9 Steel Shear Diaphragms and Shell Roof Structures

9.1 GENERAL REMARKS

During the past several decades a large number of research projects conducted throughout the world have concentrated on the investigation of the structural behavior not only of individual cold-formed steel components but also of various structural systems. Shear diaphragms and shell roof structures (including folded plate and hyperbolic paraboloid roofs) are some examples of the structural systems studied in the past.

As a result of the successful studies of shear diaphragms and shell roof structures accompanied by the development of new steel products and fabrication techniques, the applications of steel structural assemblies in building construction have been increased rapidly.

In this chapter the research work and the design methods for the use of shear diaphragms and shell roof structures are briefly discussed on the basis of the available information. For details, the reader is referred to the related references.

9.2 STEEL SHEAR DIAPHRAGMS

9.2.1 Introduction

In building construction it has been a common practice to provide a separate bracing system to resist horizontal loads due to wind load, blast force, or earthquake. However, steel floor and roof panels (Fig. 1.11), with or without concrete fill, are capable of resisting horizontal loads in addition to the beam strength for gravity loads if they are adequately interconnected to each other and to the supporting frame.\(^1\)\(^6\)\(^9\)\(^1\)\(^9\)\(^2\) The effective use of steel floor and roof panels can therefore eliminate separate bracing systems and result in a reduction of building costs (Fig. 9.1).

For the same reason, wall panels can provide not only enclosure surfaces and support normal loads, but they can also provide diaphragm action in their own planes.

In addition to the utilization of diaphragm action, steel panels used in floor, roof, and wall construction can be used to prevent the lateral buckling of
beams and the overall buckling of columns.\textsuperscript{4.115–4.120} Previous studies made by Winter have shown that even relatively flexible diaphragm systems can provide sufficient horizontal support to prevent the lateral buckling of beams in floor and roof construction.\textsuperscript{4.111} The load-carrying capacities of columns can also be increased considerably if they are continuously braced with steel diaphragms.\textsuperscript{4.116}

9.2.2 Research on Shear Diaphragms

Because the structural performance of steel diaphragms usually depends on the sectional configuration of panels, the type and arrangement of connections, the strength and thickness of the material, span length, loading function, and concrete fill, the mathematical analysis of shear diaphragms is complex. At the present time, the shear strength and the stiffness of diaphragm panels can be determined either by tests or by analytical procedures.

Since 1947 numerous diaphragm tests of cold-formed steel panels have been conducted and evaluated by a number of researchers and engineers. The diaphragm tests conducted in the United States during the period of 1947 through 1960 were summarized by Nilson in Ref. 9.2. Those tests were primarily sponsored by individual companies for the purpose of developing design data for the diaphragm action of their specific panel products. The total thickness of the panels tested generally range from 0.04 to 0.108 in. (1 to 2.8 mm). Design information based on those tests has been made available from individual companies producing such panels.

In 1962 a research project was initiated at Cornell University under the sponsorship of the AISI to study the performance of shear diaphragms constructed of corrugated and ribbed deck sections of thinner materials, from
0.017 to 0.034 in. (0.4 to 0.9 mm) in total thickness. The results of diaphragm tests conducted by Luttrell and Apparao under the direction of George Winter were summarized in Refs. 9.3, 9.4, and 9.5. Recommendations on the design and testing of shear diaphragms were presented in the AISI publication, "Design of Light Gage Steel Diaphragms," which was issued by the institute in 1967.9.6

Since 1967 additional experimental and analytical studies of steel shear diaphragms have been conducted throughout the world. In the United States, research projects on this subject have been performed by Nilson, Ammar, and Atrek,9.7–9.9 Luttrell, Ellifritt, and Huang,9.10–9.13 Easley and McFarland,9.14–9.16 Miller,9.17 Libove, Wu, and Hussain,9.18–9.21 Chern and Jorgenson,9.22 Liedtke and Sherman,9.23 Fisher, Johnson, and LaBoube,9.24–9.26 Jankowski and Sherman,9.90 Heagler,9.91 Luttrell,9.92 and others. The research programs that have been carried out in Canada include the work of Ha, Chockalingam, Fazio, and El-Hakim,9.27–9.30 and Abdel-Sayed.9.31

In Europe, the primary research projects on steel shear diaphragms have been conducted by Bryan, Davies, and Lawson.9.32–9.38 The utilization of the shear diaphragm action of steel panels in framed buildings has been well illustrated in Davies and Bryan’s book on stressed skin diaphragm design.9.39 In addition, studies of tall buildings using diaphragms were reported by El-Dakhakhni in Refs. 9.40 and 9.41.


In addition to the shear diaphragm tests mentioned above, lateral shear tests of steel buildings and tests of gabled frames with covering sheathing have been performed by Bryan and El-Dakhakhni.4.113,4.114 The structural behavior of columns and beams continuously braced by diaphragms has also been studied by Pincus, Fisher, Errera, Apparao, Celebi, Pekoz, Winter, Rockey, Nethercot, Trahair, Wikstrom, and others.4.115–4.120,4.135,4.136 This subject is discussed further in Art. 9.3.

In order to understand the structural behavior of shear diaphragms, the shear strength and the stiffness of steel diaphragms are briefly discussed in subsequent sections.

### 9.2.2.1 Shear Strength of Steel Diaphragms

Results of previous tests indicate that the shear strength per foot of steel diaphragm is usually affected by the panel configuration, the panel span and purlin or girt spacing, the material thickness and strength, acoustic perforations, types and arrangements of fasteners, and concrete fill, if any.

**Panel Configuration.** The height of panels has considerable effect on the shear strength of the diaphragm if a continuous flat plate element is not provided. The deeper profile is more flexible than are shallower sections. There-
fore the distortion of the panel, in particular near the ends, is more pronounced for deeper profiles. On the other hand, for panels with a continuous flat plate connected to the supporting frame, the panel height has little or no effect on the shear strength of the diaphragm.

With regard to the effect of the sheet width within a panel, wider sheets are generally stronger and stiffer because there are fewer side laps.

Panel Span and Purlin Spacing. Shorter span panels could provide a somewhat larger shear strength than longer span panels, but the results of tests indicate that the failure load is not particularly sensitive to changes in span.

The shear strength of panels is increased by a reduction of purlin spacing; the effect is more pronounced in the thinner panels.

Material Thickness and Strength. If a continuous flat plate is welded directly to the supporting frame, the failure load is nearly proportional to the thickness of the material. However, for systems with a formed panel, the shear is transmitted from the support beams to the plane of the shear-resisting element by the vertical ribs of the panels. The shear strength of such a diaphragm may be increased by an increase in material thickness, but not linearly.

When steels with different material properties are used, the influence of the material properties on diaphragm strength should be determined by tests or analytical procedures.

Acoustic Perforations. The presence of acoustic perforations may slightly increase the deflection of the system and decreases the shear strength.

Types and Arrangement of Fasteners. The shear strength of steel diaphragms is affected not only by the types of fasteners (welds, bolts, sheet metal screws, and others) but also by their arrangement and spacing. The shear strength of the connection depends to a considerable degree on the configuration of the surrounding metal.

Previous studies indicate that if the fasteners are small in size or few in number, failure may result from shearing or separation of the fasteners or by localized bearing or tearing of the surrounding material. If a sufficient number of fasteners are closely spaced, the panel may fail by elastic buckling, which produces diagonal waves across the entire diaphragm.

The shear strength will be increased considerably by the addition of intermediate side lap fasteners and end connections.

Concrete Fill. Steel panels with a concrete fill provide a much more rigid and effective diaphragm. The stiffening effect of the fill depends on the thickness, strength, and density of the fill and the bond between the fill and the panels.

The effect of lightweight concrete fill on shear diaphragms has been studied by Luttrell.9,11 It was found that even though this type of concrete may have a very low compressive strength of 100 to 200 psi (0.7 to 1.4 MPa), it can
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significantly improve the diaphragm performance. The most noticeable influence is the increase in shear stiffness. The shear strength can also be increased, but to a lesser extent.

9.2.2.2 Stiffness of Shear Diaphragms

In the use of shear diaphragms, deflection depending upon the stiffness of the shear diaphragms is often a major design criterion. Methods for predicting the deflection of cold-formed steel panels used as diaphragms have been developed on the basis of the specific panels tested. In general the total deflection of the diaphragm system without concrete fill is found to be a combination of the following factors:9.2.9.39

1. Deflection due to flexural stress
2. Deflection due to shear stress
3. Deflection due to seam slip
4. Deflection due to local distortion of panels and relative movement between perimeter beams and panels at end connections

The deflection due to flexural stress can be determined by the conventional formula using the moment of inertia due to perimeter beams and neglecting the influence of the diaphragm acting as a web of a plate girder.

For simplicity the combined deflections due to shear stress, seam slip, and local distortion can be determined from the results of diaphragm tests. If shear transfer devices are provided, the deflection due to relative movement between marginal beams and shear web will be negligible.

The above discussion is based on the test results of shear diaphragms without concrete fill. The use of concrete fill will increase the stiffness of shear diaphragms considerably, as discussed in the preceding section. When the advantage of concrete fill is utilized in design, the designer should consult individual companies or local building codes for design recommendations.

9.2.3 Tests of Steel Shear Diaphragms

In general, shear diaphragms are tested for each profile or pattern on a reasonable maximum span which is normally used to support vertical loads. The test frame and connections should be selected properly to simulate actual building construction if possible. Usually the mechanical properties of the steel used for the fabrication of the test panels should be similar to the specified values. If a substantially different steel is used, the test ultimate shear strength may be corrected on the basis of Ref. 9.42.

During the past, cantilever, two-bay, and three-bay steel test frames have been generally used. Another possible test method is to apply compression forces at corners along a diagonal. Nilson has shown that the single-panel cantilever test will yield the same shear strength per foot as the three-bay frame and that the deflection of an equivalent three-bay frame can be com-
puted accurately on the basis of the single-panel test. It is obvious that the use of a cantilever test is economical, particularly for long-span panels. References 9.42–9.44 contain the test procedure and the method of evaluation of the test results.

The test frame used for the cantilever test is shown in Fig. 9.2a, and Fig. 9.2b shows the cantilever beam diaphragm test.

The three-bay simple beam test frame is shown in Fig. 9.3a, and Fig. 9.3b shows the test setup for a simple beam diaphragm test.

The test results can be evaluated on the basis of the average values obtained from the testing of two identical specimens, if the deviation from the average value does not exceed 10%. Otherwise the testing of a third identical specimen is required by Refs. 9.42–9.44. The average of the two lower values obtained from the tests is regarded as the result of this series of tests. According to Ref. 9.43, if the frame has a stiffness equal to or less than 2% of that of the total diaphragm assembly, no adjustment of test results for frame resistance need be made. Otherwise, the test results should be adjusted to compensate for frame resistance.

The ultimate shear strength \( S_u \) in pounds per foot can be determined from

\[
S_u = \frac{(P_{ult})_{avg}}{b} \tag{9.1}
\]

where \((P_{ult})_{avg} = \) average value of maximum jack loads from either cantilever or simple beam tests, lb

\( b = \) depth of beam indicated in Figs. 9.2a and 9.3a, ft

The computed ultimate shear strength divided by the proper load factor gives the allowable design shear \( S_{des} \) in pounds per linear foot. (See Fig. 9.4 for the tested ultimate shear strength of standard corrugated steel diaphragms.)

According to Ref. 9.42, the shear stiffness \( G' \) is to be determined on the basis of an applied load of 0.4\((P_{ult})_{avg} \) for use in deflection determination.* For the evaluation of shear stiffness, the measured deflections at the free end of the cantilever beam or at one-third the span length of the simple beam for each loading increment can be corrected by the following equations if the support movements are to be taken into account:

1. For cantilever tests,

\[
\Delta = D_3 - \left[ D_1 + \frac{a}{b} (D_2 + D_4) \right] \tag{9.2}
\]

*Reference 9.44 suggests that the shear stiffness \( G' \) is to be determined on the basis of a reference level of 0.33\((P_{ult})_{avg} \). If the selected load level is beyond the proportional limit, use a reduced value less than the proportional limit.
Figure 9.2  (a) Plan of cantilever test frame.  (b) Cantilever beam diaphragm test.
Figure 9.3  (a) Plan of simple beam test frame. (b) Simple beam diaphragm test.
2. For simple beam tests,

\[ \Delta = \frac{D_2 + D_3 - D_1 - D_4}{2} \]  

(9.3)

where \( D_1, D_2, D_3, \) and \( D_4 \) are the measured deflections at locations indicated in Figs. 9.2a and 9.3a, and \( a/b \) is the ratio of the diaphragm dimensions. The load-deflection curve can then be plotted on the basis of the corrected test results.

The shear deflection for the load of \( 0.4(P_{ult})_{avg} \) can be computed from

\[ \Delta'_s = \Delta' - \Delta'_b \]  

(9.4)

where \( \Delta'_s = \) shear deflection for load of \( 0.4P_{ult} \)

\( \Delta' = \) average value of deflections obtained from load-deflection curves for load of \( 0.4P_{ult} \)

\( \Delta'_b = \) computed bending deflection

In the computation of \( \Delta'_b \) the following equations may be used for cantilever beams. The bending deflection at the free end is

\[ \Delta'_b = \frac{Pa^3(12)^2}{3EI} \]  

(9.5)

For simple beam tests, the bending deflection at one third the span length is
In Eqs. (9.5) and (9.6),

\[ \Delta'_b = \frac{5Pa^3(12)^2}{6EI} \]

where

\[ P = 0.4(P_{ult})_{avg}, \text{ lb} \]
\[ E = \text{modulus of elasticity of steel, } 29.5 \times 10^6 \text{ psi (203 GPa)} \]
\[ I = \text{moment of inertia considering only perimeter members of test frame, } \]
\[ = Ab^2(12)^2/2, \text{ in.}^4 \]
\[ A = \text{sectional area of perimeter members } CD \text{ and } GE \text{ in Figs. 9.2a and 9.3a}, \text{ in.}^2 \]
\[ a, b = \text{dimensions of test frame shown in Figs. 9.2a and 9.3a, ft} \]

Finally, the shear stiffness \( G' \) of the diaphragm can be computed as

\[ G' = \frac{P}{b} = \frac{0.4(P_{ult})_{avg}}{\Delta'_a} \left( \frac{a}{b} \right) \]

The shear stiffness varies with the panel configuration and the length of the diaphragm. For standard corrugated sheets, the shear stiffness for any length may be computed by Eq. (9.8) as developed by Luttrell\(^9,5\) if the constant \( K_2 \) can be established from the available test data on the same profile:

\[ G' = \frac{Et}{[2(1 + \mu)g] + p + K_2/(Lt)^2} \]

where \( G' = \text{shear stiffness, lb/in.} \)
\[ E = \text{modulus of elasticity of steel, } = 29.5 \times 10^6 \text{ psi (203 GPa)} \]
\[ t = \text{uncoated thickness of corrugated panel, in.} \]
\[ \mu = \text{Poisson’s ratio, } = 0.3 \]
\[ p = \text{corrugation pitch, in. (Fig. 9.5)} \]
\[ g = \text{girth of one complete corrugation, in. (Fig. 9.5)} \]
\[ L = \text{length of panels from center to center of end fasteners, measured parallel to corrugations, in.} \]
\[ K_2 = \text{constant depending on diaphragm cross section and end fastener spacing, in.}^4 \]

Knowing \( G' \) from the tested sheets, the constant \( K_2 \) can be computed as

\[ \text{Figure 9.5 Cross section of corrugated sheets.}^{9,42} \]
Figure 9.6 shows graphically the tested shear stiffness for 0.0198-in. (0.5-mm) thick standard corrugated diaphragms.

9.2.4 Analytical Methods for Determining Shear Strength and Stiffness of Shear Diaphragms

In Art. 9.2.3 the test method to be used for establishing the shear strength and stiffness of shear diaphragms was discussed. During the past two decades, several analytical methods have been developed for computing the shear strength and the stiffness of diaphragms. The following five methods are commonly used:

1. Steel Deck Institute (SDI) method
2. Tri-Service method
3. European recommendations
4. Nonlinear finite-element analysis
5. Simplified diaphragm analysis

For details, the reader is referred to the referenced documents and publications.

Figure 9.6 Tested shear stiffness for $2\frac{1}{2} \times \frac{1}{2}$-in. standard corrugated steel diaphragms. Thickness of panels = 0.0198 in.
With regard to the use of the European recommendations, a design guide was prepared by Bryan and Davies in 1981.\textsuperscript{9.48} This publication contains design tables and worked examples which are useful for calculating the strength and stiffness of steel roof decks when acting as diaphragms.

In addition to the above publications, general design rules on steel shear diaphragms can also be found in Ref. 9.49.

9.2.5 Design of Shear Diaphragms

Steel shear diaphragms can be designed as web elements of horizontal analogous plate girders with the perimeter framing members acting as the flanges. The primary stress in the web is shear stress, and in the flanges the primary stresses are axial stresses due to bending applied to the plate girder.

It has been a general design practice to determine the required sections of the panels and supporting beams based on vertical loads. The diaphragm system, including connection details, needed to resist horizontal loads, can then be designed on the basis of (1) the shear strength and the stiffness of the panels recommended by individual companies for the specific products and (2) the design provisions of local building codes.

For the design of shear diaphragms according to Sec. D5 of the AISI Specification,\textsuperscript{1.314} the design shear strength can be determined as follows:

i. For the ASD method,

\[ S_d = \frac{S_n}{\Omega_d} \quad (9.10a) \]

ii. For the LRFD method,

\[ S_d = \phi_d S_n \quad (9.10b) \]

where

- \( S_d \) = design shear strength for diaphragm, lb/ft.
- \( S_n \) = in-plane diaphragm nominal shear strength established by calculation or test, lb/ft
- \( \Omega_d \) = factor of safety for diaphragm shear as specified in Table 9.1
- \( \phi_d \) = resistance factor for diaphragm shear as specified in Table 9.1

In Table 9.1, the factors of safety and resistance factors are based on the statistical studies of the nominal and mean resistances from full scale tests.\textsuperscript{4.107} This study indicated that the quality of mechanical connectors is easier to control than welded connections. As a result, the variation in the strength of mechanical connectors is smaller than that for welded connections, and their
performance is more predictable. Therefore, a smaller factor of safety, or larger resistance factor, is justified for mechanical connections.\textsuperscript{1,310} The factors of safety for earthquake loading are slightly larger than those for wind load due to the ductility demands required by seismic loading.

The stress in the perimeter framing members should be checked for the combined axial stresses due to the gravity load and the wind load or earthquake applied to the structure. According to Sec. A5.1.3 of the AISI Specification, no decrease in forces is permitted when evaluating diaphragms using the provision of Sec. D5 of the Specification.

As shown in Fig. 9.7, the axial stress in the perimeter framing members due to horizontal load (wind load or earthquake) can be determined by

\[ f = \frac{F}{A} = \frac{M}{Ab} \]  

(9.11)

where \( f \) = stress in tension or compression, psi  
\( F \) = force in tension or compression, lb, \( = \frac{M}{b} \)  
\( M \) = bending moment at particular point investigated, ft-lb  
\( A \) = area of perimeter framing member, in.\textsuperscript{2}  
\( b \) = distance between centroids of perimeter members, measured perpendicular to span length of girder, ft

Usually the shear stress in panels and the axial stress in perimeter members of such a diaphragm assembly are small, and it is often found that the framing members and panels which have been correctly designed for gravity loads
Figure 9.7  Portal frame building with wall and roof diaphragm.\textsuperscript{9,42}

will function satisfactorily in diaphragm action with no increase in size. However, special attention may be required at connections of perimeter framing members. Ordinary connections may deform in the crimping mode if subjected to heavy axial forces along connected members.

If the panels are supported by a masonry wall rather than by a steel frame, tensile and compressive reinforcements should be provided for flange action within the walls adjacent to the diaphragm connected thereto.

In addition to designing shear diaphragm and perimeter members for their strengths, the deflection of shear diaphragms must also be considered. The total deflection of shear diaphragms may be computed as:

\[
\Delta_{\text{total}} = \Delta_b + \Delta_s
\]

(9.12)

where \(\Delta_{\text{total}}\) = total deflections of shear diaphragm, in.

\(\Delta_b\) = bending deflection, in.

\(\Delta_s\) = shear deflection, including deflection due to seam slip and local distortion, in.

The bending deflection and shear deflection can be computed by the formulas given in Table 9.2 for various types of beams subjected to various loading conditions.

In Table 9.2 the formula for determining the shear deflection of a diaphragm is similar to the method of computing the shear deflection of a beam having relatively great depth. It can be derived from the following equation:\textsuperscript{4,45}
TABLE 9.2  Deflection of Shear Diaphragms

<table>
<thead>
<tr>
<th>Type of Diaphragm</th>
<th>Loading Condition</th>
<th>$\Delta_b$</th>
<th>$\Delta^*$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Simple beam</td>
<td>Uniform load</td>
<td>$5wL^4(12)^2$</td>
<td>$wL^2$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{384EI}{8G'b}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Load $P$ applied at center</td>
<td>$PL^3(12)^3$</td>
<td>$PL$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{48EI}{4G'b}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Load $P$ applied at each 1/3 point of span</td>
<td>$23PL^3(12)^3$</td>
<td>$PL$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{648EI}{3G'b}$</td>
<td></td>
</tr>
<tr>
<td>Cantilever beam</td>
<td>Uniform load</td>
<td>$wa^4(12)^3$</td>
<td>$wa^2$</td>
</tr>
<tr>
<td>(at free end)</td>
<td></td>
<td>$\frac{8EI}{2G'b}$</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Load $P$ applied at free end</td>
<td>$Pa^4(12)^3$</td>
<td>$Pa$</td>
</tr>
<tr>
<td></td>
<td></td>
<td>$\frac{3EI}{G'b}$</td>
<td></td>
</tr>
</tbody>
</table>

*When the diaphragm is constructed with two or more panels of different lengths, the term $G'b$ should be replaced by $\Sigma G_i b_i$, where $G_i$ and $b_i$ are the shear stiffness and the length of a specific panel, respectively.

$$\Delta_s = \int \frac{Vv}{G'b} \, dx \quad (9.13)$$

where $V = \text{shear due to actual loads}$
$v = \text{shear due to a load of 1 lb acting at section where deflection is derived}$
$G' = \text{shear stiffness of diaphragm}$
$b = \text{width of shear diaphragm or depth of analogous beam}$.

In practical design, the total horizontal deflection of a shear diaphragm must be within the allowable limits permitted by the applicable building code or other design provisions. The following formula for masonry walls has been proposed by the Structural Engineers Association of California:

$$\text{allowable deflection} = \frac{h^2f}{0.01Et} \quad (9.14)$$

where $h = \text{unsupported height of wall, ft}$
$t = \text{thickness of wall, in.}$
$E = \text{modulus of elasticity of wall material, psi}$
$f = \text{allowable compressive stress of wall material, psi}$

9.2.6  Special Considerations
The following are several considerations which are essential in the use of steel panels as shear diaphragms.
1. If purlins and girts are framed over the top of perimeter beams, trusses, or columns, the shear plane of the panels may cause tipping of purlin and girt members by eccentric loading. For this case, rake channels or other members should be provided to transmit the shear from the plane of the panels to the flanges of the framing member or to the chords of the truss.

2. Consideration should be given to the interruption of panels by openings or nonstructural panels. It may be assumed that the effective depth of the diaphragm is equal to the total depth less the sum of the dimensions of all openings or nonstructural panels measured parallel to the depth of the diaphragm. The type of panel-to-frame fasteners used around the openings should be the same as, and their spacing equal to or less than, that used in the tests to establish the diaphragm value.

3. When panels are designed as shear diaphragms, a note shall be made on the drawings to the effect that the panels function as braces for the building and that any removal of the panels is prohibited unless other special separate bracing is provided.

4. The performance of shear diaphragms depends strongly on the type, spacing, strength, and integrity of the fasteners. The type of fasteners used in the building should be the same as, and their spacing not larger than, that used in the test to establish the diaphragm value.

5. Diaphragms are not effective until all components are in place and fully interconnected. Temporary shoring should therefore be provided to hold the diaphragms in the desired alignment until all panels are placed, or other construction techniques should be used to make the resulting diaphragm effective. Temporary bracing should be introduced when replacing panels.

6. Methods of erection and maintenance used for the construction of shear diaphragms should be evaluated carefully to ensure proper diaphragm action. Proper inspection and quality control procedures would be established to ensure the soundness and spacing of the connections.

For other guidelines on practical considerations, erection, inspection, and other design information, see Ref. 9.45.

Example 9.1 Use the ASD method to design a longitudinal bracing system by using steel shear diaphragms for a mill building as shown in Fig. 9.8a and b. Assume that a wind load of 20 psf is applied to the end wall of the building and that corrugated steel sheets having a base metal thickness of 0.0198 in. are used as roof and wall panels.

Solution

A. Alternate I Longitudinal X-bracing is usually provided for a mill building in the planes of the roof, side walls and the lower chord of the truss,* as

*See Alternate II, in which the X-bracing is eliminated in the plane of the lower chord of the truss.
Figure 9.8  (a) Mill building. (b) End elevation. (c) Wind bracing in planes of roof and side walls, side elevation. (d) Bracing in plane of lower chord. (e) Assumed area of wind load to be carried at A, B, C, and D. (f) Shear diaphragms in planes of roof and side walls, side elevation. (g) Assumed area of wind load to be carried in planes of roof and bottom chord of truss. (h) End wall columns run all the way to roof plane.
Figure 9.8  (Continued)
shown in Fig. 9.8c and d. The intent of this example is to illustrate the use of shear diaphragms in the planes of the roof and side walls instead of the X-bracing system. The design of the roof truss and other structural members is beyond the scope of this example.

1. **Wind Panel Loads.** The wind panel loads at A, B, C, and D can be computed as follows based on the assumed area in Fig. 9.8e:

   \[
   \begin{align*}
   W_A &= 20(9.375 \times 12.5 + \frac{1}{2} \times 8.385 \times 4.193) = 2700 \text{ lb} \\
   W_B &= 20(16.77 \times 4.193) = 1410 \text{ lb} \\
   W_C &= 20(8.385 \times 4.193 + \frac{1}{2} \times 11.25 \times 7.5) = 1550 \text{ lb} \\
   W_D &= 20(20.635 \times 12.5 + 5.625 \times 7.5 + \frac{1}{2} \times 7.5 \times 15.0) = 7130 \text{ lb}
   \end{align*}
   \]

2. **Shear Diaphragm in Plane of Roof.** Considering that the planes of the side walls and the lower chord of the truss are adequately braced, the wind loads to be resisted by the roof panels used as a shear diaphragm are \(W_C\) and \(W_B\) as indicated in Fig. 9.8f. Consequently the shear developed along the eave struts is

   \[
   v = \frac{775 + 1410}{140} = \frac{2185}{140} = 15.6 \text{ lb/ft}
   \]

   From Table A.1 of Appendix A of Ref. 9.42, the average nominal shear strength for the corrugated sheets having a base metal thickness of 0.0198 in. is 370 lb/ft. Using a safety factor of 2.00 as recommended by AISI for screw connectors, the allowable shear strength for the design is

   \[
   S_{\text{des}} = \frac{370}{2.00} = 185 \text{ lb/ft}
   \]

   Since the allowable shear is much larger than the actual shear value of 15.6 lb/ft developed in the roof due to the wind load, the roof panels are adequate to resist the wind load applied to the end wall, even though no intermediate fasteners are provided. Usually intermediate fasteners are used for roof panels, and as a result, additional strength will be provided by such fasteners.

3. **Shear Diaphragm in Plane of Side Wall.** As far as the shear diaphragms in the planes of the side walls are concerned, the total load to be resisted by one side wall as shown in Fig. 9.8f, is

   \[
   P = W_A + W_B + \frac{1}{2}W_C + W_D \\
   = 2700 + 1410 + 775 + 7130 = 12,015 \text{ lb}
   \]

   \[
   S_{\text{des}} = \frac{370}{2.00} = 185 \text{ lb/ft}
   \]
The effective diaphragm width \( b_{\text{eff}} \) is the length of the building with the widths of doors and windows subtracted. This is based on the consideration that the wall panels are adequately fastened to the perimeter members around openings, that is,

\[
b_{\text{eff}} = 140 - (16.0 + 3 \times 7.5 + 7.0 + 5.0) = 89.5 \text{ ft}
\]

Therefore the shear to be resisted by the diaphragm is

\[
v = \frac{P}{b_{\text{eff}}} = \frac{12,015}{89.5} = 134 \text{ lb/ft}
\]

or the required static ultimate shear resistance should be

\[
S_u = S_d \times SF = 134 \times 2.00 = 263 \text{ lb/ft}
\]

Since the average nominal shear strength for the 0.0198-in. thick corrugated sheets spanning at 3 ft is 370 lb/ft, which is larger than the computed value of 268 lb/ft, the corrugated sheets are adequate for shear diaphragm action.

In determining the wind load to be resisted in the planes of roof and side walls, assumptions may be made as shown in Fig. 9.8g. Based on this figure, the wind load to be resisted by the shear diaphragm in the plane of the roof is 2250 lb, which is slightly larger than the load of 2185 lb used previously. The total load to be used for the design of the shear diaphragm in the planes of side walls is the same as the load computed from Fig. 9.8f.

4. **Purlin Members.** It should be noted that the shear force in the plane of roof panels can cause the tipping of purlins due to eccentricity. Rake channels or other means may be required to transmit the shear force from the plane of roof panels to chord members. This can become important in short wide buildings if purlins are framed over the top of trusses.

B. **Alternate II** When end wall columns run all the way to the roof plane, as shown in Fig. 9.8h, there is no need for X-bracing in the plane of the bottom chord. For this case, the force to be resisted by one side of the roof diaphragm is

\[
P = 20 \times \frac{1}{2}[(12.5 + 20) \times 30] = 9750 \text{ lb}
\]

and
The above shear developed along the eave struts is smaller than the allowable shear of 180 lb/ft for 0.0198-in. thick corrugated sheets. Therefore, the roof panels are adequate to act as a diaphragm.

For side walls, the shear to be resisted by the diaphragm is

\[ v = \frac{P}{b_{\text{eff}}} = \frac{9750}{89.5} = 109 \text{ lb/ft} \]

or the required static ultimate shear resistance is

\[ S_u = 109 \times 2.00 = 218 \text{ lb/ft} \]

Since the above computed \( S_u \) is less than the nominal shear strength of 370 lb/ft for the 0.0198-in. thick sheets, the wall panels are also adequate to act as a shear diaphragm. The X-bracing in the plane of the side walls can therefore be eliminated.

### 9.3 STRUCTURAL MEMBERS BRACED BY DIAPHRAGMS

#### 9.3.1 Beams and Columns Braced by Steel Diaphragms

In Art. 9.2 the application of steel diaphragms in building construction was discussed. It has been pointed out that in addition to utilizing their bending strength and diaphragm action, the steel panels and decking used in walls, roofs, and floors can be very effective in bracing members of steel framing against overall buckling of columns and lateral buckling of beams in the plane of panels. Both theoretical and experimental results indicate that the failure load of diaphragm-braced members can be much higher than the critical load for the same member without diaphragm bracing.

In the past, investigations of thin-walled steel open sections with and without bracing have been conducted by numerous investigators. Since 1961 the structural behavior of diaphragm-braced columns and beams has been studied at Cornell by Winter, Fisher, Pincus, Errera, Apparao, Celebi, Pekoz, Simaan, Soroushian, Zhang and others. In these studies, the equilibrium and energy methods have been used for diaphragm-braced beams and columns. In addition to the Cornell work, numerous studies have been conducted at other institutions and several individual steel companies.
9.3.2 Diaphragm-Braced Wall Studs

In Art. 5.10 the application of wall studs in building construction was briefly discussed. Because the shear diaphragm action of wall material can increase the load-carrying capacity of wall studs significantly, the effect of sheathing material on the design load of wall studs is considered in Sec. D4(b) and Secs. D4.1 through D4.3 of the AISI Specification. However, it should be noted that these AISI design requirements are now limited only to those studs that have identical wall material attached to both flanges. For studs with wall material on one flange only see Sec. C4.4 of the Specification. When unidentical wall materials are attached to two flanges, the reader is referred to Refs. 9.52–9.55.

In the evaluation of the load-carrying capacity of wall studs, consideration should be given to the structural strength and stiffness of the wall assembly. As far as the structural strength is concerned, the maximum load that can be carried by wall studs is governed by either (1) column buckling of studs between fasteners in the plane of the wall (Fig. 9.9) or (2) overall column buckling of studs (Fig. 9.10). The following discussion deals with the critical loads for these types of buckling.

9.3.2.1 Column Buckling of Wall Studs between Fasteners

When the stud buckles between fasteners, as shown in Fig. 9.9, the failure mode may be (1) flexural buckling, (2) torsional buckling, or (3) torsional–flexural buckling, depending on the geometric configuration of the cross section and the spacing

![Figure 9.9 Buckling of studs between fully effective fasteners.](image-url)
of fasteners. For these types of column buckling, the critical loads are based on the stud itself, without any interaction with the wall material. Therefore the design formulas given in Arts. 5.3 and 5.4 are equally applicable to these cases.

9.3.2.2 Overall Column Buckling of Wall Studs Braced by Shear Diaphragms on Both Flanges

The overall column buckling of wall studs braced by sheathing material has been studied extensively at Cornell University and other institutions. The earlier AISI provisions were developed primarily on the basis of the Cornell work.9.52–9.55 Even though the original research has considered the shear rigidity and the rotational restraint of the wall material that is attached either on one flange or on both flanges of the wall studs, for the purpose of simplicity, the AISI design requirements are provided only for the studs braced by shear diaphragms on both flanges. In addition, the rotational restraint provided by the wall material is neglected in the AISI provisions.

Based on their comprehensive studies of wall assemblies, Simaan and Pe-koz have shown several stability equations for determining the critical loads for different types of overall column buckling of wall studs.9.55 The following buckling load equations are used for channels or C-sections, Z-sections, and I-sections having wall materials on both flanges:

1. **Singly Symmetric Channels or C-sections**
   a. **Flexural buckling about y-axis**

   \[ P_{cr} = P_y + \overline{Q} \]  
   \( (9.15) \)
b. **Torsional–flexural buckling**

\[
P_{cr} = \frac{1}{2\beta} \left[ (P_x + P_{zQ}) - \sqrt{(P_x + P_{zQ})^2 - 4\beta P_x P_{zQ}} \right] \tag{9.16}
\]

where \( P_{cr} \) = critical buckling load, kips  
\( P_x \) = Euler flexural buckling load about \( x \)-axis of wall studs, kips, i.e.,

\[
\frac{\pi^2 EI_x}{(K_x L_x)^2} \tag{9.17}
\]

\( P_y \) = Euler flexural buckling load about \( y \)-axis of wall studs, kips, i.e.,

\[
\frac{\pi^2 EI_y}{(K_y L_y)^2} \tag{9.18}
\]

\( P_z \) = torsional buckling load about \( z \)-axis of wall studs, kips, i.e.

\[
\left[ \frac{\pi^2 EC_w}{(K L)^2} + GJ \right] \left( \frac{1}{r_0^2} \right) \tag{9.19}
\]

\[
P_{zQ} = P_z + \frac{Q d^2}{4r_0^2} \tag{9.20}
\]

\( \overline{Q} \) = shear rigidity for two wallboards, kips  
\( d \) = depth of channel or C-section, in.

Other symbols are as defined in Arts. 5.4 and 5.7.

2. **Z-Sections**

a. **Torsional buckling about \( z \)-axis**

\[
P_{cr} = P_{zQ} = P_z + \frac{\overline{Q} d^2}{4r_0^2} \tag{9.21}
\]

b. **Combined flexural buckling about \( x \)- and \( y \)-axes**

\[
P_{cr} = \frac{1}{2} \left[ (P_x + P_y + \overline{Q}) - \sqrt{(P_x + P_y + \overline{Q})^2 - 4(P_x P_y + P_x \overline{Q} - P_{xy})} \right] \tag{9.22}
\]
where

\[ P_{xy} = \frac{\pi^2 EI_{xy}}{(K_x K_y L)^2} \] (9.23)

and \( I_{xy} \) is the product of the inertia of wall studs, in.^4

3. **Doubly Symmetric I-Sections**
   a. **Flexural buckling about y-axis**

\[ P_{cr} = P_y + \bar{Q} \] (9.24)

b. **Flexural buckling about x-axis**

\[ P_{cr} = P_x \] (9.25)

c. **Torsional buckling about z-axis**

\[ P_{cr} = P_z \bar{Q} = P_z + \frac{\bar{Q} d^2}{4 r_0^2} \] (9.26)

By using the above equations for critical loads, the critical elastic buckling stress \( \sigma_{cr} \) can be computed as

\[ \sigma_{cr} = \frac{P_{cr}}{A} \] (9.27)

9.3.2.3 **AISI Design Criteria for Wall Studs** The following excerpts are adopted from Sec. D4 of the AISI Specification for the design of wall studs:1,314

**D4 Wall Studs and Wall Stud Assemblies**

Wall studs shall be designed either on the basis of an all steel system in accordance with Section C or on the basis of sheathing in accordance with Section D4.1 through D4.3. Both solid and perforated webs shall be permitted. Both ends of the stud shall be connected to restrain rotation about the longitudinal stud axis and horizontal displacement perpendicular to the stud axis.

(a) **All Steel Design**

Wall stud assemblies using an all steel design shall be designed neglecting the structural contribution of the attached sheathings and shall comply with the requirements of Section C. In the case of circular web perforations, see Section B2.2, and for non-circular web perforations, the effective area shall be determined as follows:

The effective area, \( A_{cr} \), at a stress \( F_n \), shall be determined in accordance
with Sec. B, assuming the web to consist of two unstiffened elements, one on each side of the perforation, or the effective area, \(A_e\), shall be determined from stub-column tests.

When \(A_e\) is determined in accordance with Section B, the following limitations related to the size and spacing of perforations and the depth of the stud shall apply:

1. The center-to-center spacing of web perforations shall not be less than 24 inches (610 mm).
2. The maximum width of web perforations shall be the lesser of 0.5 times the depth, \(d\), of the section or 2-1/2 inches (63.5 mm).
3. The length of web perforations shall not exceed 4-1/2 inches (114 mm).
4. The section depth-to-thickness ratio, \(d/t\), shall not be less than 20.
5. The distance between the end of the stud and the near edge of a perforation shall not be less than 10 inches (254 mm).

(b) Sheathing Braced Design

Wall stud assemblies using a sheathing braced design shall be designed in accordance with Sections D4.1 through D4.3 and in addition shall comply with the following requirements:

In the case of perforated webs, the effective area, \(A_e\), shall be determined as in (a) above.

Sheathing shall be attached to both sides of the stud and connected to the bottom and top horizontal members of the wall to provide lateral and torsional support to the stud in the plane of the wall.

Sheathing shall conform to the limitations specified under Table D4. Additional bracing shall be provided during construction, if required.

The equations given are applicable within the following limits:

\[
\text{Yield strength, } F_y \leq 50 \text{ ksi (345 MPa)}
\]
\[
\text{Section depth, } d \leq 6.0 \text{ in. (152 mm)}
\]
\[
\text{Section thickness, } t \leq 0.075 \text{ in. (1.91 mm)}
\]
\[
\text{Overall length, } L \leq 16 \text{ ft (4.88 mm)}
\]
\[
\text{Stud spacing, 12 in. (305 mm) minimum; 24 in. (610 mm) maximum}
\]

**D4.1 Wall Studs in Compression**

For studs having identical sheathing attached to both flanges, and neglecting any rotational restraint provided by the sheathing, the nominal axial strength, \(P_n\), shall be calculated as follows:

\[
P_n = A_e F_n
\]

\[
\Omega_c = 1.80 \text{ (ASD)}
\]

\[
\phi_c = 0.85 \text{ (LRFD)}
\]
where $A_e$ = Effective area determined at $F_n$

$F_n$ = The lowest value determined by the following three conditions:

(a) To prevent column buckling between fasteners in the plane of the wall, $F_n$ shall be calculated according to Section C4 with $KL$ equal to two times the distance between fasteners.

(b) To prevent flexural and/or torsional overall column buckling, $F_n$ shall be calculated in accordance with Section C4 with $F_e$ taken as the smaller of the two $\sigma_{cr}$ values specified for the following section types, where $\sigma_{cr}$ is the theoretical elastic buckling stress under concentric loading.

1. Singly symmetric C-sections

$$\sigma_{cr} = \sigma_{cy} + \overline{Q}_a$$  \hspace{1cm} (9.29)

$$\sigma_{cr} = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_{eq}) - \sqrt{(\sigma_{ex} + \sigma_{eq})^2 - (4\beta\sigma_{ex}\sigma_{eq})} \right]$$  \hspace{1cm} (9.30)

2. Z-Sections

$$\sigma_{cr} = \sigma_t + \overline{Q}_t$$  \hspace{1cm} (9.31)

$$\sigma_{cr} = \frac{1}{2} \left\{ (\sigma_{ex} + \sigma_{ey} + \overline{Q}_a) - \sqrt{(\sigma_{ex} + \sigma_{ey} + \overline{Q}_a)^2 - 4(\sigma_{ex}\sigma_{ey} + \sigma_{ex}\overline{Q}_a - \sigma_{exy})} \right\}$$  \hspace{1cm} (9.32)

3. I-Sections (doubly symmetric)

$$\sigma_{cr} = \sigma_{cy} + \overline{Q}_a$$  \hspace{1cm} (9.33)

$$\sigma_{cr} = \sigma_{ex}$$  \hspace{1cm} (9.34)

In the above formulas:

$$\sigma_{ex} = \frac{\pi^2E}{(L/r_y)^2}$$  \hspace{1cm} (9.35)

$$\sigma_{exy} = (\pi^2EI_{ey})/(AL^2)$$  \hspace{1cm} (9.36)

$$\sigma_{ey} = \frac{\pi^2E}{(L/r_y)^2}$$  \hspace{1cm} (9.37)

$$\sigma_t = \frac{1}{Ar_0^2} \left[ GJ + \frac{\pi^2EC_w}{(L)^2} \right]$$  \hspace{1cm} (9.38)

$$\sigma_{eq} = \sigma_t + \overline{Q}_t$$  \hspace{1cm} (9.39)

$$\overline{Q} = \overline{Q}_o(2 - s/s')$$  \hspace{1cm} (9.40)
where \( s = \) fastener spacing, in. (mm); \( 6 \) in. (152 mm) \( \leq s \leq 12 \) in. (305 mm)

\[ s' = 12 \text{ in. (305 mm)} \]

\[ \bar{Q} = \text{See Table D4} \]

\[ \bar{Q}_a = \frac{Q}{A} \quad (9.41) \]

\( A = \) area of full unreduced cross section

\( L = \) length of stud

\[ Q_1 = \frac{(Qd^2)}{4A r_0^2} \quad (9.42) \]

\( d = \) depth of section

\( I_{xy} = \) product of inertia

(c) To prevent shear failure of the sheathing, a value of \( F_n \) shall be used in the following equations so that the shear strain of the sheathing, \( \gamma \), does not exceed the permissible shear strain, \( \bar{\gamma} \). The shear strain, \( \gamma \), shall be determined as follows:

\[ \gamma = (\pi/L)[C_1 + (E_id/2)] \quad (9.43) \]

where

\( C_1 \) and \( E_1 \) are the absolute values of \( C_1 \) and \( E_1 \) specified below for each section type:

1. Singly Symmetric C-Sections

\[ C_1 = \frac{(F_nC_0)}{(\sigma_{ey} - F_n + \bar{Q}_a)} \quad (9.44) \]

\[ E_1 = \frac{F_n[(\sigma_{ex} - F_n)r_0^2E_0 - x_0D_0] - F_n x_0(D_0 - x_0E_0)}{(\sigma_{ex} - F_n)r_0^2(\sigma_{Q} - F_n) - (F_n x_0)^2} \quad (9.45) \]

2. Z-Sections

\[ C_1 = \frac{F_n[C_0(\sigma_{ex} - F_n) - D_0\sigma_{exy}]}{(\sigma_{ey} - F_n + \bar{Q}_a)(\sigma_{ex} - F_n) - \sigma_{exy}^2} \quad (9.46) \]

\[ E_1 = \frac{(F_nE_0)}{(\sigma_{Q} - F_n)} \quad (9.47) \]

3. I-Sections

\[ C_1 = \frac{(F_nC_0)}{(\sigma_{ey} - F_n + \bar{Q}_a)} \quad (9.48) \]

\[ E_1 = 0 \]

where \( x_0 = \) distance from shear center to centroid along principal \( x \)-axis, in.

(absolute value)

\( C_0, E_0, \) and \( D_0 \) are initial column imperfections which shall be assumed to be at least

\[ C_0 = L/350 \text{ in a direction parallel to the wall} \quad (9.49) \]
$D_0 = L/700$ in a direction perpendicular to the wall

$E_0 = L/(d \times 10,000)$, rad, a measure of the initial twist of the stud from the initial ideal, unbuckled shape

If $F_n > 0.5 F_y$, then in the definitions for $\sigma_{ov}$, $\sigma_{mv}$, $\sigma_{ev}$, and $\sigma_{Q\theta}$, the parameters $E$ and $G$ shall be replaced by $E'$ and $G'$, respectively, as defined below:

$$E' = 4EF_n(F_y - F_n)/F_y^2$$

$$G' = G(E'/E)$$

Sheathing parameters $\bar{Q}_o$ and $\bar{\gamma}$ shall be permitted to be determined from representative full-scale tests, conducted and evaluated as described by published documented methods (see Commentary), or from the small-scale-test values given in Table D4.

### D4.2 Wall Studs in Bending

For studs having identical sheathing attached to both figures, and neglecting any rotational restrain provided by the sheathing, the nominal flexural strengths are $M_{nxo}$ and $M_{nyo}$, where

For sections with stiffened or partially stiffened compression flanges:

$$\Omega_b = 1.67 \text{ (ASD)}$$

$$\phi_b = 0.95 \text{ (LRFD)}$$

For sections with unstiffened compression flanges:

$$\Omega_b = 1.67 \text{ (ASD)}$$

$$\phi_b = 0.90 \text{ (LRFD)}$$

$M_{nxo}$ and $M_{nyo}$ = Nominal flexural strengths about the centroidal axes determined in accordance with Section C3.1, excluding the provisions of Section C3.1.2 (lateral buckling)

### D4.3 Wall Studs with Combined Axial Load and Bending

The required axial strength and flexural strength shall satisfy the interaction equations of Section C5 with the following redefined terms:

$P_n$ = Nominal axial strength determined according to Section D4.1

$M_{nx}$ and $M_{ny}$ in Equations C5.2.1-1, C5.2.1-2 and C5.2.1-3 for ASD or C5.2.2-1, C5.2.2-2 and C5.2.2-3 shall be replaced by nominal flexural strengths, $M_{nxo}$ and $M_{nyo}$, respectively.


TABLE D4 Sheathing Parameters\(^{(1)}\)

<table>
<thead>
<tr>
<th>Sheathing(^{(2)})</th>
<th>(\overline{Q})(^{0})</th>
<th>(k)</th>
<th>(kN^{\prime})</th>
<th>Length/length</th>
</tr>
</thead>
<tbody>
<tr>
<td>3/8 in. (9.5 mm) to 5/8 in. (15.9 mm) thick gypsum</td>
<td>24.0</td>
<td>107.0</td>
<td>0.008</td>
<td></td>
</tr>
<tr>
<td>Lignocellulosic board</td>
<td>12.0</td>
<td>53.4</td>
<td>0.009</td>
<td></td>
</tr>
<tr>
<td>Fiberboard (regular or impregnated)</td>
<td>7.2</td>
<td>32.0</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td>Fiberboard (heavy impregnated)</td>
<td>14.4</td>
<td>64.1</td>
<td>0.010</td>
<td></td>
</tr>
</tbody>
</table>

\(^{(1)}\) The values given are subject to the following limitations:
- All values are for sheathing on both sides of the wall assembly.
- All fasteners are No. 6, type S-12, self-drilling drywall screws with pan or bugle head, or equivalent.

\(^{(2)}\) All sheathing is 1/2 in. (12.7 mm) thick except as noted.
- For other types of sheathing, \(Q\) shall be permitted to be determined conservatively from representative small-specimen tests as described by published documented methods (see Commentary).

It should be noted that in 1996, the AISI design provisions for wall studs were revised to permit (a) all-steel design and (b) sheathing braced design of wall studs with either solid or perforated webs. For sheathing braced design, in order to be effective, sheathing must retain its design strength and integrity for the expected service life of the wall.

For the case of all-steel design, the approach of determining effective areas in accordance with Specification Section D4(a) is being used in the RMI Specification\(^{1.165}\) for the design of perforated rack columns. The validity of this approach for wall studs was verified in a Cornell project on wall studs reported by Miller and Pekoz.\(^{9.98}\) The limitations for the size and spacing of perforations and the depth of studs are based on the parameters used in the program. For sections with perforations which do not meet these limits, the effective area can be determined by stub column tests.

In the foregoing AISI design provisions, Sec. D4.1(a) is based on the discussion given in Art. 9.3.2.1, except that the effective length \(KL\) is taken as two times the distance between fasteners. Thus even if an occasional attachment is defective to a degree that it is completely inoperative, the allowable design load will still be sufficient.

In Sec. D4.1(b) of the specification, Eqs. (9.29) through (9.34) were derived from Eqs. (9.15) through (9.25) with \(K_x = K_y = K_t = 1.0\). The type of torsional buckling of doubly symmetric I-sections [Eq. (9.26)] is not considered in the AISI requirements because it is not usually a failure mode.

The design shear rigidity \(Q\) for two wallboards was determined in the 1980 and 1986 editions of the Specification as
\( \overline{Q} = \overline{q} B \) \hspace{1cm} (9.55)

in which the value \( \overline{q} \) was defined as the design shear rigidity for two wall-boards per inch of stud spacing. Based on the discussions presented in Ref. 9.55, \( \overline{q} \) can be determined by

\[ \overline{q} = \frac{2G'}{SF} \] \hspace{1cm} (9.56)

where \( G' = \) diaphragm shear stiffness of a single wallboard for a load of 0.8\( P_{ult} \)

\[ \frac{0.8P_{ult}}{b} = \frac{0.8P_{ult}}{ \Delta_d / a} = \frac{0.8P_{ult}}{ \Delta_d / b}, \text{ kips/in.} \] \hspace{1cm} (9.57)

\( P_{ult} = \) ultimate load reached in shear diaphragm test of a given wallboard, kips (Fig. 9.11)

\( \Delta_d = \) shear deflection corresponding to a load of 0.8\( P_{ult} \), in. (Fig. 9.11)

\( a, b = \) geometric dimensions corresponding to shear diaphragm test frame, ft (Fig. 9.11)

\( SF = \) safety factor, = 1.5

The reason for using 0.8\( P_{ult} \) for \( G' \) is that the shear deflection and thus the shear rigidity at the ultimate load \( P_{ult} \) are not well defined and reproducible. A safety factor of 1.5 was used to avoid premature failure of the wallboard.

---

**Figure 9.11** Determination of shear rigidity, \( \overline{Q}^{0.55} \)
9.3 STRUCTURAL MEMBERS BRACED BY DIAPHRAGMS

By substituting the equation of $G'$ and the safety factor into Eq. (9.56), the design shear rigidity for wallboards on both sides of the stud can be evaluated as

$$\bar{q} = \frac{0.53 \, P_{\text{ult}} \left( \frac{a}{b} \right)}{\Delta_d}$$

(9.58)

Based on the results of a series of shear diaphragm tests using different wallboards with No. 6, type S-12, self-drilling drywall screws at 6- to 12-in. (152- to 305-mm) spacing, some typical values of $\bar{q}_0$ have been developed and were given in Table D4 of the 1980 and 1986 editions of the AISI Specification. In this table, the value of $\bar{q}_0$ was computed by

$$\bar{q}_0 = \frac{\bar{q}}{2 - s/12}$$

(9.59)

where $s$ is the fastener spacing, in.

In the 1996 edition of the AISI Specification, the equation for the design shear rigidity $Q$ for sheathing on both sides of the wall was rewritten on the basis of a recent study of gypsum-sheathed cold-formed steel wall studs. In Ref. 9.108, Miller and Pekoz indicated that the strength of gypsum wall board-braced studs was observed to be rather intensive to stud spacing. Moreover, the deformations of gypsum wallboard panel (in tension) were observed to be localized at the fasteners, and not distributed throughout the panel as in a shear diaphragm. The $Q_o$ values listed in Table D4 were determined from $Q_o = 12\bar{q}_0$, in which the $\bar{q}_0$ values were obtained from the 1986 edition of the AISI Specification. The values given in Table D4 for gypsum are based on dry service conditions.

In addition to the requirements discussed above, the AISI Specification considers the shear strain requirements as well. In this regard, Sec. D4.1(c) specifies that the computed shear strain $\gamma$ according to Eq. (9.43) and for a value of $F_n$, should not exceed the permissible shear strain of the wallboard $\bar{\gamma}$ given in Table D4 of the specification. From Eqs. (9.43) through (9.48) it can be seen that the shear strain in the wallboard is affected by the initial imperfections of wall studs, for which some minimum values for sweep, camber, and possible twist of studs, are recommended in Eqs. (9.49) through (9.51) to represent the general practice.

Example 9.2 Use the ASD and LRFD methods to compute the allowable axial load for the C-section shown in Fig. 9.12 if it is to be used as wall studs having a length of 12 ft. Assume that the studs are spaced at every 12 in. and that ½-in. thick gypsum boards are attached to both flanges of the stud. All fasteners are No. 6, type S-12, self-drilling drywall screws at 12-in. spacing. Use $F_y = 33$ ksi. Assume that the dead-to-live load ratio is 1/5.
Solution

A. ASD Method

1. Sectional Properties. Using the methods discussed in Chaps. 4 and 5 and the AISI Design Manual the following sectional properties can be computed on the basis of the full area of the given C-section:

\[
\begin{align*}
A &= 0.651 \text{ in.}^2 \\
I_x &= 2.823 \text{ in.}^4 \\
I_y &= 0.209 \text{ in.}^4 \\
r_x &= 2.08 \text{ in.} \\
r_y &= 0.566 \text{ in.} \\
J &= 0.00109 \text{ in.}^4 \\
C_w &= 1.34 \text{ in.}^6 \\
x_0 &= 1.071 \text{ in.} \\
r_0 &= 2.41 \text{ in.}
\end{align*}
\]
2. **Allowable Axial Load.** According to Sec. D4.1 of the AISI Specification, the allowable axial load for the given stud having identical sheathing attached to both flanges and neglecting any rotational restraint provided by the sheathing can be determined by Eq. (9.28) as follows:

\[ P_a = A_e F_n / \Omega_c \]

In the above equation, the nominal buckling stress \( F_n \) is the lowest value determined by the following three conditions:

\( (F_n)_1 \) = nominal buckling stress for column buckling of stud between fasteners in the plane of the wall

\( (F_n)_2 \) = nominal buckling stress for flexural and/or torsional-flexural overall column buckling

\( (F_n)_3 \) = nominal buckling stress to limit shear strain of wallboard to no more than the permissible value

a. **Calculation of \( (F_n)_1 \).** In order to prevent column buckling of the stud between fasteners in the plane of the wall, consideration should be given to flexural buckling and torsional-flexural buckling of the singly symmetric C-section. In the calculation of the elastic buckling stress, the effective length \( KL \) is taken to be two times the distance between fasteners.

i. **Nominal buckling stress for flexural buckling**

\[ KL = 2 \times \text{(spacing of screws)} \]

\[ = 2 \times 12 = 24 \text{ in.} \]

\[ KL/r = 24/r_y = 42.40 \]

Using Eq. (5.56)

\[ F_e = \frac{\pi^2 E}{(KL/r)^2} = \frac{\pi^2(29,500)}{(42.40)^2} = 161.95 \text{ ksi} \]

\[ \lambda_c = \frac{F_y}{\sqrt{F_e}} = \sqrt{\frac{33}{161.95}} = 0.451 < 1.5 \]

From Eq. (5.54),
ii. Nominal buckling stress for torsional–flexural buckling. From Eq. (5.57),

$$F_e \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_t) - \sqrt{(\sigma_{ex} + \sigma_t)^2 - 4 \beta \sigma_{ex} \sigma_t} \right]$$

where $\beta = 1 - \left( \frac{x_0}{r_0} \right)^2 = 0.803$

$$\sigma_{ex} = \frac{\pi^2 E}{(KL/r_0)^2} = \frac{\pi^2 (29,500)}{(24/2.08)^2} = 2187 \text{ ksi}$$

$$\sigma_t = \frac{1}{Ar_0^2} \left[ GJ + \frac{\pi^2 EC_w}{(KL)^2} \right]$$

$$= \frac{1}{0.651(2.41)^2} \left[ 11,300 \times 0.00109 
+ \frac{\pi^2 (29,500)(1.34)}{(24)^2} \right]$$

$$= 182.4 \text{ ksi}$$

Therefore

$$F_e = 179.2 \text{ ksi}$$

$$\lambda_e = \sqrt{\frac{F_y}{F_e}} = \sqrt{\frac{33}{179.2}} = 0.429 < 1.5$$

From Eq. (5.54),

$$F_n = (0.658^{3/2}) F_y = (0.658^{0.429^2})(33)$$

$$= 30.55 \text{ ksi}$$

The governing nominal buckling stress is the smaller of the values $F_n$ computed in items (i) and (ii) above, i.e.,

$$(F_n)_{(1)} = 30.31 \text{ ksi}$$

b. Calculation of $(F_n)_{(2)}$. In order to prevent flexural and/or torsional–flexural overall column buckling, the theoretical elastic buckling stress $F_e$ should be the smaller of the two $\sigma_{cr}$ values computed for the singly symmetric C-section as follows:
For flexural overall column buckling,

\[ \sigma_{CR} = \sigma_{ey} + \bar{Q}_a \]  \hspace{1cm} (9.29)

For torsional–flexural overall column buckling,

\[ \sigma_{CR} = \frac{1}{2\beta} \left[ (\sigma_{ex} + \sigma_{tQ}) - \sqrt{(\sigma_{ex} + \sigma_{tQ})^2 - 4\beta \sigma_{ex}\sigma_{tQ}} \right] \]  \hspace{1cm} (9.30)

In Eq. (9.29)

\[ \sigma_{ey} = \frac{\pi^2 E}{(KL/r_e)^2} = \frac{\pi^2(29,500)}{(12 \times 12/0.566)^2} = 4.50 \text{ ksi} \]

\[ \bar{Q}_a = \frac{\bar{Q}}{A} \]

From Eq. (9.40) and Table D4 of the AISI Specification,

\[ \bar{Q} = \bar{Q}_o (2 - s/s') = (24.0)(2 - 12/12) \]

\[ = 24 \text{ kips} \]

\[ \bar{Q}_a = \frac{24}{0.651} = 36.87 \text{ ksi} \]

According to Eq. (9.29), the theoretical elastic critical buckling stress is

\[ \sigma_{CR} = 4.50 + 36.87 = 41.37 \text{ ksi} \]  \hspace{1cm} (9.29a)

In Eq. (9.30),

\[ \beta = 0.803 \]

\[ \sigma_{ex} = \frac{\pi^2 E}{(KL/r_e)^2} = \frac{\pi^2(29,500)}{(12 \times 12/2.08)^2} = 60.75 \text{ ksi} \]

\[ \sigma_{tQ} = \sigma_t + \bar{Q}_t \]

where \( \sigma_t = \frac{1}{A r_0^2} \left( G J + \frac{\pi^2 E C_w}{L^2} \right) \)

\[ = \frac{1}{0.651(2.41)^2} \left[ 11,300 \times 0.00109 + \frac{\pi^2(29,500)(1.34)}{(12 \times 12)^2} \right] \]

\[ = 8.23 \text{ ksi} \]
\[
\overline{Q}_t = \frac{Qd^2}{4Ar_0^2} = \frac{(24)(5.50)^2}{4(0.651)(2.41)^2} = 48.00 \text{ ksi}
\]

and

\[
\sigma_{iQ} = 8.23 + 48.00 = 56.23 \text{ ksi}
\]

From Eq. (9.30),

\[
\sigma_{CR} = \frac{1}{2(0.803)} \left[ (60.75 + 56.23) - \sqrt{(60.75 + 56.23)^2 - 4(0.803)(60.75)(56.23)} \right] = 40.41 \text{ ksi}
\]

Use the smaller value given in Eqs. (9.29a) and (9.30a)

\[
F_e = 40.41 \text{ ksi}
\]

\[
\lambda_e = \sqrt[3]{\frac{F_y}{F_e}} = \sqrt[3]{\frac{33}{40.41}} = 0.904 < 1.5
\]

From Eq. (5.54),

\[
(F_n)_{x^2} = (0.658^{0.5})F_y = (0.658^{0.9042})(33)
\]

\[
= 23.44 \text{ ksi}
\]

c. *Calculation of \( (F_n)_{x^3} \).* In items (a) and (b) it was found that in order to prevent column buckling, the nominal buckling stress should not exceed 23.44 ksi. According to Sec. D4.1(c) of the AISI Specification, in order to prevent shear failure of the sheathing, a value of \( F_n \) should be used in the given equations so that the shear strain of the sheathing, \( \gamma \), computed by Eq. (9.43) does not exceed the permissible value of \( \gamma = 0.008 \) in./in., which is given in Table D4 of the AISI Specification for ½-in.-thick gypsum board.

Based on Eq. (9.43), the shear strain of the sheathing can be computed as follows:

\[
\gamma = \frac{\pi}{L} \left( C_1 + E_1 \frac{d}{2} \right)
\]
where

\[
C_1 = \frac{F_nC_0}{\sigma_{ey} - F_n + Q_a} \quad (9.44)
\]

\[
E_1 = \frac{F_n[\sigma_{ex} - F_n(r_0E_0 - x_0D_0) - F_nx_0(D_0 - x_0E_0)]}{(\sigma_{ex} - F_n)\sigma_0^2(\sigma_{equ} - F_n) - (F_nx_0)^2} \quad (9.45)
\]

As the first approximation, let

\[
F_n = (F_n)_{23.44} \text{ ksi}
\]

\[
C_0 = \frac{L}{350} = \frac{12 \times 12}{350} = 0.411 \text{ in.}
\]

\[
E_0 = \frac{L}{d \times 10,000} = \frac{12 \times 12}{5.50 \times 10,000} = 0.0026 \text{ rad.}
\]

\[
D_0 = \frac{L}{700} = \frac{12 \times 12}{700} = 0.206 \text{ in.}
\]

Therefore, from Eqs. (9.44) and (9.45),

\[
C_1 = \frac{23.44 \times 0.411}{4.50 - 23.44 + 36.87} = 0.537
\]

\[
E_1 = -0.0462
\]

Use an absolute value, \(E_1 = 0.0462\)

Substituting the values of \(C_1\), \(E_1\), \(L\), and \(d\) into Eq. (9.43), the shear strain is

\[
\gamma = \left(\frac{\pi}{12 \times 12}\right)\left[0.537 + \frac{0.0462 \times 5.5}{2}\right]
\]

\[
= 0.0145 \text{ in./in.} > (\tilde{\gamma} = 0.008 \text{ in./in.})
\]

Since the computed \(\gamma\) value for \(F_n = 23.44\) ksi is larger than the permissible \(\tilde{\gamma}\) value, a smaller \(F_n\) value should be used. After several trials, it was found that a value of \(F_n = 17.50\) ksi would give the permissible shear strain of 0.008 in./in. as shown below.
Try $F_n = 17.50$ ksi

Since $F_n > (F_y/2 = 16.5$ ksi), use $E'$ and $G'$ to compute the values of $\sigma_{ex}$, $\sigma_{ey}$, $\sigma_t$, and $\sigma_{tQ}$.

$$E' = 4E\frac{F_y - F_n}{F_y^2}$$
$$= 4(E)(17.50)(33 - 17.50)/(33)^2 = 0.996E, \text{ ksi}.$$ $G' = G(E'/E) = 0.996G$ ksi

$\sigma_{ex} = 60.75 \left( \frac{E'}{E} \right) = 60.51$ ksi

$\sigma_{ey} = 4.50 \left( \frac{E'}{E} \right) = 4.48$ ksi

$\sigma_t = 8.23 \left( \frac{E'}{E} \right) = 8.20$ ksi

$\sigma_{tQ} = \sigma_t + \bar{Q_t} = 8.20 + 48.00 = 56.20$

$C_1 = 0.302$ in.

$E_1 = 0.0238$ rad.

and $\gamma = 0.008$ in./in. = ($\overline{\gamma} = 0.008$ in./in.) O.K.

Therefore $(F_n)_3 = 17.50$ ksi.

d. Determination of $F_n$. From items (a), (b), and (c), the following three values of nominal buckling stress were computed for different design considerations:

$$(F_n)_1 = 30.31$ ksi

$$(F_n)_2 = 23.44$ ksi

$$(F_n)_3 = 17.50$ ksi

The smallest value of the above three stresses should be used for computing the effective area and the allowable axial load for the given C-section stud. i.e.,

$$F_n = 17.50$$ ksi

e. Calculation of the effective area. The effective area should be computed for the governing nominal buckling stress of 17.50 ksi.
9.3 STRUCTURAL MEMBERS BRACED BY DIAPHRAGMS

i. Effective width of compression flanges (Art. 3.5.3.2)

\[ S = 1.28\sqrt{E/f} = 1.28\sqrt{29,500/17.50} = 53.55 \]
\[ S/3 = 17.52 \]
\[ w_1/t = [1.625 - 2(0.136 + 0.071)]/0.071 \]
\[ = 1.211/0.071 = 17.06 \]

Since \( w_1/t < S/3 \), \( b_1 = w_1 = 1.211 \) in.
The flanges are fully effective.

ii. Effective width of edge stiffeners (Art. 3.5.3.2)

\[ w_2/t = [0.500 - (0.136 + 0.071)]/0.071 \]
\[ = 0.293/0.071 = 4.13 < 14 \quad \text{O.K.} \]
\[ \lambda = \frac{1.052}{\sqrt{1/k}} \left( \frac{w_2}{t} \right) \sqrt{\frac{f}{E}} = \frac{1.052}{\sqrt{1/0.43}} \left( \frac{17.50}{29,500} \right) \]
\[ = 0.161 < 0.673 \]
\[ d'_s = w_2 = 0.293 \text{ in.} \]
\[ d_s = d'_s = 0.293 \text{ in.} \]

The edge stiffeners are fully effective.

iii. Effective width of webs (Art. 3.5.1.1)

\[ w_3/t = [5.50 - 2(0.136 + 0.071)]/0.071 \]
\[ = 5.086/0.071 = 71.63 \]
\[ \lambda = \frac{1.052}{\sqrt{4}} \left( \frac{71.63}{1/k} \right) \sqrt{\frac{17.50}{29,500}} = 0.918 > 0.673 \]
\[ \rho = (1 - 0.22/\lambda)/\lambda = (1 - 0.22/0.918)/0.918 \]
\[ = 0.828 \]
\[ b_3 = \rho w_3 = 0.828 (5.086) = 4.211 \text{ in.} \]

iv. Effective area, \( A_e \)

\[ A_e = A - (w_3 - b_3)(t) = 0.651 - (5.086 - 4.211)(0.071) \]
\[ = 0.589 \text{ in.}^2 \]
f. *Nominal axial load and allowable axial load.* Based on $F_n = 17.50$ ksi and $A_e = 0.589$ in.$^2$, the nominal axial load is

$$P_n = A_e F_n = (0.589)(17.50) = 10.31 \text{ kips}$$

The allowable axial load is

$$P_a = \frac{P_n}{\Omega_e} = \frac{10.31}{1.80} = 5.73 \text{ kips}$$

**B. LRFD Method**

For the LRFD method, the design strength is

$$\phi_c P_n = (0.85)(10.31) = 8.76 \text{ kips}$$

Based on the load combinations given in Art. 3.3.2.2, the governing required axial load is

$$P_u = 1.2P_D + 1.6P_L = 1.2P_D + 1.6(5P_D) = 9.2P_D$$

Using $P_u = \phi_c P_n$,

$$P_D = 8.76/9.2 = 0.95 \text{ kips}$$

$$P_L = 5P_D = 4.75 \text{ kips}$$

The allowable axial load is

$$P_a = P_D + P_L = 5.70 \text{ kips}$$

It can be seen that both ASD and LRFD methods give approximately the same result for this particular case. It should also be noted that the C-section stud used in this example is selected from page I-12 of the 1996 edition of the *AISI Cold-Formed Steel Design Manual*. This C-section with lips is designated as 5.5CS1.625 × 071. The North American Steel Framing Alliance (NASFA) has recently announced the new universal designator systems for cold-formed steel studs, joists and track sections. For details, see Ref. 9.109 or visit the NASFA website: www.steel-framingalliance.com.

The preceding discussion and Example 9.2 dealt with the wall studs under concentric loading. For studs subjected to axial load and bending moment, the design strength of the studs should be determined according to Sec. D4.3 of the AISI Specification. A study of wall studs with combined compression and lateral loads was reported in Ref. 9.66. Additional studies on the behavior
9.4 SHELL ROOF STRUCTURES

9.4.1 Introduction

Steel folded-plate and hyperbolic paraboloid roof structures have been used increasingly in building construction for churches, auditoriums, gymnasiums, classrooms, restaurants, office buildings, and airplane hangars. This is because such steel structures offer a number of advantages as compared with some other types of folded-plate and shell roof structures to be discussed. Since the effective use of steel panels in roof construction is not only to provide an economical structure but also to make the building architecturally attractive and flexible for future change, structural engineers and architects have paid more attention to steel folded-plate and hyperbolic paraboloid roof structures during recent years.

The purpose of this discussion is mainly to describe the methods of analysis and design of folded-plate and hyperbolic paraboloid roof structures which are currently used in engineering practice. In addition, it is intended to review briefly the research work relative to steel folded-plate and shell roof structures and to compare the test results with those predicted by analysis.

In this discussion, design examples will be used for illustration. The shear strength of steel panels, the empirical formulas to determine deflection, and the load factors used in various examples are for illustrative purposes only. Actual design values and details of connections should be based on individual manufacturers’ recommendations on specific products.

9.4.2 Folded-Plate Roofs

9.4.2.1 General Remarks  A folded-plate structure is a three-dimensional assembly of plates. The use of steel panels in folded-plate construction started in this country about 1960. Application in building construction has increased rapidly during recent years. The design method used in engineering practice is mainly based on the successful investigation of steel shear diaphragms and cold-formed steel folded-plate roof structures.

9.4.2.2 Advantages of Steel Folded-Plate Roofs  Steel folded-plate roofs are being used increasingly because they offer several advantages in addition to the versatility of design:
1. Reduced Dead Load. A typical steel folded plate generally weighs about 11 lb/ft$^2$ (527 N/m$^2$), which is substantially less than some other types of folded plates.

2. Simplified Design. The present design method for steel plates roofs is simpler than the design of some other types of folded plates, as discussed later.

3. Easy Erection. Steel folded-plate construction requires relatively little scaffolding and shoring. Shoring can be removed as soon as the roof is welded in place.

9.4.2.3 Types of Folded-Plate Roofs Folded-plate roofs can be classified into three categories: single-bay, multiple-bay, and radial folded plate, as shown in Fig. 9.13. The folded plates can be either prismatic or nonprismatic.

The sawtooth folded-plate roof shown in Fig. 9.13$b$ has been found to be the most efficient multiple-bay structure and is commonly used in building construction. Figure 9.14 shows a folded-plate structure of the sawtooth type used for schools.

Figure 9.13 Types of folded-plate roofs. (a) Single-bay. (b) Multiple-bay. (c) Radial.
9.4 SHELL ROOF STRUCTURES

9.4.2.4 Analysis and Design of Folded Plates  A folded-plate roof structure consists mainly of three components, as shown in Fig. 9.15:

1. Steel roof panels.
2. Fold line members at ridges and valleys.
3. End frames or end walls.

In general, the plate width (or the span length of roof panels) ranges from about 7 to 12 ft (2.1 to 3.7 m), the slope of the plate varies from about 20° to 45°, and the span length of the folded plate may be up to 100 ft (30.5 m).
Since unusually low roof slopes will result in excessive vertical deflections and high diaphragm forces, it is not economical to design a roof structure with low slopes.

In the analysis and design of folded-plate roofs, two methods are available to engineers. They are the simplified method and the finite-element method. The former provides a direct technique that will suffice for use in the final design for many structures. The latter permits a more detailed analysis for various types of loading, support, and material.

In the simplified method, steel roof panels as shown in Fig. 9.15 are designed as simply supported slabs in the transverse direction between fold lines. The reaction of the panels is then applied to fold lines as a line loading, which resolves itself into two components parallel to the two adjacent plates, as shown in Fig. 9.16. These load components are then carried by an inclined deep girder spanned between end frames or end walls (Fig. 9.15). These deep girders consist of fold line members as flanges and steel panels as a web element. The longitudinal flange force in fold line members can be obtained by dividing the bending moment of the deep girder by its depth. The shear force is resisted by the diaphragm action of the steel roof panels. Therefore the shear diaphragm discussed in Art. 9.2 is directly related to the design of the folded-plate structure discussed in this article.

In the design of fold line members, it is usually found that the longitudinal flange force is small because of the considerable width of the plate. A bent plate or an angle section is often used as the fold line member.

Referring to Fig. 9.15, an end frame or end wall must be provided at the ends of the folded plates to support the end reaction of the inclined deep girder. In the design of the end frame or end wall, the end reaction of the plate may be considered to be uniformly distributed through the entire depth of the girder. Tie rods between valleys must be provided to resist the horizontal thrust. If a rigid frame is used, consideration should be given to such a horizontal thrust.

When a masonry bearing wall is used, a steel welding plate should be provided at the top of the wall for the attachment of panels. It should be capable of resisting the force due to the folded-plate action.

Along the longitudinal exterior edge, it is a general practice to provide a vertical edge plate or longitudinal light framing with intermittent columns to

![Figure 9.16](image_url)  
**Figure 9.16**  Force components along fold lines.
carry vertical loads. If an exterior inclined plate is to cantilever out from the fold line, a vertical edge plate will not be necessary.

In addition to the consideration of beam strength, the deflection characteristics of the folded-plate roof should also be investigated, particularly for long-span structures. It has been found that a method similar to the Williot diagram for determining truss deflections can also be used for the prediction of the deflection of a steel folded-plate roof. In this method the in-plane deflection of each plate should first be computed as a sum of the deflections due to flexure, shear, and seam slip, considering the plate temporarily separated from the adjacent plates. The true displacement of the fold line can then be determined analytically or graphically by a Williot diagram. When determining flexural deflection, the moment of inertia of the deep girder may be based on the area of the fold line members only. The shear deflection and the deflection due to seam slip should be computed by the empirical formulas recommended by manufacturers for the specific panels and the system of connection used in the construction. In some cases it may be found that the deflection due to seam slip is negligible.

**Example 9.3** Discuss the procedures to be used for the design of an interior plate of a multiple-bay folded-plate roof (Fig. 9.17) by using the simplified ASD method.

**Given:**
- Uniform dead load \( w_D \), psf (along roof surface).
- Uniform live load \( w_L \), psf (on horizontal projection).
- Span \( L \), ft.
- Unit width \( B \), ft.
- Slope distance \( b \) between fold lines, ft (or depth of analogous plate girder).

**Solution**

1. **Design of Steel Panels—Slab Action in Transverse Direction**

   \[ M_1 = \frac{1}{8} \times w_L B^2 + \frac{1}{8} w_D b B \text{ ft-lb} \]

   Select a panel section to meet the requirements of beam design and deflection criteria.

**Figure 9.17** Example 9.3.
2. **Design of Fold Line Members.** The vertical line loading is

\[ w = w_l B + w_t b \text{ lb/ft} \]

The load component in the direction of the inclined girder is \( w' \). The total load applied to the inclined deep girder \( EF \) is \( 2w' \).

\[ M_2 = \frac{1}{8} \times 2w'L^2 \text{ ft-lb} \]

Select a fold line member to satisfy the required moment.

3. **Design for Plate Shear**

\[ V = 2w' \times \frac{L}{2} = w'L \text{ lb} \]

\[ v = \frac{V}{b} \text{ lb/ft} \]

Select an adequate welding system on the basis of the manufacturer’s recommendations.

4. **Deflection**

   a. **In-plane flexural deflection** (considering that the inclined plate is temporarily separated)

\[ \Delta_b = \frac{5}{384} \times \frac{2w'L^4}{EI} \text{ in.} \quad (12) \]

where

\[ I = 2A_t \left( \frac{b}{2} \right)^2 \]

\[ E = 29.5 \times 10^6 \text{ psi} \]

b. **In-plane shear deflection** (including the deflection due to seam slip)

\[ \Delta_s = \frac{2w'L^2}{2G'b} \text{ in.} \]

where \( G' \) is the shear stiffness of steel panels obtained from diaphragm tests, lb/in. See Art. 9.2.
c. Total in-plane deflection

\[ \Delta = \Delta_b + \Delta_i \text{ in.} \]

d. Total vertical deflection. After the in-plane deflection is computed, the maximum vertical deflection of fold line members can be determined by a Williot diaphragm, as shown in Fig. 9.18.

9.4.2.5 Research on Folded-Plate Roofs Full-size folded-plate assemblies have been tested by Nilson at Cornell University\textsuperscript{1.77} and by Davies, Bryan, and Lawson at the University of Salford.\textsuperscript{9.77} The following results of tests were discussed in Ref. 1.77 for the Cornell work.

The test assembly used by Nilson was trapezoidal in cross section and was fabricated from 1\(\frac{1}{2}\)-in. (38-mm) deep cold-formed steel panels as plates (five plates) and 3\(\frac{3}{8}\) -in. (89-mm) bent plate as fold line members. The span length of the test structure was 42 ft 6 in. (13 m), and the width of the assembly was 14 ft (4.3 m), as shown in Fig. 9.19. The test setup is shown in Fig. 9.20. It should be noted that the jack loads were applied upward because of the convenience of testing.

In Ref. 1.77 Nilson indicated that the experimental structure performed in good agreement with predictions based on the simplified method of analysis, which was used in Example 9.3. It was reported that the tested ultimate load was 11% higher than that predicted by analysis and that the observed stresses in fold line members were about 20% lower than indicated by analysis due to the neglect of the flexural contribution of the steel panel flat plate elements. In view of the fact that this difference is on the safe side and the size of the fold line members is often controlled by practical considerations and clearance requirements rather than by stress, Nilson concluded that no modification of the design method would be necessary. As far as the deflection of the structure is concerned, the measured values were almost exactly as predicted.

In the 1960s, AISI sponsored a research project on cold-formed steel folded plates at Arizona State University to study further the methods of analysis and design of various types of folded-plate roofs, including rectangular and

![Figure 9.18 Williot diagram used for determining total vertical deflection.](image)
circular planforms. In this project, both the simplified analysis and the finite-element approach were studied in detail by Schoeller, Pian, and Lundgren.\textsuperscript{1.81}

For a multiple sawtooth folded-plate structure with a span of 40 ft (12.2 m), the analytical results obtained from the simplified method and the finite-element method are compared as follows:\textsuperscript{1.81}
9.4 SHELL ROOF STRUCTURES

9.4.2.6 Truss-Type Folded-Plate Roofs  The above discussion is related to the analysis and design of membrane-type folded-plate roofs in which the steel roof panels not only support normal loads but also resist shear forces in their own planes. This type of structure is generally used for spans of up to about 100 ft (30.5 m).

For long-span structures, a folded-plate roof may be constructed by utilizing inclined simple trusses as basic units, covering them with steel panels. In this case steel panels will resist normal loads only. The design of basic trusses is based on the conventional method. Additional information on the design and use of folded-plate roofs can be found in Refs. 1.84 and 9.78.

9.4.3 Hyperbolic Paraboloid Roofs

9.4.3.1 General Remarks  The hyperbolic paraboloid roof has also gained increasing popularity during recent years due to the economical use of materials and its attractive appearance. The hyperbolic paraboloid shell is a doubly curved surface which seems difficult to construct from steel but in fact can be built easily with either single-layer or double-layer standard steel roof deck panels. This is so because the doubly-curved surface of a hyperbolic paraboloid has the practical advantage of straight line generators as shown in Fig. 9.21.

Figure 1.14 shows the Frisch restaurant building in Cincinnati, Ohio, which consists of four paraboloids, each 33.5 ft (10.2 m) square, having a common column in the center and four exterior corner columns, giving a basic building of 67 ft (20.4 m) square. The roof of the building is constructed of laminated 1 1/2-in. (38-mm) steel deck of 26-in. (660-mm) wide panels. The lower layer

<table>
<thead>
<tr>
<th></th>
<th>Simplified Method</th>
<th>Finite-Element Method</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum fold line force, lb</td>
<td>22,500</td>
<td>23,400</td>
</tr>
<tr>
<td>Maximum plate shear, lb/ft</td>
<td>1,024</td>
<td>958</td>
</tr>
</tbody>
</table>

Figure 9.21  Surface of hyperbolic paraboloid.9.79
is 0.0516-in. (1.3-mm) thick steel panels, and the upper layer is 0.0396-in. (1-mm)-thick steel panels placed at right angles and welded together. The reason for using a two-layer laminated hyperbolic paraboloid roof was to achieve additional stiffness and resistance to point loading. The use of 0.0516-in. (1.3-mm)-thick steel panels in the lower layer was for ease of welding. The roof plan and the structure details of the Frisch restaurant building, designed by H. T. Graham, are shown in Fig. 9.22.

In 1970 Zetlin, Thornton, and Tomasetti used hyperbolic paraboloids to construct the world’s largest cold-formed steel superbay hangars for the American Airlines Boeing 747s in Los Angeles and San Francisco, California (Fig. 1.15). The overall dimensions of the building shown in Fig. 9.23 are 450 ft (137 m) along the door sides and 560 ft (171 m) at the end wall. The central core of the building is 100 ft (30.5 m) wide and 450 ft (137 m) long. The hangar area is covered by a 230-ft (70-m) cantilever on each side of the core.

Figure 9.22  Roof plan and structural details of Frisch Restaurant, Cincinnati, Ohio, of hyperbolic paraboloid construction. (Reprinted from Architectural Record, March 1962, copyrighted by McGraw-Hill Book Co., Inc.)
As shown in Figs. 9.23 and 9.24, the roof system is composed of 16 basic structural modules. Each roof module consists of a ridge member, two valley members, edge members, and the warped hyperbolic paraboloids, as indicated in Fig. 9.24. The ridge and valley members are hot-rolled steel shapes. The hyperbolic paraboloids were made of cold-formed steel decking consisting of a flat 0.0934-in. (2.4-mm)-thick sheet, 26 in. (660 mm) wide, with two 9-in. (229-mm) wide by 7 1/2-in. (191-mm) deep 0.0516-in. (1.3-mm)-thick steel hat sections welded to the flat sheets. Figure 9.25 shows typical cross section of the steel deck used.

In order to be able to use this type of structural system in any area of the world, prestressed cables are incorporated into the shell structures (Fig. 9.26). Since the structural strand cables induce a prestress in the shell, the system is readily adaptable to any geographic site.
A comparison of various types of designs indicate that this type of building with its hyperbolic paraboloids weighs approximately 40% less than a conventional steel construction.

9.4.3.2 *Types of Hyperbolic Paraboloid Roofs* The surface of a hyperbolic paraboloid may be defined by two methods. As shown in Fig. 9.21, with the x-, y-, and z-axes mutually perpendicular in space, the surface is formu-
lated by two straight lines called generators. One line, parallel to the xz plane, rotates about and moves along the y-axis; the other, parallel to the yz plane, rotates about and moves along the x-axis. The intersection of the generators is contained in the surface of the hyperbolic paraboloid.

Figure 9.27 shows several types of hyperbolic paraboloid roofs which may be modified or varied in other ways to achieve a striking appearance.

In general, type I is the most pleasing of the shapes available. The edge beams are in compression and are usually tubular members. For this type of roof, the most serious problem is the horizontal thrust at the supporting columns. Usually the columns are kept short in order to transfer the thrust down to the floor where tie rods can be hidden. Four units of this type with a common center column probably provide the most rigid roof structure, as shown in Fig. 1.14.

Type II is an inverted umbrella, which is the easiest and the least expensive to build. The edge members of this type are in tension, and engineers usually use angles as edge members.

Type III is the most useful type for canopy entrance structures. The edge members connected to the columns are in compression and are usually tubes, while the outside edge members are in tension and could be angles or channels. In some cases, one half of the roof may be kept horizontal and the other half tilts up.

Generally speaking, type IV is the most useful of all the available shapes. The entire building can be covered with a completely clear span. The horizontal ties between columns on four sides can be incorporated in the wall construction.

\[ \text{Figure 9.27 Types of hyperbolic paraboloid roofs.} \]
the span-to-corner-depression ratio \( a/h \) shown in Fig. 9.21 is less than or equal to approximately 5.0), the membrane theory may be used. For the cases of a deep shell subjected to unsymmetrical loading and a shallow shell, the finite-element method will provide accurate results.\textsuperscript{9.80}

In the membrane theory, the equation of the surface of a hyperbolic paraboloid can be defined from Fig. 9.28. Since

\[
\frac{h}{c} = \frac{b}{x} \quad \text{and} \quad \frac{c}{z} = \frac{a}{y}
\]

\[
z = \frac{cy}{a} = \frac{hxy}{ab} = kxy
\]

where \( k = h/ab \), in which \( h \) is the amount of corner depression of the surface having the horizontal projections \( a \) and \( b \).

If we rotate coordinate axes \( x \) and \( y \) by 45°, as shown in Fig. 9.29, the equations for two sets of parabolas can be obtained in terms of the new coordinate system using \( x' \) and \( y' \).\textsuperscript{9.81} Substituting

\[
x = x' \cos 45° - y' \sin 45°
\]

\[
y = y' \cos 45° + x' \sin 45°
\]

into Eq. (9.61), one can obtain a new equation for \( z \) in terms of \( x' \) and \( y' \),

\[
z = \frac{1}{2} k[x'^2 - y'^2]
\]

In Eq. (9.64), if the value of \( x' \) remains constant, as represented by line \( a'-a' \) in Fig. 9.30, the equation for the parabolic curve can be written as

---

**Figure 9.28** Dimensions used for defining the surface of hyperbolic paraboloid roof.
in which the negative sign indicates concave downward.

When the value of \( y' \) remains constant, Eq. (9.66) can be obtained for a concave upward parabola:

\[
z' = \frac{1}{2} ky'^2
\]  

(9.66)

For a constant value of \( z \), the hyperbolic curve can be expressed by

\[
\frac{2z}{k} = x'^2 - y'^2
\]  

(9.67)

Figure 9.31 shows a concave downward parabolic arch subjected to a uni-
form load of \( w/2 \), where \( w \) is the roof load per square foot. Since the bending moment throughout a parabolic arch supporting only a uniform load equals zero,

\[
H(-h) = \frac{\frac{w}{2}L^2}{8} \quad (9.68)
\]

\[
H = -\frac{wL^2}{16h} \quad (9.69)
\]

Use Eq. (9.65) for \( y' = L/2 \) and \( z' = h \) then

\[
h = -\frac{kL^2}{8} \quad (9.70)
\]

Substituting the value of \( h \) in Eq. (9.69), one obtains Eq. (9.71) for the horizontal thrust \( H \),

\[
H = \frac{wab}{2h} \quad (9.71)
\]

The above analogy can also be used for the concave upward parabolic tie. It can be seen that if the load is applied uniformly over the horizontal projection of the surface, compressive membrane stress results in the concave downward parabolas, and tensile membrane stress results in the concave upward parabolas. These tensile and compressive membrane stresses are uniform throughout the surface and are equal to \( wab/2h \), in which \( w \) is the applied load per unit surface area (Fig. 9.32). Since the compressive and tensile membrane stresses are equal in magnitude and are perpendicular to each other, a state of pure shear occurs in planes of 45\(^\circ\) from the direction of either membrane stress. Thus only shear stress need be transmitted along the joints of the panels. The force in edge beams resulting from the shear of \( q = \frac{wab}{2h} \) along the edge members is shown in Fig. 9.32.

Example 9.4 illustrates the design of an inverted hyperbolic umbrella by using the membrane theory.
Example 9.4  Use the ASD method to design the inverted steel hyperbolic paraboloid umbrella roof shown in Fig. 9.33 for \( w = 30 \text{ psf} \) on horizontal projection. Use A36 steel for framing members.

Solution

1. Determination of Panel Shear. The panel shear can be determined as follows:

\[
v = \frac{wa^2}{2h} = \frac{30 \times (15)^2}{2 \times 4} = 845 \text{ lb/ft}
\]

In accordance with Sec. D5 of the AISI Specification (Table 9.1), the factor of safety for gravity load alone is 2.45 for welded connections. The required ultimate shear strength of the roof deck system is therefore
\[ v_u = 845(2.45) = 2070 \text{ lb/ft} \]

2. Selection of Steel Roof Deck. Select a type of steel roof deck from catalogs to satisfy the following requirements:
   a. Flexural strength
   b. Shear strength
   c. Deflection

3. Design of Framing Members
   a. Tension members AB and AC. Considering the quadrant ABCD as shown in Fig. 9.34, the tensile force is

   \[ T = va = \left( \frac{wa^2}{2h} \right) a = 845(15) = 12,680 \text{ lb} \]

   Using the AISC ASD Specification with an allowable stress of 22 ksi for A36 steel, the required area is

   \[ A = \frac{T}{F} = \frac{12.68}{22} = 0.58 \text{ in.}^2 \]

   Using L4 \( \times \) 3 \( \times \) \( \frac{1}{4} \) in., the actual area is 1.69 in.\(^2\)

   b. Compression members BD and CD. Considering sec. 1-1 of Fig. 9.34, the compressive force is

   \[ C = vc = \left( \frac{wa^2}{2h} \right) c = 845(15.55) = 13,140 \text{ lb} \]

   Since the compression member will support two quadrants, the total compressive force for design is

   \[ C' = 2C = 2(13,140) = 26,280 \text{ lb} \]

   Using the AISC ASD Specification with an allowable stress of 22

\[ \text{Figure 9.34 Forces in framing members.} \]
ksi, the required area is

\[ A = \frac{C'}{22} = 1.195 \text{ in.}^2 \]

Selecting two L4 \( \times 3 \times \frac{1}{4} \) in., the actual area is 3.38 in.\(^2\) The use of an allowable stress of 22 ksi is due to the fact that the compressive force varies from zero at point B to the maximum value at point D and that the member is continuously braced by steel panels.

The above discussion is based on the requirement of strength. In some cases the deflection of hyperbolic paraboloid roofs at unsupported corners may be excessive and therefore will control the design of this type of structure. Research work\(^9,80\) indicates that the corner deflection is affected by (1) the shear and bending stiffness of steel panels, (2) the bending and axial stiffness of edge members, (3) the type and arrangement of connections used to join steel panels and to connect steel panels to edge members, and (4) the eccentricity of the shear force transmitted from the deck to edge members. The finite-element method may be used to predict deflection.

If the corner deflection is found to be critical, the design may be improved by increasing the bending stiffness of the edge members and steel panels and the curvature of the shell.

In the design of hyperbolic paraboloid roofs, consideration should also be given to the possible buckling of steel deck and the overall buckling of edge members. The determination of the buckling load can be made either by the energy approach or by the finite-element method.\(^9,80\)

### 9.4.3.4 Research on Cold-Formed Steel Hyperbolic Paraboloid Roofs

A full-scale hyperbolic paraboloid shell constructed of one-layer standard steel roof deck sections has been tested at Cornell University by Nilson.\(^9,79\) The test setup is shown in Fig. 9.35. Note that the jack loads are applied upward because of convenience of testing. The specimen was 15 \( \times \) 15 ft (4.6 \( \times \) 4.6 m) with 3-ft (0.92-m) rise, which represents one of the four quarter-surfaces of an inverted umbrella (type II), as shown in Fig. 9.27. The panels used in the specimen were standard 0.0635-in. (1.6-mm)-thick steel roof deck sections, 18 in. (457 mm) wide, with ribs 6 in. (152 mm) on centers and 1\(\frac{1}{2}\) in. (38 mm) high. Panels were welded to each other along the seam joints and to the perimeter frame members.

The structural feasibility of the hyperbolic paraboloid shell structure has been demonstrated by Nilson’s test. The results indicate that shear values obtained from horizontal diaphragm testing can be used conservatively in the design of curved shear surfaces.

The structural behavior of hyperbolic paraboloid shell has been studied further at Cornell University by Winter, Gergely, Muskat, Parker, and Banavalkar under the sponsorship of the AISI. Findings of this study are reported
in Refs. 9.80, 9.82–9.87. In addition, investigations of hyperbolic paraboloid roofs have been conducted by individual steel companies.\textsuperscript{9.88} It is expected that additional design information may be developed from the previous research.

9.4.3.5 Curvilinear Grid Frame Type Hyperbolic Paraboloids\textsuperscript{9.72,9.75} The above discussion is made on the basis that the cold-formed panels are stressed as a membrane in the hyperbolic paraboloid roof. This membrane type of hyperbolic paraboloid structure is mostly used for relatively small structures. For larger structures a hyperbolic paraboloid roof may be constructed with structural members as a curvilinear grid frame. The construction shown in Fig. 9.36 was developed by Hutton of Purdue University and consists of struts

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**Figure 9.35** Testing of hyperbolic paraboloid shell.\textsuperscript{9.79}

**Figure 9.36** Curvilinear grid frame type hyperbolic paraboloid.\textsuperscript{9.76}
and ties. The ties of the system can be designed as cables. Either analytical or graphic solutions are possible.

With regard to the design of ribs, the top ribs are designed for compression and bending, and the bottom ribs are designed for compression only. The fascia is designed for combined axial and bending stress. In addition, the ground tie should be provided as a tension member.

In the curvilinear grid frame type, cold-formed steel decks do not participate in shear membrane stresses; rather a grid of structural steel members supports the loads.

Two grid frame methods are possible. They are the double-arch and the orthogonal systems. For the double-arch grid system, the positive arches are compressive elements and the negative arches are curved tensile members. The analysis of this system for concentrated load and unbalanced load can be complex.

For the orthogonal grid system, as shown in Fig. 9.36, the ribs are straight members parallel to the edge members and supported by a series of ties. Two layers of corrugated steel panels may be used perpendicular to each other to replace the ribs.

In addition to the types of shell roofs discussed above, two structural systems (barrel shells and composite truss and sheet panels) generally used for lightweight utility and farm buildings have been discussed by Abdel-Sayed in Ref. 9.89.