allowable story drift; \(0.025 \times h_{xx} = 0.025(13.5 \text{ ft}) = 0.338 \text{ in.}\)  

Diaphragm, rigid. Lateral deformation \(\leq 2\) (story drift)  

\[ \Delta = 0.0126 \text{ in.} \leq 2(0.338 \text{ in.}) = 0.675 \text{ in.} \]

Therefore, the floor diaphragm is rigid

\textit{Second floor}

Floor deck: 3 in. deep, 22 gage, Composite deck with normal weight concrete,  
support fasteners: \(\frac{3}{8}\) in. puddle welds and button punched sidelap fasteners  
Beam spacing = 10 ft  
Diaphragm length = 210 ft  
Diaphragm width = 120 ft  
Because of the open atrium in the floor diaphragm, an effective diaphragm depth of 75 ft, will be used for the deflection calculations

Load to diaphragm = \((46 \text{ kips}) / (210 \text{ ft}) = 0.221 \text{ klf}\)

Diaphragm shear load = \(\frac{221 \text{ plf}(210 \text{ ft})}{2(120 \text{ ft})} = 193 \text{ plf}\)

Fastener layout = 36 / 4; 1 button punched sidelap fastener  
\(K_1 = 0.729 \quad K_2 = 870 \quad K_3 = 2,380\)  
Shear strength = 1,588 plf  

Steel Deck Institute  
Diaphragm Design Manual

\[ G' = \frac{K_2}{3.590 + 3K_1 \text{ span}} + K_3 = \frac{870}{3.50 + 3(0.729)(6)} + 2,380 = 2,533 \text{ k/in} \]

\[ \Delta = \frac{wL^2}{8BG'} = \frac{(0.221 \text{ klf})(210 \text{ ft})^2}{8(75 \text{ ft})(2,533 \text{ k/in})} = 0.00641 \text{ in.} \]

Allowable story drift; \(0.025 \times h_{xx} = 0.025(13.5 \text{ ft}) = 0.338 \text{ in.}\)  

Diaphragm, rigid. Lateral deformation \(\leq 2\) (story drift)  

\[ \Delta = 0.00641 \text{ in.} \leq 2(0.338 \text{ in.}) = 0.675 \text{ in.} \]

Therefore, the floor diaphragm is rigid
**Horizontal shear distribution and torsion:**

**Load to Grids 1 and 8**

<table>
<thead>
<tr>
<th></th>
<th>Load to Frame</th>
<th>Accidental Torsion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>%</td>
<td>kips</td>
</tr>
<tr>
<td>Roof</td>
<td>64.6</td>
<td>50</td>
<td>32.3</td>
</tr>
<tr>
<td>Fourth</td>
<td>146</td>
<td>50</td>
<td>73.0</td>
</tr>
<tr>
<td>Third</td>
<td>97.1</td>
<td>50</td>
<td>48.6</td>
</tr>
<tr>
<td>Second</td>
<td>46.4</td>
<td>50</td>
<td>23.2</td>
</tr>
<tr>
<td>Base</td>
<td>195</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Load to Grids A and F**

<table>
<thead>
<tr>
<th></th>
<th>Load to Frame</th>
<th>Accidental Torsion</th>
<th>Total</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>kips</td>
<td>%</td>
<td>kips</td>
</tr>
<tr>
<td>Roof</td>
<td>64.6</td>
<td>50</td>
<td>32.3</td>
</tr>
<tr>
<td>Fourth</td>
<td>146</td>
<td>50</td>
<td>73.0</td>
</tr>
<tr>
<td>Third</td>
<td>97.1</td>
<td>50</td>
<td>48.6</td>
</tr>
<tr>
<td>Second</td>
<td>46.4</td>
<td>50</td>
<td>24.0</td>
</tr>
<tr>
<td>Base</td>
<td>196</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: In this example, Grids A and F have both been conservatively designed for the slightly higher load on Grid A due to the atrium opening.

(1) At the Second Floor slab, because of the atrium opening, the center of mass is not at the center of rigidity.

<table>
<thead>
<tr>
<th>Area</th>
<th>Mass</th>
<th>x-dist</th>
<th>Mx</th>
<th>y-dist</th>
<th>My</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>10,950</td>
<td>931</td>
<td>45.25</td>
<td>42,118</td>
<td>60.5</td>
</tr>
<tr>
<td>II</td>
<td>2,265</td>
<td>193</td>
<td>105.5</td>
<td>20,311</td>
<td>37.75</td>
</tr>
<tr>
<td>III</td>
<td>10,950</td>
<td>931</td>
<td>165.8</td>
<td>154,279</td>
<td>60.5</td>
</tr>
</tbody>
</table>

\[
x = \frac{216,709 \text{ k-ft}}{2,054 \text{ kips}} = 105.5 \text{ ft}
\]

\[
105.5 \text{ ft} / 211 \text{ ft} = 50\%
\]

\[
y = \frac{119,894 \text{ k-ft}}{2,054 \text{ kips}} = 58.37 \text{ ft}
\]

\[
(121 \text{ ft} - 58.37 \text{ ft}) / 121 \text{ ft} = 51.8\%
\]
MOMENT FRAME MODEL

Grids 1 and 8 were modeled in conventional structural analysis software as two-dimensional models. The second-order option in the structural analysis program was not used. Rather, for illustration purposes, second-order effects are calculated separately, using the $B_1-B_2$ approximation method given in Specification Section C2.1b.

The column and beam layouts for the moment frames follow. Although the frames on Grids A and F are the same, slightly heavier seismic loads accumulate on grid F, after accounting for the atrium area on Grid A and accidental torsion. The models are half-building models. The frame was originally modeled with W14×82 interior columns and W21×44 non-composite beams. This model had a drift that substantially exceeded the 0.025$h_s$ story drift allowed per IBC Table 1617.3.1, Seismic Use Group I. The column size was incremented up to a W14×99 and the W21×44 beams were upsized to W24×55 (with minimum composite studs) and the beams were modeled with a stiffness of $I_{eq} = I_s$. Alternatively, the beams could be modeled as $I_{eq} = 0.6I_{lb} + 0.4I_s$ (Formula 24). This equation is given in AISC Design Guide 8. These changes resulted in a drift that satisfied the $L / 400$ limit. This layout is shown in the Grid A and F, Frame (½ building) layout that follows.

All of the vertical loads on the frame were modeled as point loads on the frame. As noted in the description of the dead load, $W$, in IBC Section 1617.5, 0.010 kip/ft² is included in the dead load combinations. The remainder of the half-building model gravity loads were accumulated in the leaning column, which was connected to the frame portion of the model with pinned ended links. See Geschwindner, AISC Engineering Journal, Fourth Quarter 1994, A Practical Approach to the “Leaning” Column. The dead load and live load are shown in the load cases that follow. The wind and the seismic loads are modeled and distributed 1/14 to exterior columns and 1/7 to the interior columns. This approach minimizes the tendency to accumulate too much load in the lateral system nearest an externally applied load.

There are four horizontal load cases. Two are the wind load and seismic load, per the previous discussion. In addition, notional loads of $N_i = 0.002Y_i$ were established. These load cases are shown in the load cases that follow.

The same modeling procedures were used in the braced frame analysis. If column bases are not fixed in construction, they should not be fixed in the analysis.
The model layout, dead loads, live loads, wind loads, seismic loads, dead load notional loads, and live load notional loads for the moment frame are given as follows:
FRAME (1/2 BLDG)

SNOW LOADS

FRAME (1/2 BLDG)

LIVE LOADS
FRAME (1/2 BLDG)

10 lbs/SQ FT SEISMIC PARTITION LOADS

FRAME (1/2 BLDG)

WIND LOADS
FRAME (1/2 BLDG)

SEISMIC LOADS

FRAME (1/2 BLDG)

NOTIONAL DL LOADS
CALCULATION OF REQUIRED STRENGTH - THREE METHODS

Three methods of determining the required strength including second-order effects are included below. A fourth method – second-order analysis by amplified first-order analysis – is found in Section C2.2b or the Specification. The method requires the inclusion of notional loads in the analysis, but all required strengths can be determined from a first-order analysis. For guidance on applying these methods, see the discussion in Manual Part 2 titled Required Strength, Effective Length, and Second-Order Effects.

GENERAL INFORMATION FOR ALL THREE METHODS

Seismic load dominates over winds loads in the moment frame direction of this example building. Although the frame analysis that follows was run for all LRFD and ASD load combinations, the column unity design check is highly dependent on the moment portion, and therefore, the controlling equations are those with the load combinations shown below. A typical column near the middle of the frame is analyzed, but the first interior columns and the end columns were also checked. Beam analysis is covered after the three different methods are shown for the typical interior column.

Note: The second-order analysis and the unity checks are based on the moments and column loads for the particular load combination being checked, and not on the envelope of maximum values.

METHOD 1. EFFECTIVE LENGTH METHOD

This method accounts for second-order effects in frames by amplifying the axial forces and moments in members and connections from a first-order analysis.

A first-order frame analysis is run using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for any load case for which the lateral load is not already greater. The general load combinations are in ASCE 7 and are summarized in Part 2 of the Manual.

A summary of the axial loads, moments and 1st floor drifts from the first-order computer analysis is shown below:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2D ± 1.0E + 0.5L + 0.2S</td>
<td>D ± (W or 0.7 E) (Controls for columns)</td>
</tr>
<tr>
<td>(Controls for columns and beams)</td>
<td>D + 0.75(W or 0.7E) + 0.75L + 0.75(L or S or R) (Controls for beams)</td>
</tr>
<tr>
<td>For Interior Column Design:</td>
<td>For Interior Column Design:</td>
</tr>
<tr>
<td>$P_n = 335$ kips</td>
<td>$P_n = 247$ kips</td>
</tr>
<tr>
<td>$M_{1n} = 157$ kip-ft (from first-order analysis)</td>
<td>$M_{1n} = 110$ kip-ft (from first-order analysis)</td>
</tr>
<tr>
<td>$M_{2n} = 229$ kip-ft (from first-order analysis)</td>
<td>$M_{2n} = 161$ kip-ft (from first-order analysis)</td>
</tr>
<tr>
<td>First-order first floor drift = 0.562 in.</td>
<td>First-order first floor drift = 0.394 in.</td>
</tr>
</tbody>
</table>

The required second-order flexural strength, $M_r$, and axial strength, $P_r$, are as follows:
For typical interior columns the gravity-load moments are approximately balanced, therefore, $M_{nr} = 0.0$ kip-ft
Determine \(B_1\)

\[P_r = \text{required second-order axial strength using LRFD or ASD load combinations, kips.}\]

Note that for members in axial compression \(P_r\) may be taken as \(P_r = P_{nt} + P_{lt}\). For a long frame, such as this one, the change in load to the interior columns, associated with lateral load is negligible.

Therefore, \(P_r = 335 \text{ kips}\) (from previous calculations) and

\[
P_{el} = \frac{\pi^2 EI}{(K_iL)^2} = \frac{\pi(29,000 \text{ ksi})(1,110 \text{ in}^4)}{(1.0)((13.5 \text{ ft})(12 \text{ in/ft}))^2} = 12,100 \text{ kips}
\]

\[C_m = 0.6 - 0.4(M_1 / M_2) = 0.6 - 0.4 (157 \text{ kip-ft} / 229 \text{ kip-ft}) = 0.326\]

\[\alpha = 1.0,\]

\[B_1 = \frac{C_m}{1 - \left(\frac{\alpha P_r}{P_{el}}\right)} = \frac{0.326}{1 - \left(\frac{(1.0)(335 \text{ kips})}{12,100 \text{ kips}}\right)} = 0.335 \geq 1; \text{ Use 1.0}\]

\[M_r = B_1 M_{nt} + B_2 M_{lt}\]

Determine \(B_1\)

\[P_r = \text{required second-order axial strength using LRFD or ASD load combinations, kips.}\]

Note that for members in axial compression \(P_r\) may be taken as \(P_r = P_{nt} + P_{lt}\). For a long frame, such as this one, the change in load to the interior columns, associated with lateral load is negligible.

Therefore, \(P_r = 247 \text{ kips}\) (from previous calculations) and

\[
P_{el} = \frac{\pi^2 EI}{(K_iL)^2} = \frac{\pi(29,000 \text{ ksi})(1,110 \text{ in}^4)}{(1.0)((13.5 \text{ ft})(12 \text{ in/ft}))^2} = 12,100 \text{ kips}
\]

\[C_m = 0.6 - 0.4(M_1 / M_2) = 0.6 - 0.4 (110 \text{ kip-ft} / 161 \text{ kip-ft}) = 0.326\]

\[\alpha = 1.6,\]

\[B_1 = \frac{C_m}{1 - \left(\frac{\alpha P_r}{P_{el}}\right)} = \frac{0.326}{1 - \left(\frac{(1.6)(247 \text{ kips})}{12,100 \text{ kips}}\right)} = 0.337 \geq 1; \text{ Use 1.0}\]
**Calculate $B_2$**

\[
B_2 = \frac{1}{1 - \left( \frac{\alpha \sum P_{\alpha}}{\sum P_{e2}} \right)}
\]

where:

\[
\alpha = 1.0,
\]

\[
\sum P_{\alpha} = 5,250 \text{ kips (from computer output)}
\]

and

\[
\sum P_{e2} \text{ may be taken as } = R_M \frac{HL}{\Delta_H}
\]

where $R_M$ is taken as 0.85 for moment frames

\[
\sum H = 1.2D + 1.0E + 0.5L + 0.2S
\]

= 195 kips (Horizontal)

(from previous seismic force distribution calculations)

\[
\Delta_H = 0.562 \text{ in. (from computer output)}
\]

\[
\sum P_{e2} = 0.85 \left( \frac{195 \text{ kips}}{0.562 \text{ in.}} \right)(13.5 \text{ ft})(12 \text{ in/ft}) = 47,800 \text{ kips}
\]

\[
B_2 = \frac{1}{1 - \left( \frac{\alpha \sum P_{\alpha}}{\sum P_{e2}} \right)}
\]

\[
= \frac{1}{1 - \left( \frac{1.0}{47,800 \text{ kips}} \right)(5,250 \text{ kips})}
\]

\[
= 1.12 \geq 1
\]

**Calculate amplified moment**

\[
= (1.0)(0.0 \text{ kip-ft}) + (1.12)(229 \text{ kip-ft})
\]

\[
= 256 \text{ kip-ft}
\]

**Calculate amplified axial load**

\[
P_r = 335 \text{ kips}
\]

(from computer analysis)

---

**Calculate $B_2$**

\[
B_2 = \frac{1}{1 - \left( \frac{\alpha \sum P_{\alpha}}{\sum P_{e2}} \right)}
\]

where:

\[
\alpha = 1.6,
\]

\[
\sum P_{\alpha} = 3,750 \text{ kips (from computer output)}
\]

and

\[
\sum P_{e2} \text{ may be taken as } = R_M \frac{HL}{\Delta_H}
\]

where $R_M$ is taken as 0.85 for moment frames

\[
\sum H = D + 0.7E
\]

= 137 kips (Horizontal)

(from previous seismic force distribution calculations)

\[
\Delta_H = 0.394 \text{ in. (from computer output)}
\]

\[
\sum P_{e2} = 0.85 \left( \frac{137 \text{ kips}}{0.394 \text{ in.}} \right)(13.5 \text{ ft})(12 \text{ in/ft}) = 47,900 \text{ kips}
\]

\[
B_2 = \frac{1}{1 - \left( \frac{\alpha \sum P_{\alpha}}{\sum P_{e2}} \right)}
\]

\[
= \frac{1}{1 - \left( \frac{1.6}{47,900 \text{ kips}} \right)(3,750 \text{ kips})}
\]

\[
= 1.14 \geq 1
\]

**Calculate amplified moment**

\[
= (1.0)(0.0 \text{ kip-ft}) + (1.14)(161 \text{ kip-ft})
\]

\[
= 184 \text{ kip-ft}
\]

**Calculate amplified axial load**

\[
P_r = 247 \text{ kips}
\]

(from computer analysis)
\[
P_r = P_w + B_2 P_l
\]
\[
= 335 \text{ kips} + (1.12)(0.0 \text{ kips})
\]
\[
= 335 \text{ kips}
\]

**Determine the controlling effective length**

For out-of-plane buckling in the braced frame

\[K_y = 1.0\]

For in-plane buckling in the moment frame, use the nomograph

\[K_x = 1.43\]

To account for leaning columns in the controlling load case

For leaning columns,

\[
\sum Q = 3010 \text{ kips}
\]
\[
\sum P = 2490 \text{ kips}
\]

\[
K = K_y \sqrt{1 + \frac{\sum Q}{\sum P}}
\]
\[
= 1.43 \sqrt{1 + \frac{3010}{2490}} = 2.13
\]

\[
\frac{K L_x}{r_x / r_y} = 2.13(13.5) = 28.29
\]

\[
\frac{K L_x}{r_x / r_y} = 17.3
\]

\[
\frac{K L_x}{r_x / r_y} = 2.13(13.5) = 28.29
\]

\[
\frac{K L_x}{r_x / r_y} = 17.3
\]
\[
\begin{align*}
P_c &= 1,041 \text{ kips} \quad (W14 \times 99 \ @ \ KL = 17.3 \text{ ft}) \\
\frac{P}{P_c} &= \frac{335 \text{ kips}}{1,041 \text{ kips}} = 0.321 \geq 0.2 \\
M_{cs} &= 646 \text{ kip-ft} \quad (W14 \times 99) \\
\frac{P}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cs}}{M_{cs}} + \frac{M_{cs}}{M_{cy}} \right) &= 0.321 + \left( \frac{8}{9} \right) \left( \frac{256 \text{ kip-ft}}{646 \text{ kip-ft}} \right) \\
&= 0.673 \leq 1.0 \quad \text{o.k.}
\end{align*}
\]

\[
\begin{align*}
P_c &= 692 \text{ kips} \quad (W14 \times 99 \ @ \ KL = 17.4 \text{ ft}) \\
\frac{P}{P_c} &= \frac{247 \text{ kips}}{692 \text{ kips}} = 0.357 \geq 0.2 \\
M_{cs} &= 430 \text{ kip-ft} \quad (W14 \times 99) \\
\frac{P}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cs}}{M_{cs}} + \frac{M_{cs}}{M_{cy}} \right) &= 0.357 + \left( \frac{8}{9} \right) \left( \frac{184 \text{ kip-ft}}{430 \text{ kip-ft}} \right) \\
&= 0.737 \leq 1.0 \quad \text{o.k.}
\end{align*}
\]
**METHOD 2. SIMPLIFIED DETERMINATION OF REQUIRED STRENGTH**

A method of second-order analysis based upon drift limits and other assumptions is described in Chapter 2 of the Manual. A first-order frame analysis is run using the load combinations for LRFD or ASD. A minimum lateral load (notional load) equal to 0.2% of the gravity loads is included for any load case for which the lateral load is not already greater.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2D ± 1.0E + 0.5L + 0.2S (Controls</td>
<td>D ± (W or 0.7 E) (Controls columns)</td>
</tr>
<tr>
<td>columns and beams)</td>
<td></td>
</tr>
<tr>
<td></td>
<td>D ± 0.75(W or 0.7E) + 0.75L + 0.75(Lc or</td>
</tr>
<tr>
<td></td>
<td>S or R)</td>
</tr>
<tr>
<td>For a first-order analysis</td>
<td></td>
</tr>
<tr>
<td>For Interior Column Design:</td>
<td>For a first-order analysis</td>
</tr>
<tr>
<td>( P_a = 335 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>( M_{1a} = 157 \text{ kip-ft} ) (from</td>
<td>( M_a = 247 \text{ kips}</td>
</tr>
<tr>
<td>first-order analysis)</td>
<td></td>
</tr>
<tr>
<td>( M_{2a} = 229 \text{ kip-ft} ) (from</td>
<td>( M_{1a} = 110 \text{ kip-ft} ) (from</td>
</tr>
<tr>
<td>first-order analysis)</td>
<td>first-order analysis)</td>
</tr>
<tr>
<td>First-floor first-order drift = 0.562 in.</td>
<td>First-floor first-order drift = 0.394 in.</td>
</tr>
</tbody>
</table>

Then the following steps are executed.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Step 1:</strong></td>
<td><strong>Step 1:</strong></td>
</tr>
<tr>
<td>Lateral load = 195 kips</td>
<td>Lateral load = 140 kips</td>
</tr>
<tr>
<td>Deflection due to first-order elastic</td>
<td>Deflection due to first-order elastic</td>
</tr>
<tr>
<td>analysis</td>
<td>analysis</td>
</tr>
<tr>
<td>( \Delta = 0.562 \text{ in. between first and second floor} )</td>
<td>( \Delta = 0.394 \text{ in. between first and second floor} )</td>
</tr>
<tr>
<td>Floor height = 13.5 ft</td>
<td>Floor height = 13.5 ft</td>
</tr>
<tr>
<td>Drift ratio = (13.5 ft)(12 in/ft) / 0.562 in</td>
<td>Drift ratio = (13.5 ft)(12 in/ft) / 0.394 in</td>
</tr>
<tr>
<td>( = 288 )</td>
<td>( = 411 )</td>
</tr>
<tr>
<td><strong>Step 2:</strong></td>
<td><strong>Step 2:</strong></td>
</tr>
<tr>
<td>Design story drift limit = 0.025 ( h_{ss} )</td>
<td>Design story drift limit = 0.025 ( h_{ss} )</td>
</tr>
<tr>
<td>( = h/400 )</td>
<td>( = h/400 )</td>
</tr>
<tr>
<td>Adjusted Lateral load = (288/400)(195 kips)</td>
<td>Adjusted Lateral load = (411/400)(137 kips)</td>
</tr>
<tr>
<td>( = 141 \text{ kips} )</td>
<td>( = 141 \text{ kips} )</td>
</tr>
</tbody>
</table>

IBC Table 1617.3.1
Step 3: Load ratio = \( \frac{1.0 \times \text{total story load}}{\text{lateral load}} \)

\[ = (1.0) \frac{5,250 \text{ kips}}{141 \text{ kips}} \]

\[ = 37.2 \]

Interpolating from the table: \( B_2 = 1.1 \)
Which matches the value obtained in the first method to the 2 significant figures of the table

Step 3: (for an ASD design the ratio must be factored by 1.6)
Load ratio = \( \frac{1.6 \times \text{total story load}}{\text{lateral load}} \)

\[ = (1.6) \frac{3,750 \text{ kips}}{141 \text{ kips}} \]

\[ = 42.6 \]
Interpolating from the table: \( B_2 = 1.1 \)
Which matches the value obtained in the first method to the 2 significant figures of the table

Note: Because the table is intentionally based on two significant figures, this value is taken as 1.1 rather than an interpolated value > 1.1. This convenient selection is within the accuracy of the method. Since the selection is in the shaded area of the chart, \( K = 1.0 \).

Step 4. Multiply all the forces and moment from the first-order analysis by the value obtained from the table.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>[ M_r = B_2(M_{nt} + M_{lt}) ]</td>
<td>[ M_r = B_2(M_{nt} + M_{lt}) ]</td>
</tr>
<tr>
<td>[ = 1.1(0 \text{ kip-ft} + 229 \text{ kip-ft}) = 252 \text{ kip-ft} ]</td>
<td>[ = 1.1(0 \text{ kip-ft} + 161 \text{ kip-ft}) = 177 \text{ kip-ft} ]</td>
</tr>
<tr>
<td>[ P_r = 1.1(P_{nt} + P_{lt}) ]</td>
<td>[ P_r = 1.1(P_{nt} + P_{lt}) ]</td>
</tr>
<tr>
<td>[ = 1.1(335 \text{ kips} + 0.0 \text{ kips}) = 368 \text{ kips} ]</td>
<td>[ = 1.1(247 \text{ kips} + 0.0 \text{ kips}) = 272 \text{ kips} ]</td>
</tr>
</tbody>
</table>

For \( \frac{P_r}{P_c} = \frac{368 \text{ kips}}{1,140 \text{ kips}} = 0.323 \geq 0.2 \)

For \( \frac{P_r}{P_c} = \frac{272 \text{ kips}}{759 \text{ kips}} = 0.358 \geq 0.2 \)

where \( P_c = 1,140 \text{ kips} \) (W14 × 99 @ KL = 13.5 ft)

\[ M_{cx} = 646 \text{ kip-ft} \) (W14 × 99)

\[ \frac{P_r}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cx}}{M_{cy}} + \frac{M_{cy}}{M_{cx}} \right) \]

\[ = 0.323 + \left( \frac{8}{9} \right) \left( \frac{252 \text{ kip-ft}}{646 \text{ kip-ft}} \right) \]

\[ = 0.670 \leq 1.0 \quad \text{o.k.} \]

\[ \frac{P_r}{P_c} + \left( \frac{8}{9} \right) \left( \frac{M_{cx}}{M_{cy}} + \frac{M_{cy}}{M_{cx}} \right) \]

\[ = 0.358 + \left( \frac{8}{9} \right) \left( \frac{177 \text{ kip-ft}}{430 \text{ kip-ft}} \right) \]

\[ = 0.724 \leq 1.0 \quad \text{o.k.} \]
METHOD 3. DIRECT ANALYSIS METHOD

Second-order analysis by the direct analysis method is found in Appendix 7 of the Specification. This method requires that both the flexural stiffness and axial stiffness be reduced and that 0.2% notional lateral loads be applied in the analysis. The combination of these two modifications account for the second-order effects and the results for design can be taken directly from the analysis. A summary of the axial loads, moments and 1st floor drifts from first-order analysis is shown below:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.2D ± 1.0E + 0.5L + 0.2S (Controls columns and beams)</td>
<td>D ± (W or 0.7 E) (Controls columns)</td>
</tr>
<tr>
<td>For a 1st order analysis with notional loads and reduced stiffness:</td>
<td>For a 1st order analysis with notional loads and with reduced stiffness:</td>
</tr>
<tr>
<td>For Interior Column Design:</td>
<td>For Interior Column Design:</td>
</tr>
<tr>
<td>$P_u = 335$ kips</td>
<td>$P_a = 247$ kips</td>
</tr>
<tr>
<td>$M_{1u} = 157$ kip-ft (from first-order analysis)</td>
<td>$M_{1a} = 110$ kip-ft</td>
</tr>
<tr>
<td>$M_{2u} = 229$ kip-ft (from first-order analysis)</td>
<td>$M_{2a} = 161$ kip-ft</td>
</tr>
<tr>
<td>First-floor drift due to reduced stiffnesses = 0.703 in.</td>
<td>First-floor drift due to reduced stiffnesses = 0.493 in.</td>
</tr>
</tbody>
</table>

Note: For ASD, this method requires multiplying the ASD load combinations by a factor of 1.6 in analyzing the drift of the structure, and then dividing the results by 1.6 to obtain the required strengths. The ASD forces shown above include the multiplier of 1.6.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_v = 1,140$ kips (W14x99 @ $KL = 13.5$ ft)</td>
<td>$P_v = 759$ kips (W14x99 @ $KL = 13.5$ ft)</td>
</tr>
<tr>
<td>$P_v = 335$ kips $P_v = 1,140$ kips $= 0.294 \geq 0.2$</td>
<td>$P_v = 247$ kips $P_v = 759$ kips $= 0.325 \geq 0.2$</td>
</tr>
<tr>
<td>$M_v = 646$ kip-ft (W14x99)</td>
<td>$M_v = 430$ kip-ft (W14x99)</td>
</tr>
<tr>
<td>$M_r = B_1M_{1u} + B_2M_{2u}$</td>
<td>$M_r = B_1M_{1a} + B_2M_{2a}$</td>
</tr>
<tr>
<td>Determine $B_1$</td>
<td>Determine $B_1$</td>
</tr>
<tr>
<td>$P_r = $ required second-order axial strength using LRFD or ASD load combinations, kips.</td>
<td>$P_r = $ required second-order axial strength using LRFD or ASD load combinations, kips.</td>
</tr>
<tr>
<td>Note that for members in axial compression $P_r$ may be taken as $P_r = P_{n} + P_{l}$. For a long frame, such as this one, the change in load to the interior columns, associated with lateral load is negligible.</td>
<td>Note that for members in axial compression $P_r$ may be taken as $P_r = P_{n} + P_{l}$. For a long frame, such as this one, the change in load to the interior columns, associated with lateral load is negligible.</td>
</tr>
<tr>
<td>Therefore, $P_r = 335$ kips (from previous calculations) and</td>
<td>Therefore, $P_r = 247$ kips (from previous calculations) and</td>
</tr>
</tbody>
</table>

Manual Table 4-1
Specification Sect H1.1
Manual Table 3-2
\[
I = 1,110 \text{ in}^4 (W_{14} \times 99)
\]

\[
P_{el} = \frac{\pi^2 EI}{(K,L)^2}
\]

\[
= \pi (29,000 \text{ ksi}) \left(1,110 \text{ in}^4\right)
\]

\[
= \frac{1}{(1.0)\left((13.5 \text{ ft})(12 \text{ in/ft})\right)^2}
\]

\[
= 12,106 \text{ kips}
\]

\[
C_m = 0.6 - 0.4(M_1 / M_2)
\]

\[
= 0.6 - 0.4 \left(157 \text{ kip-ft} / 229 \text{ kip-ft}\right)
\]

\[
= 0.326
\]

\[
\alpha = 1.0,
\]

\[
B_1 = \frac{C_m}{1 - \left(\frac{\alpha P}{P_{el}}\right)}
\]

\[
= \frac{0.326}{1 - \left(\frac{1.0}(335 \text{ kips})}{12,100 \text{ kips}}\right)
\]

\[
= 0.335 \geq 1; \text{ Use } 1.0
\]

**Calculate \(B_2\)**

\[
B_2 = \frac{1}{1 - \left(\frac{\alpha \sum P_{w2}}{\sum P_{w2}}\right)}
\]

where:

\[
\alpha = 1.0,
\]

\[
\sum P_{w1} = 5,250 \text{ kips (from computer output)}
\]

\[
\sum P_{w2} \text{ may be taken as } R_m \frac{HL}{\Delta_H}
\]

where \(R_m\) is taken as 0.85 for moment frames

\[
\sum H = 1.2D + 1.0E + 0.5L + 0.2S
\]

\[
= 195 \text{ kips (Horizontal)}
\]

(from previous seismic force distribution calculations)

\[
I = 1,110 \text{ in}^4 (W_{14} \times 99)
\]

\[
P_{el} = \frac{\pi^2 EI}{(K,L)^2}
\]

\[
= \pi (29,000 \text{ ksi}) \left(1,110 \text{ in}^4\right)
\]

\[
= \frac{1}{(1.0)\left((13.5 \text{ ft})(12 \text{ in/ft})\right)^2}
\]

\[
= 12,100 \text{ kips}
\]

\[
C_m = 0.6 - 0.4(M_1 / M_2)
\]

\[
= 0.6 - 0.4 \left(110 \text{ kip-ft} / 161 \text{ kip-ft}\right)
\]

\[
= 0.327
\]

\[
\alpha = 1.6,
\]

\[
B_1 = \frac{C_m}{1 - \left(\frac{\alpha P}{P_{el}}\right)}
\]

\[
= \frac{0.327}{1 - \left(\frac{1.6}((247 \text{ kips})}{12,100 \text{ kips}}\right)
\]

\[
= 0.338 \geq 1; \text{ Use } 1.0
\]

**Calculate \(B_2\)**

\[
B_2 = \frac{1}{1 - \left(\frac{\alpha \sum P_{w2}}{\sum P_{w2}}\right)}
\]

where:

\[
\alpha = 1.6,
\]

\[
\sum P_{w1} = 3,750 \text{ kips (from computer output)}
\]

\[
\sum P_{w2} \text{ may be taken as } R_m \frac{HL}{\Delta_H}
\]

where \(R_m\) is taken as 0.85 for moment frames

\[
\sum H = D + 0.7E
\]

\[
= 137 \text{ kips (Horizontal)}
\]

(from previous seismic force distribution calculations)
<table>
<thead>
<tr>
<th>Condition</th>
<th>Value</th>
<th>Calculation</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\Delta_{u}$ = 0.703 in.</td>
<td></td>
<td></td>
</tr>
<tr>
<td>$\sum P_{c2} = 0.85 \frac{(195 \text{ kips})(13.5 \text{ ft})(12 \text{ in/ft})}{0.703 \text{ in.}}$</td>
<td>$\sum P_{c2} = 0.85 \frac{(137 \text{ kips})(13.5 \text{ ft})(12 \text{ in/ft})}{0.493 \text{ in.}}$</td>
<td>$B_2 = \frac{1}{1 - \left(1.0 \frac{5,250 \text{ kips}}{38,200 \text{ kips}}\right)}$ = 1.16 $\geq 1$</td>
</tr>
<tr>
<td>$B_2 = \frac{1}{1 - \left(\frac{\alpha \sum P_{n}^2}{\sum P_{c2}}\right)}$</td>
<td>$B_2 = \frac{1}{1 - \left(1.6 \frac{3,750 \text{ kips}}{38,300 \text{ kips}}\right)}$ = 1.19 $\geq 1$</td>
<td></td>
</tr>
<tr>
<td>Calculate amplified moment</td>
<td>$(1.0)(0.0 \text{ kip-ft}) + (1.16)(229 \text{ kip-ft})$</td>
<td>$(1.0)(0.0 \text{ kip-ft}) + (1.19)(161 \text{ kip-ft})$</td>
</tr>
<tr>
<td>$= 266 \text{ kip-ft}$</td>
<td>$= 192 \text{ kip-ft}$</td>
<td></td>
</tr>
<tr>
<td>Calculate amplified axial load</td>
<td>$P_r = 335 \text{ kips}$ (from computer analysis)</td>
<td>$P_r = 247 \text{ kips}$ (from computer analysis)</td>
</tr>
<tr>
<td>$P_r = P_{nt} + B_2 P_{lt}$</td>
<td>$P_r = P_{nt} + B_2 P_{lt}$</td>
<td>$P_r = 335 \text{ kips} + (1.16)(0.0 \text{ kips})$ = 335 kips</td>
</tr>
<tr>
<td>$= 335 \text{ kips}$</td>
<td>$= 247 \text{ kips} + (1.19)(0.0 \text{ kips})$ = 247 kips</td>
<td></td>
</tr>
<tr>
<td>$P_r = 335 \text{ kips}$ (from computer analysis)</td>
<td>$P_r = 247 \text{ kips}$ (from computer analysis)</td>
<td>$P_r = 335 \text{ kips} + (1.16)(0.0 \text{ kips})$ = 335 kips</td>
</tr>
<tr>
<td>$P_r$, $P_{nt}$, $B_2$, $P_{lt}$</td>
<td>$P_r = 247 \text{ kips} + (1.19)(0.0 \text{ kips})$ = 247 kips</td>
<td></td>
</tr>
<tr>
<td>$P_r = \frac{8}{9} \frac{M_{x} + M_{y}}{M_{cy}}$</td>
<td>$P_r = \frac{8}{9} \frac{M_{x} + M_{y}}{M_{cy}}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.294 + \frac{8}{9} \frac{266 \text{ kip-ft}}{646 \text{ kip-ft}}$</td>
<td>$= 0.325 + \frac{8}{9} \frac{192 \text{ kip-ft}}{430 \text{ kip-ft}}$</td>
<td></td>
</tr>
<tr>
<td>$= 0.660 \leq 1.0$</td>
<td>$= 0.722 \leq 1.0$</td>
<td></td>
</tr>
</tbody>
</table>

Specification

Eqn H1-1b
BEAM ANALYSIS IN THE MOMENT FRAME

The controlling load combinations for the beams in the frames are shown below and evaluated for the second floor beam. The dead load, live load and seismic moments were taken from computer analysis.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1.2D + 1.0E + 0.5L$</td>
<td>$D + 0.7E$</td>
</tr>
<tr>
<td>$= 1.2(153 \text{kip-ft}) + 1.0(117 \text{kip-ft})$</td>
<td>$= 153 \text{kip-ft} + 0.7(112 \text{kip-ft})$</td>
</tr>
<tr>
<td>$+ 0.5(80.5 \text{kip-ft})$</td>
<td>$= 153 \text{kip-ft} + 78.4 \text{kip-ft}$</td>
</tr>
<tr>
<td>$= 184 \text{kip-ft} + 117 \text{kip-ft} + 40.3 \text{kip-ft}$</td>
<td>$= 231 \text{kip-ft}$</td>
</tr>
<tr>
<td>$= 341 \text{kip-ft}$ controls</td>
<td>$D + 0.75(0.7E) + 0.75L$ controls</td>
</tr>
<tr>
<td>$= 153 \text{kip-ft} + 0.75(0.7)(112 \text{kip-ft})$</td>
<td>$= 153 \text{kip-ft} + 58.8 \text{kip-ft} + 60.4 \text{kip-ft}$</td>
</tr>
<tr>
<td>$+ 0.75(80.5 \text{kip-ft})$</td>
<td>$= 272 \text{kip-ft}$ controls</td>
</tr>
</tbody>
</table>

The available moment is 503 kip-ft

503 kip-ft > 341 kip-ft o.k.

The available moment is 334 kip-ft

334 kip-ft > 272 kip-ft o.k.

Manual Table 3-2

Note: because the typical exterior beam was increased to a W24×55 to help with drift, all load combinations resulted in less demand than the available moment strength of the beam.

The ends of these beams can be designed by one of the techniques illustrated in the Chapter IIB of the design examples.
BRACED FRAME ANALYSIS

The braced frames at Grids 1 and 8 were analyzed for their lateral loads. The same stability design requirements from Chapter C were applied to this system.

Second-order analysis by amplified first-order analysis

The following is a method to account for second-order effects in frames by amplifying the axial forces and moments in members and connections from a first-order analysis.

First a first-order frame analysis is run using the load combinations for LRFD and ASD. From this analysis the critical axial loads, moments, and deflections are obtained.

The required second-order flexural strength, $M_r$, and axial strength, $P_r$, are as follows:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{e1} = \frac{\pi^2 EI}{(K, L)^2}$</td>
<td>$P_{e2} = \frac{\pi^2 EI}{(K, L)^2}$</td>
</tr>
<tr>
<td>$\pi \left( \frac{29,000 \text{ ksi}}{1.0} \right) \left( \frac{425 \text{ in}^4}{(13.5 \text{ ft})(12 \text{ in/ft})^2} \right)$</td>
<td>$\pi \left( \frac{29,000 \text{ ksi}}{1.0} \right) \left( \frac{425 \text{ in}^4}{(13.5 \text{ ft})(12 \text{ in/ft})^2} \right)$</td>
</tr>
<tr>
<td>$= 4,635 \text{ kips}$</td>
<td>$= 4,635 \text{ kips}$</td>
</tr>
<tr>
<td>$I = 425 \text{ in}^4 \ (W12 \times 58)$</td>
<td>$I = 425 \text{ in}^4 \ (W12 \times 58)$</td>
</tr>
<tr>
<td>$B_2 = \frac{1}{1 - \left( \frac{\sum P_{at}}{\alpha \sum P_{e2}} \right)}$</td>
<td>$B_2 = \frac{1}{1 - \left( \frac{\alpha \sum P_{at}}{\sum P_{e2}} \right)}$</td>
</tr>
<tr>
<td>$= \frac{1}{1 - \left( \frac{1.0(5,452 \text{ kips})}{150,722 \text{ kips}} \right)}$</td>
<td>$= \frac{1}{1 - \left( \frac{1.6(3,894 \text{ kips})}{150,722 \text{ kips}} \right)}$</td>
</tr>
<tr>
<td>$= 1.04 \geq 1$</td>
<td>$= 1.04 \geq 1$</td>
</tr>
<tr>
<td>$\sum P_{at} = 5,452 \text{ kips} \ (\text{from computer output})$</td>
<td>$\sum P_{at} = 3,894 \text{ kips} \ (\text{from computer output})$</td>
</tr>
<tr>
<td>$\sum P_{e2} = R_{af} \sum \frac{HL}{\Delta_{hf}}$</td>
<td>$\sum P_{e2} = R_{af} \sum \frac{HL}{\Delta_{hf}}$</td>
</tr>
<tr>
<td>$= 1.0 \left( \frac{195 \text{ kips}}{13.5 \text{ ft}} \right) \left( \frac{12 \text{ in/ft}}{0.210 \text{ in.}} \right)$</td>
<td>$= 1.0 \left( \frac{137 \text{ kips}}{13.5 \text{ ft}} \right) \left( \frac{12 \text{ in/ft}}{0.147 \text{ in.}} \right)$</td>
</tr>
<tr>
<td>$= 150,722 \text{ kips}$</td>
<td>$= 150,722 \text{ kips}$</td>
</tr>
</tbody>
</table>
\[ \sum H = 1.2D + 1.0E + L + 0.2S \]
\[ = 195 \text{ kips} \quad \text{(from previous calculations)} \]
\[ \Delta_n = 0.210 \text{ in. (from computer output)} \]
\[ P_r = P_n + B_2P_l \]
\[ = 242 \text{ kips} + (1.04)(220 \text{ kips}) \]
\[ = 470 \text{ kips} \]
\[ P_c = 553 \text{ kips (W12 \times 58)} \]
\[ \text{For } \frac{P_r}{P_c} = \frac{470 \text{ kips}}{553 \text{ kips}} = 0.850 \leq 1.0 \]

\[ \sum H = D + 0.7E \]
\[ = 137 \text{ kips} \quad \text{(from previous calculations)} \]
\[ \Delta_n = 0.147 \text{ in. (from computer output)} \]
\[ P_r = P_n + B_2P_l \]
\[ = 173 \text{ kips} + (1.04)(179 \text{ kips}) \]
\[ = 360 \text{ kips} \]
\[ P_c = 368 \text{ kips (W12 \times 58)} \]
\[ \text{For } \frac{P_r}{P_c} = \frac{360 \text{ kips}}{368 \text{ kips}} = 0.977 \leq 1.0 \]

Note: Notice that the lower displacements of the braced frame produce much lower values for \( B_2 \). Similar values could be expected for the other two methods of analysis.
ANALYSIS OF DRAG STRUTS

The fourth floor has the highest force to the ends of the building at \( E = 80.3 \text{ kips} \) (from previous calculations). The beams at the end of the building which span 22.5 ft are W16×31

The loads for these edge beams with a DL of 0.75 ksf (5.5 ft) and exterior wall at 0.503 kip/ft and a LL of 0.80 ksf (5.5 ft) are \( DL_{\text{tot}} = 0.916 \text{ kip/ft} \), \( LL_{\text{tot}} = 0.440 \text{ kip/ft} \).

The controlling load combinations are LRFD (1.2\( D+1.0E+0.50L \)) and ASD (1.0\( D+0.75(0.7E)+0.75L \)) or (\( D+0.7E \))

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( M_d = 1.2(58.0 \text{ kip-ft}) + 0.50(27.8 \text{ kip-ft}) )</td>
<td>( M_d = 1.0(58.0 \text{ kip-ft}) + 0.75(27.8 \text{ kip-ft}) )</td>
</tr>
<tr>
<td>= 83.5 kip-ft</td>
<td>= 78.9 kip-ft</td>
</tr>
<tr>
<td>or ( M_d = 58.0 \text{ kip-ft} )</td>
<td></td>
</tr>
<tr>
<td>Load from the diaphragm shear</td>
<td>Load from the diaphragm shear</td>
</tr>
<tr>
<td>( F_p = 80.3 \text{ kips} )</td>
<td>( F_p = 0.75(0.70)(80.3 \text{ kips}) = 42.2 \text{ kips} )</td>
</tr>
<tr>
<td>or ( F_p = 0.70(80.3 \text{ kips}) = 56.2 \text{ kips} )</td>
<td></td>
</tr>
<tr>
<td>Load to the drag struts</td>
<td>Load to the drag struts</td>
</tr>
</tbody>
</table>

Only the two 45 ft long segments on either side of the brace can transfer load into the brace, because the stair opening is in front of the brace.

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V = 80.3 \text{ kips} /2(45 \text{ ft}) = 0.892 \text{ kip/ft} )</td>
<td>( V = 42.2 \text{ kips} /2(45 \text{ ft}) = 0.469 \text{ kip/ft} )</td>
</tr>
<tr>
<td>or ( V = 56.2 \text{ kips} /2(45 \text{ ft}) = 0.624 \text{ kip/ft} )</td>
<td></td>
</tr>
<tr>
<td>The top flange stress due to bending</td>
<td>The top flange stress due to bending</td>
</tr>
<tr>
<td>( = 83.5 \text{ kip-ft}(12 \text{ in/ft}) / 47.2 \text{ in}^3 )</td>
<td>( = 78.9 \text{ kip-ft}(12 \text{ in/ft}) / 47.2 \text{ in}^3 )</td>
</tr>
<tr>
<td>( = 21.2 \text{ ksi} )</td>
<td>( = 20.1 \text{ ksi} )</td>
</tr>
<tr>
<td>or ( )</td>
<td></td>
</tr>
<tr>
<td>( = 58.0 \text{ kip-ft}(12 \text{ in/ft}) / 47.2 \text{ in}^3 )</td>
<td>( = 14.7 \text{ ksi} )</td>
</tr>
</tbody>
</table>

Note: It is often possible to resist the drag strut force using the slab directly. For illustration purposes, this solution will instead use the beam to resist the force independently of the slab. The full cross-section can be used to resist the force if the member is designed as a column braced at one flange only (plus any other intermediate bracing present, such as from filler beams). Alternatively, a reduced cross-section consisting of the top flange plus a portion of the web can be used. Arbitrarily use the top flange and 8 times the web thickness as an area to carry the drag strut component.
Area = 5.53 in.\(^2\) + 8(0.275 in.\(^2\)) = 2.43 in.\(^2\) + 0.605 in.\(^2\) = 3.04 in.\(^2\)

Ignoring the small segment of the beam between Grid C and D, the stress due to the drag strut force is:

<table>
<thead>
<tr>
<th>LRFD</th>
<th>ASD</th>
</tr>
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<tbody>
<tr>
<td>(= \left( \frac{80.3 \text{ ft}}{2} \right) \left( \frac{0.892 \text{ kip/ft}}{3.04 \text{ in.}^2} \right) = 13.2 \text{ ksi} )</td>
<td>(= \left( \frac{80.3 \text{ ft}}{2} \right) \left( \frac{0.469 \text{ kip/ft}}{3.04 \text{ in.}^2} \right) = 6.94 \text{ ksi} )</td>
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<tr>
<td>Total top flange stress</td>
<td>Total top flange stress</td>
</tr>
<tr>
<td>= 21.2 ksi + 13.2 ksi</td>
<td>= 20.1 ksi + 6.49 ksi</td>
</tr>
<tr>
<td>= 34.4 ksi.</td>
<td>= 26.6 ksi controls</td>
</tr>
</tbody>
</table>

For bending or compression \( \phi = 0.90 \)

\( \phi F_y = 45 \text{ ksi} > 34.4 \text{ ksi} \) o.k.

For bending or compression \( \Omega = 1.67 \)

\( F_y / \Omega = 29.9 \text{ ksi} > 26.6 \text{ ksi} \) o.k.

Note: Because the drag strut load is a horizontal load, notations indicating the method of transfer into the strut, and the extra horizontal load which must be accommodated by the beam end connections should be indicated on the drawings.
APPENDIX A

CROSS-REFERENCE LIST FOR THE
2005 AISC SPECIFICATION FOR STRUCTURAL STEEL BUILDINGS TO PAST AISC SPECIFICATIONS

The following cross-reference list relates the new 2005 Specification for Structural Steel Buildings table of contents to the corresponding sections, where applicable, of past AISC specifications. Cross references are given to the five standards that the new 2005 AISC Specification replaces:

### CROSS-REFERENCE LIST FOR THE 2005 AISC SPECIFICATION

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<th>A. GENERAL PROVISIONS</th>
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<td>4. Anchor Rods and Threaded Rods</td>
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<td>5. Filler Metal and Flux for Welding</td>
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<td>6. Stud Shear Connectors</td>
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| A4. Structural Design Drawings and Specifications | A7 | A7 | NEW | NEW | NEW |

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| B2. Loads and Load Combinations | A4 | A4 | 1.3 | NEW | NEW |

| B3. Design Basis | NEW | A5.1 | NEW | NEW | NEW |

| 3. Design for Strength Using Load and Resistance Factor Design (LRFD) | NEW | A5.2 | NEW | NEW | NEW |

| 4. Design for Strength Using Allowable Strength Design (ASD) | A5.3 | NEW | NEW | NEW | NEW |

| 5. Design for Stability | B4 | B4 | NEW | NEW | NEW |

| 6. Design of Connections | J | J1.1 | 9 | NEW | NEW |

| 6a. Simple Connections | J1.2 | J1.2 | NEW | NEW | NEW |

| 6b. Moment Connections | J1.3 | A2 | NEW | NEW | NEW |

| 7. Design for Serviceability | A5.4 | A5.4 | NEW | NEW | NEW |

| 8. Design for Fatigue | K2 | K2 | NEW | NEW | NEW |

| 9. Design for Fatigue | K3 | K3 | NEW | NEW | NEW |

| 10. Design for Fire Conditions | NEW | NEW | NEW | NEW | NEW |

| 11. Design for Corrosion Effects | L5 | L5 | NEW | NEW | NEW |

| 12. Design Wall Thickness for HSS | NEW | NEW | 1.2 | NEW | NEW |

| 13. Gross and Net Area Determination | NEW | NEW | NEW | NEW | NEW |

| a. Gross Area | B1 | B1 | NEW | NEW | NEW |

| b. Net Area | B2 | B2 | NEW | NEW | NEW |

| B4. Classification of Sections for Local Buckling | B5.1 | NEW | NEW | NEW | NEW |

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| 2. Stiffened Elements | B5.1 | NEW | NEW | NEW | NEW |

| B5. Fabrication, Erection and Quality Control | M | M | NEW | NEW | NEW |

| B6. Evaluation of Existing Structures | NEW | N | NEW | NEW | NEW |

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| C1. Stability Design Requirements | NEW | NEW | NEW | NEW | NEW |

| 2. Members Stability Design Requirements | NEW | NEW | NEW | NEW | NEW |

| 3. System Stability Design Requirements | NEW | NEW | NEW | NEW | NEW |

| 3a. Braced-Frame and Shear-Wall Systems | C2.1 | C2.1 | NEW | NEW | NEW |

| 3b. Moment-Frame Systems | C2.2 | C2.2 | NEW | NEW | NEW |

| 3c. Gravity Framing Systems | NEW | NEW | NEW | NEW | NEW |

| 3d. Combined Systems | NEW | NEW | NEW | NEW | NEW |
### CROSS-REFERENCE LIST FOR THE 2005 AISC SPECIFICATION

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- Lateral-Torsional Buckling
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- Weak Axis Shear in Singly and Doubly Symmetric Shapes
- Beams and Girders with Web Openings
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<td>1.6 Columns and Other Compression Members</td>
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<td>1.7 Beams and Others Flexural Members</td>
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<td>1.9 Connections</td>
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### Appendix 2. DESIGN FOR PONDING

#### 2.1 Simplified Design for Pounding

- K2
- NEW
- App. K2
- NEW
- NEW

#### 2.2 Improved Design for Ponding

- NEW
- App. K2
- NEW
- NEW

### Appendix 3. DESIGN FOR FATIGUE

#### 3.1 General

- K4, App. K4
- K3, App. K3.1
- NEW
- NEW
- NEW

#### 3.2 Calculation of Maximum Stresses and Stress Ranges

- App. K4.2
- App. K3.2
- NEW
- NEW
- NEW

#### 3.3 Design Stress Range

- App. K4.2
- App. K3.3
- NEW
- NEW
- NEW

#### 3.4 Bolts and Threaded Parts

- App. K4.3
- App. K3.4
- NEW
- NEW
- NEW

### Appendix 4. STRUCTURAL DESIGN FOR FIRE CONDITIONS

- NEW
- NEW
- NEW
- NEW
- NEW

### Appendix 5. EVALUATION OF EXISTING STRUCTURES

#### 5.1 General Provisions

- NEW
- N1
- NEW
- NEW
- NEW

#### 5.2 Material Properties

1. Determination of Required Tests
   - NEW
   - N2.1
   - NEW
   - NEW
   - NEW

2. Tensile Properties
   - N2.2

3. Chemical Composition
   - N2.3

4. Base Metal Notch Toughness
   - N2.4

5. Weld Metal
   - N2.5

6. Bolts and Rivets
   - N2.6

#### 5.3 Evaluation by Structural Analysis

1. Dimensional Data
   - NEW
   - N3.1
   - NEW
   - NEW
   - NEW

2. Strength Evaluation
   - N3.2

3. Serviceability Evaluation
   - N3.3

#### 5.4 Evaluation by Load Tests

1. Determination of Load Rating by Testing
   - NEW
   - N4.1
   - NEW
   - NEW
   - NEW

2. Serviceability Evaluation
   - N4.2

#### 5.5 Evaluation Report

- NEW

### Appendix 6. STABILITY BRACING FOR COLUMNS AND BEAMS

- NEW
- C3
- NEW
- NEW
- NEW

### Appendix 7. DIRECT ANALYSIS METHOD

- NEW
- NEW
- NEW
- NEW
- NEW